# JOHANNES KEPLER
# NEW ASTRONOMY

Translated by
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# Foreword

Owen Gingerich

Kepler's *Astronomia nova*, together with Copernicus's *De revolutionibus* and Newton's *Principia*, belongs in the select group of the most important books of the Scientific Revolution. Kepler's formidable treatise contains the first statement of elliptical orbits – a radical departure from the previous exclusive pre-eminence of the circle in astronomical hypotheses. The *Astronomia nova* also contains a powerful, but flawed, statement of the law of areas. But perhaps more important, it is the first published account wherein a scientist documents how he has coped with the multiplicity of imperfect data to forge a theory of surpassing accuracy.

For centuries, Kepler and his extraordinary creative genius have been overshadowed by his contemporary, Galileo. Like the *Astronomia nova*, both the *Sidereus nuncius* and the *Dialogo sopra i due massimi sistemi del mondo* played major roles in bringing about the acceptance of the heliocentric world view. Galileo's works are eminently readable and have long been accessible in English translation. Not so for Kepler's pioneering study.

Unlike Galileo's *Dialogo*, which was written in the vernacular and aimed at a general intellectual audience that extended far beyond academia, Kepler's book is a technical treatise written for the specialist in celestial mechanics. Nevertheless, like Galileo's *Dialogo*, the *Astronomia nova* is a polemical work; it is crafted to convince Kepler's readers that his revolutionary solution to the ancient problem of planetary motions is the only viable alternative. He apologizes at the outset for being too prolix, but its expansive presentation serves his purpose. In fact, 80% of the book was drafted – including the

introduction, the title page, and dedicatory poem – before he had even arrived at the elliptical shape of the Martian orbit. Kepler touched up the introduction with a marginal reference to the ellipse, and tried, with only partial success, to make the new final chapters seamless with what had gone before. If he stumbled and left out an essential paragraph here or there, as D. T. Whiteside has assured me is so, no one seemed to notice – the rhetoric, planned or unwitting, had the desired effect. When Kepler plaintively sought the reader's pity because one wearying iterative procedure was carried out more than 70 times, Delambre remarked that the complaint was its own reward.

Kepler's *Astronomia nova* is more a book to be mined than to be read. Nevertheless it abounds with Kepler's sly humor. 'I am going to give you a clown show,' he says as he explains his fumbling attempt to make an observation at a geometrical configuration of Mars that his mentor, Tycho Brahe, had neglected. Elsewhere he writes, 'Who would believe it! The hypothesis . . . goes up in smoke.' Chapter 7, entitled 'How I First Came to Work on Mars,' contains one of Kepler's most important biographical statements, recounting the events that led to his working in Prague for Tycho.

The introduction to Kepler's treatise, salvaged and reworked from his earlier *Mysterium cosmographicum* where it had been censored out by the Tübingen University Senate, is one of the most interesting defenses of the Copernican theory in the entire seventeenth century. In fact, for over three centuries, the introduction to the *Astronomia nova* was the only significant piece of Keplerian writing turned into English – it was partially translated by Thomas Salusbury in his *Mathematical Collections and Translations* of 1661. In those decades when the heliocentric arrangement was still problematic, Kepler's discussion of the relation of Holy Scripture to the Copernican doctrine, found in this introduction, was an influential statement. In its Latin original, it was reprinted over and over with Galileo's *Dialogo*. Undoubtedly Galileo profited from it, though it would have been suicidal for the Italian Catholic to have acknowledged such a Lutheran source.

As long as Latin remained a working language for astronomers, it was unnecessary to translate the *Astronomia nova*. On the other hand, after the seventeenth century, astronomers had little reason to read Kepler's original text except as a historical curiosity. Its style was not easy to grasp – Kepler in a hurry is not a notably clear writer – and his allusions and puns and occasionally idiosyncratic punctua-

tion contribute to the problem. Add to this erroneous diagrams – such as the one in Chapter 16 that Donahue is here the first to notice in print – and the difficulty becomes even more apparent.

The fine Latin edition prepared by Max Caspar in the *Johannes Kepler Gesammelte Werke* series in 1937 provided a good start for those determined to penetrate Kepler's prose. In the late 1960s we achieved a rough English translation with the aid of several Harvard students, most notably Ann Wegner, and the entry in the *Dictionary of Scientific Biography* indicates that a proper translation was on its way. After some years William Donahue challenged us to finish it, saying that otherwise he would undertake the task himself. As events unfolded, it became much the more sensible idea to let him profit at least in a minor way from our rough translation, and indeed, it provided Donahue with an occasional felicitous phrase or insight.

Today, students who are less interested in the specific results of Kepler's 'Warfare on Mars' and far more intrigued by the process of scientific discovery can find an archeological treasure-trove in the strata of Kepler's treatise. We see a visionary driven by a search for physical causes, unsatisfied by the purely geometrical 'hypotheses' used in the time-honored tool kit of astronomical procedures. As a thorough-going Copernican, Kepler believed the sun simply had to be near the center of the planetary system, for only the sun could serve as a source of the driving power for the planetary motions. Copernicus had already noticed that the nearer a planet was to the sun, the shorter its period – in fact, this must have been one of the primary reasons for considering the sun-centered arrangement. He pointed to the sun 'seated upon its throne as a royal governor' and wrote that 'only in this arrangement do we find a sure connection between the motion and the size of the orbit'. For Kepler, the sun was the seat of a mysterious force, related to magnetism, that not only propelled the planets but which interacted with the planets' own magnetism to guide each one in its path of approach and regression to the hearth of the universe. With this translation a far wider audience can relive Kepler's excitement when he found that an ellipse seemed to answer to such a magnetic configuration. Appropriately, the sun fell at the focus – Kepler's word, its usage derived from its ancient meaning, 'hearth'.

Kepler, with his enthusiasm and his burning desire to publish, had already begun writing his treatise even before he had discovered the ellipse. Now, in 1605, he hastily finished it. Printing, however, was delayed by dissension with Brahe's heirs. The eccentric Danish observer had died in 1601 after Kepler had worked with him for only

ten months. Emperor Rudolph had agreed to buy the observation books for Kepler's use, but had not made good on his promise. Finally, after the heirs were given an opportunity to include their own fulsome introduction, the tall, handsome volume was published in Heidelberg in 1609.

The Warfare on Mars was the anvil on which Brahe's observations were forged into a radical new celestial mechanics. But in those years between 1600 and 1605, Kepler himself passed through the refiner's fire. The youthful speculations of his *Mysterium cosmographicum* were now behind him, and, having acquired the finest treasury of observations that had ever existed, he could no longer be satisfied with the rather imprecise fit between model and data that he had put forward in 1596. His on-going task was to hammer the planetary theory into practical tables, and from the tables to make calculations of daily planetary positions – both a challenge and a drudgery, against which he exclaimed, 'Don't sentence me completely to the treadmill of mathematical calculations. Leave me time for philosophical speculations, my sole delight!' And in that framework of cosmological speculation tempered by a newly won respect for the data itself, he completed his *Harmonice mundi*, an extension and refinement of his earlier ideas. In the course of this work he discovered, in 1618, the numerical relationship between the distance of a planet from the sun and its period, today called Kepler's third law. The *Harmonice* was followed by the *Epitome astronomiae Copernicanae*, Kepler's longest work and the clarifying summary of all his discoveries in celestial mechanics. And finally, the *Tabulae Rudolphinae* (1627) and the *Ephemerides* (1617, 1630) appeared.

Meanwhile Kepler had been busy with many other topics. His *Astronomiae pars optica* (1604) had prepared the way for a prompt theoretical treatment of the just-invented telescope, the *Dioptrice* (1611). His thin pamphlet on the six-cornered snowflake (*Strena*, 1611) is considered a foundation work in mineralogy, and his *Stereometria doliorum vinariorum* (1615) is a significant antecedent to the integral calculus. Having heard about logarithms, he independently figured out how to calculate them and published his own table in 1625. Nevertheless, as we survey this prodigious output, the *Astronomia nova* stands as the high-water mark, the achievement where he not only established his own professional credentials, but where he made his most lasting contribution to astronomy. His results, which swept away nearly two millennia of imperfect assumptions about planetary motions, were truly the new astronomy.

# Acknowledgements

It is a sobering thought that the work of translating this book has required somewhat more time than Kepler took, from the beginning of his work on Mars to actual publication, including all the false leads, interruptions, and the seemingly interminable haggling with Tycho Brahe's heirs. Kepler's warning to the reader (in his Introduction) about the bramble-infested path of his argument seems in retrospect an accurate characterization. During those years of sporadic work, I have been helped and encouraged by more people than I could possibly thank here. However, there are a few individuals and organizations that deserve special mention.

Foremost among these is Professor Owen Gingerich, of the Harvard-Smithsonian Center for Astrophysics. Professor Gingerich had hoped that he would be the one to produce the first English translation of this book, and was well on his way to doing so. Yet when I appeared with what was then a rival endeavour, he not only gave over his own hopes very graciously, but also has assisted me over the years in a great many ways. These range from giving me a copy of Ann Wegner's draft translation, through assistance in obtaining financial support and access to research materials, to help in finding a publisher. Indeed, although the translation does not bear his name, it is, in a sense, still very much his project.

I am also deeply grateful to the faculty, students, and staff of St. John's College in Santa Fe, New Mexico. The college provided the initial impetus for the translation by giving me the opportunity to lead a discussion class on the *Astronomia nova*. Subsequently, the library

staff, and Tracey Kimball in particular, were most helpful, both in arranging interlibrary loans and in great forbearance regarding overdue books.

Crucial funding for the translation was provided by the United States National Science Foundation. This grant came at a time when work on the project had nearly stopped because of other responsibilities, and allowed me to concentrate my attention once again upon completing the work.

Several other individuals who were particularly helpful deserve special mention. Ann Wegner, in association with Professor Gingerich, wrote a draft translation of all but a few chapters of the *Astronomia nova,* which proved most useful in spotting omissions and in suggesting particularly apt translations of several awkward terms. Curtis Wilson of St. John's College in Annapolis has devoted considerable time and effort to a number of technical questions I asked him. His replies were always both interesting and helpful. Barbara Welther of the Harvard-Smithsonian Center for Astrophysics generously provided computed longitudes for Mars corresponding to the Tychonic observations in chapter 15. And Simon Mitton, of the Cambridge University Press, has been singularly helpful and patient in seeing the book through to publication. I would also like to thank an anonymous reviewer for the Cambridge University Press whose comments on part of the translation were of great assistance in the final revision of the entire work.

To my wife, Dana Densmore, go very special thanks. She has unfailingly supported me in this apparently endless project, and has also been an inspiration in the high standards of excellence she sets for her own work.

This translation is dedicated to Mary Corinne Rosebrook of the Sidwell Friends School in Washington D.C., who never gave up on a most unpromising Latin student.

# Translator's introduction

## The title and its significance

When Kepler boldly chose the words '*Astronomia nova*' to head the title of this book, he must have had some sense of how apt they were. Although the *New Astronomy* is not lacking in historical antecedents (as Kepler himself was keenly aware), it is an astonishing book, utterly unlike anything that had appeared before. Astronomy is no longer seen as a deductive science, founded mainly upon geometry, whose aim is to construct an ideal system that matches the appearances of the celestial motions as closely as possible. Instead, it is an adventure in which the human being uses all the means at his disposal to explore the creation in which God has placed him as His image. Error is no longer equated with failure, but is seen as an indication of the way to the truth. Thus the rules of the game are no longer fixed, but are to be discovered in the playing. And so the hypotheses of the ancients, the uniform circular motions previously thought to be indispensable to astronomy, were tested, found wanting, and rejected. The question was no longer 'How can the appearances be accounted for?', but 'How does God make things move?'

It is nevertheless not certain that *New Astronomy* is the title by which Kepler intended the work to be known. On the title page of the first edition, the largest typeface is reserved for the name of Kepler's patron, the Emperor Rudolph II, and that of Kepler's subject, the planet Mars. This would suggest that the intended short title was '*De motibus stellae Martis*' (that is, '*On the motions of the star Mars*'). And it would indeed have been somewhat awkward to refer to the

work as *'New Astronomy'*, as I have discovered in telling others about the translation. Kepler himself usually referred to it modestly as *'Commentaries on Mars'*. Nevertheless, the book is now commonly called *'Astronomia nova'*, and so it seems fitting to give its English version the English equivalent of that short title. It is no more than the book deserves.

## On the composition of the *New Astronomy*

About the history of the work, little will be said here: there is already both too much and too little in other sources. Especially noteworthy are Owen Gingerich's articles and papers, particularly his article on Kepler in the *Dictionary of Scientific Biography,* C. C. Gillispie, editor (New York, 1973), Vol. 7 pp. 289–312. Many other useful sources are mentioned in the bibliography to this article, among them two excellent articles by Curtis Wilson. Also highly recommended are D. T. Whiteside's 'Keplerian Planetary Eggs, Laid and Unlaid, 1600–1605', *Journal for the History of Astronomy* 5, 1974, pp. 1–21, and E. J. Aiton, 'Infinitesimals and the Area Law', in F. Kraft, K. Meyer, and B. Sticker, editors, *Internationales Kepler-Symposium Weil der Stadt 1971,* (Hildesheim, 1973), pp. 285–305 (both of which are too recent to appear in the Gingerich bibliography). However, studies of the relevant manuscripts by Gingerich[1] and my own reading of the work itself[2] suggest that there is much yet to be learned about the stages of composition. So it seems best to include, instead of the usual historical sketch, only the few remarks on the history of composition of the work that will assist the reader's understanding, and that may help guide further scholarly enquiry. These concern a certain obligation Kepler felt towards Tycho Brahe, as well as the ample evidence of frustration and even despair that frequently arose in the original investigations.

To treat the second point first, Kepler often takes note of errors that waylaid him in his campaign: the whole of chapter 58 is a good example. But there is a great deal that, out of consideration for the reader, Kepler did not report. Sometimes (as in chapter 16) he makes

1 'Kepler's Treatment of Redundant Observations; or, the Computer Versus Kepler Revisited', in F. Kraft, K. Meyer, and B. Sticker, editors, *Internationales Kepler-Symposium Weil der Stadt 1971,* (Hildesheim, 1973), pp. 307–314; also 'Johannes Kepler and the New Astronomy', *Quarterly Journal of the Royal Astronomical Society* (1972) **13**, 346–373, especially p. 352.

2 See for example the notes to chapter 53 of this work.

this clear, but not always. In particular, he never hints that he had mostly completed at least one draft even before discovering the elliptical shape of the orbit. The project had, indeed, progressed so far that Kepler had actually sent the manuscript to the Emperor as a preliminary to publication[3]. We know of the existence and contents of this 'Proto-*Astronomia nova*' only from manuscript sources, especially a letter to his teacher Michael Maestlin (of March 5, 1605)[4] and a (much earlier) draft table of contents[5]. This work was not simply abandoned; rather, it was revised, the polished form of the elliptical orbit being superimposed upon earlier attempts, leaving nonetheless a few vestigial rough edges and mismatched seams[6]. This tendency of Kepler's to gloss over many of the difficulties he confronted has two main consequences for the reader. First, the computations presented were often intended as examples, to give an idea of the procedure used. This is most conspicuously so in chapter 16, which would have at least doubled the size of the book had the entire iterative process been presented. Less obvious, but equally revealing, are the sample computations in chapter 53, in comparison with the entirely different results displayed in the table at the end of that chapter. Second, the selection of data and arguments, and sometimes the data themselves, were determined by the conclusions. That is, although Kepler often seems to have been chronicling his researches, the *New Astronomy* is actually a carefully constructed argument that skillfully interweaves elements of history and (it should be added) of fiction[7]. Taken as history, it is often demonstrably false, but Kepler never intended it as history. His introduction to the 'Summaries of the Individual Chapters' makes his intentions abundantly clear. *Caveat lector!*

There is another feature of the work that may seem puzzling, that may at least partly be explained by Kepler's sense of obligation to Brahe. It will be seen that in the earlier parts (through chapter 26), wherever a geometrical proof is carried out on the planetary orbit,

3 Letter number 325, in KGW 15 pp. 145–147. Kepler remarks, however, that 'certain chapters are still missing'.
4 Letter number 335, in KGW 15 pp. 170–176.
5 In KGW 3 pp. 457–60. For the date, see Gingerich, 'Kepler's Treatment of Redundant Operations', *op. cit.*, note 20, p. 313.
6 For example, at the beginning of chapter 54, Kepler refers to a 15′ correction of the mean longitudes supposedly made in chapter 53. No such correction appears in that chapter as it presently stands, and it seems clear that Kepler later revised chapter 53 without correcting the cross reference.
7 The entire table at the end of chapter 53, for example, is based upon computed longitudes presented as observations.

the demonstration is presented in all three forms of hypotheses (Ptolemaic, Copernican, and Tychonic). In Kepler's words[8]:

> I, in the demonstrations that follow, shall link together all three authors' forms. For Tycho, too, whenever I suggested this, answered that he was about to do this on his own initiative even if I had kept silent (and he would have done it had he survived), and on his death bed asked me, whom he knew to be of the Copernican persuasion, that I demonstrate everything in his hypotheses.

There is nevertheless more to this cumbersome triple demonstration than the fulfillment of a solemn promise. Kepler's intention, as is shown in chapter 6, was to establish the perfect geometrical equivalence of the three forms of hypotheses in order to show that geometry alone cannot decide which is correct. This prepares the reader for the climactic Part IV, in which the 'first inequality' (the inequality in the heliocentric longitudes) is treated 'from physical causes and the author's own ideas'[9]. It would of course have been possible to treat the elliptical orbit generated in that part as just another geometrical hypothesis, and to show how it could have been accommodated to the Ptolemaic and Tychonic forms. That Kepler did not present such a demonstration is a consequence of his belief that to separate the geometry from the physics upon which it is based turns it into nonsense. His promise to Tycho was a promise about hypotheses, and is no longer valid in an 'astronomy without hypotheses'[10].

## Historical background and salient features of the *New Astronomy*

As for the work itself, it is my intention to allow Kepler to speak for himself, providing notes and comments where his sense might not be clear to the modern reader, or where a reference or allusion might have been obscured by the passing of time. However, the opportunity to make a few remarks on the work as a whole is hard to resist. Also, one who has made a careful study of the book may have some insights that will prove useful. So I beg the reader's indulgence as I make a brief excursion into issues Kepler raises in this book and how he approaches them.

[8] Chapter 6, p. 157.
[9] Part IV, title.
[10] See the passage from Ramus on the verso of the title page, and Kepler's reply.

*Historical context*

The central question that guides Kepler's investigations throughout the *New Astronomy* is, 'What is the actual path of the planet through space?' This question occurs to us so naturally and seems so fundamental to planetary astronomy that its revolutionary implications come as something of a surprise. But at the time, for an astronomer to pose this question as insistently as Kepler did was unprecedented. Why was this so?

To find the answer, we need to look at the aims and methods of pre-Keplerian astronomy[11]. There was by no means a general consensus as to what these were, but certain characteristics are conspicuous. Planetary astronomy was the application of geometry to motion. On the relationship between the geometry and the motion, there was a broad spectrum of viewpoints, ranging from an extreme nominalism or fictionalism to a variety of realist interpretations, However, even though there were many who held that an adequate geometrical model must somehow reflect physical reality, the reality they saw was nearly always an expression of the model, and not the independent motion of a body through space. This distinction is conveniently illustrated in Reinhold's edition of Peurbach's *Theoricae novae planetarum* (Paris, 1558 and other editions), as well as in Copernicus. In the Reinhold/Peurbach version of the Ptolemaic theory of Mercury, the deferent is depicted as an oval, the curve resulting from Ptolemy's crank mechanism[12]. And when Copernicus presents his general model of planetary motion[13], he remarks that as a result of his epicyclet on an eccentric, 'the planet does not describe a perfect circle in accordance with the theory of the ancient mathematicians but a curve differing imperceptibly from one.' In both instances, the

[11] The following account is based upon my research into the physics of celestial motion in the sixteenth and seventeenth centuries, and upon Nicholas Jardine's more recent studies. For detailed references, I would direct the reader to my article, 'The Solid Planetary Spheres in Post-Copernican Natural Philosophy', in Robert S. Westman, editor, *The Copernican Achievement* (University of California, 1975), pp. 244–275, or to my more detailed doctoral dissertation, published as *The Dissolution of the Celestial Spheres, 1595–1650* (New York: Arno Press, 1981). The former is clearer and more concise, while the latter is much more thorough, although the general analysis is confused in places. Pertinent works of Jardine are 'The Significance of the Copernican Orbs', *Journal for the History of Astronomy* 13 (1982), pp. 168–194, and *The Birth of History and Philosophy of Science*, Cambridge University Press, Cambridge 1984.

[12] See the Mercury diagram in Peurbach's *Theoricae* (in the 1588 edition, it precedes fol. 69). This diagram is included in Owen Gingerich, 'Kepler's Place in Astronomy,' *Vistas in Astronomy*, 18 (Pergamon Press, Oxford and New York, 1975), p. 274. Kepler himself refers to it in a marginal note at the end of chapter 46 of the present work.

[13] *De revolutionibus*, V. 4, fol. 142 b.

curves were conceived as expressions of the underlying reality of the orbs.

An early printed example of how these physical orbs were conceived is again provided by Peurbach. In his models, the Ptolemaic epicycles were contained in eccentric channels in the planetary regions. Peurbach was not the originator of this idea, which is found in Islamic astronomy along with many other quasi-mechanical arrangements of spheres. But his work was widely read and reprinted, and so this kind of model became well known and was associated with his name. Thus Kepler says, in chapter 2 of this book,

> Ptolemy has described these circles to us in their bare form, as geometry applied to the observations shows them. Peurbach set up a way for them to move around which follows Aristotle [i. e., makes sense in the context of Aristotelian physics].

Looking at this diagram, a modern reader immediately sees a mechanism, a solid transparent device perhaps made of some crystalline substance[14]. A contemporary of Peurbach, however, would see something quite different. For mediaeval natural philosophy had surpassed even Aristotle in separating the heavens from earth. The most widely accepted view was that even the material of which the heavens are made is wholly different from elementary matter (Aristotle had never explicitly distinguished them materially, although he did believe in a 'quintessence' that was different from the elements in its form). This material was described as intermediate between the eternal and the temporal, and was compared to the human intellect. In more extreme views, it had 'no potential for existence, but only for location'[15]. And although mathematical quantities are the same everywhere, real quantities of the celestial material are utterly different in kind from quantities of elements. In other words, spatial relations in the heavens are not like those we experience on earth. Dante gives a vivid and (to the modern mind) startling account of such views in Canto 2 of the *Paradiso*.

In the sixteenth century, however, neoclassical humanism, Reformation theology, and a renascence of several non-Aristotelian

[14] It should be remarked that the property of crystallinity was not applied to the planetary spheres until it was no longer usual to take them seriously, and was mostly attributed to them by those who did not believe in real spheres. In the heyday of real orbs, the *sphaera crystallina* was invariably the starless ninth sphere, beyond the sphere of the fixed stars, which was given that name following Ezekiel 1. 22: 'And the likeness of the firmament upon the heads of the living creatures was as the colour of the terrible crystal, stretched forth over their heads above.'

[15] Iacopo Zabarella, *De rebus naturalibus* . . . (Cologne, 1590), *De natura coeli*, cap. 6 col. 252.

natural philosophies, combined to bring the heavens closer to earth. As a result of this, the previously ideal celestial spheres acquired a decidedly un-Aristotelian solidity: the properties of non-interpenetrability, rigidity, and hardness were creeping into the ethereal regions. By the 1570's, when Tycho Brahe (philosophically a Paracelsian) applied his formidable instruments to the measurement of celestial parallax, the spheres had congealed enough to be highly vulnerable. So when his observations suggested that the comet of 1577 had passed through a region that should have been filled with solid spheres, instead of recalling Dante's verses describing dimensions interpenetrating each other, he threw out the whole apparatus. By the time Kepler was writing the *New Astronomy,* the notion that the heavens are filled with very pure air was gaining respect, particularly among educated men outside the universities. To Kepler, the fluidity of this region, which he called 'aethereal air', was a perfectly obvious consequence of Tycho's parallax observations[16].

*How the context shaped Kepler's views*

Therefore, when he approached the question of the physical nature of the planets' motions, Kepler saw a world dramatically different from Peurbach's. Epicycles no longer made sense, because they were no longer supported by any substance. In Kepler's words, they would require that the planet's mover 'imagine for itself the centre of its orb and its distance from it, or be assisted by some other distinguishing property of a circle in order to lay out its own circle.'[17] The motion of the centre of the epicycle upon the deferent is even more improbable:

> For it is also incredible in itself that an immaterial power reside in a non-body, move in space and time, but have no subject, moving itself (as I said) from place to place.[18]

What Kepler saw instead is depicted in the second diagram in chapter 1: a series of interlocking spirals, each slightly different from the others, that never quite repeats itself. The appearance of this diagram is a dramatic moment in the history of thought. Nothing like it had ever been published before. For, unlike Peurbach's and Copernicus's resultant curves, epicyclic motion had become the motion of a body, now freed from its spheres, and space had become a uniform medium in which this motion is performed. The astro-

[16] See, for example, chapter 2 p. 126
[17] Chapter 2 p. 128.
[18] Chapter 2 p. 128.

nomer's task was no longer to find a geometrical model to represent these spirals: he had to separate illusion from reality, and find the paths that the planets really traverse in that uniform medium. But to find what is illusory and what is real, one must go beyond the arcs and loops to find out what really moves the planets. And thus the 'new astronomy' was born: the astronomy built upon physics.

*Kepler's response to the challenge*

The task Kepler had set himself was far more demanding than anything an astronomer had previously undertaken. No physical theory of the celestial motions had ever been required to yield accurate predictions[19], nor had any predictive apparatus been required to satisfy the requirements of physics without the use of real spheres or orbs. Indeed, it was not even clear how to begin. Between the qualitative gropings of physics and the all-but-inscrutable records of observations lay a void. Something was needed to express the pattern hidden in the observations so as to make it accessible to physical explanation. In another stroke of genius, Kepler realized that this role could be played by geometry. For even if it was no longer thought to represent physical reality, geometry could simultaneously express the position of the path in space, and the exact cyclical nature of the path in time. Accordingly, throughout the *New Astronomy*, one sees a parallel development of theory on three levels: the physical theory, the geometrical model, and the predictive apparatus.

Like the dancing Graces in Renaissance art, these three distinct theoretical levels interact as the argument proceeds, coming in turns to the foreground while keeping the others ever within reach. Thus the 'vicarious hypothesis' developed in chapter 16, accurate for the longitudes though demonstrably false, guides the formation of the geometrical oval hypothesis of chapters 46–50, which in turn was developed from physical principles in chapter 45. And thus the physical conjecture that the sun is the source of power spurs an enquiry on the observational level, in chapters 22–28, whose aim is the construction of a new geometrical model of the earth's motion. And the new geometrical model in turn confirms the original conjecture, which is then expanded in subsequent chapters. Kepler's goal,

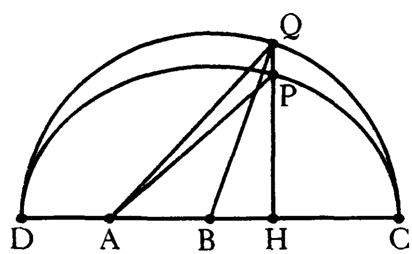
[19] Although Copernicus certainly intended his construction to represent reality, it is what I would call a geometrically based system rather than a physically based one. Contemporary physically based systems were developed by Telesio, Patrizi, Campanella, Aslachus, Lydiat, and others, but they could only represent the motions qualitatively, if at all.

which he very nearly attained, was to intertwine the three into a single comprehensive theory: the interaction of physical forces and powers produces the elliptical orbit, which, together with the 'Second Law' (areas swept out are proportional to the time), predicts the planet's position with perfect accuracy. It must be perfect, because it is true.

**Technical aspects of Kepler's astronomy and mathematics**

Those bees having been liberated from the translator's bonnet, a return can now be made to the business of preparing the reader for the fray. A campaign against the war-god himself is not to be undertaken lightly, and as Kepler says, 'the difficulties and thorns of *my* discoveries infest the very reading.'[20] This is perhaps even more so for the modern reader, for the mathematics and astronomy are unfamiliar, and many terms are used that do not have exact modern equivalents. Most of the unfamiliar or problematical terms are discussed in the Glossary. The few remaining points that need a more general explanation will be taken up here. These include the meaning of 'anomaly' and the different kinds of anomaly, Kepler's trigonometry and arithmetic, and the use of tables.

In the traditional geometrical astronomy, the term '**anomaly**' denotes a planet's angular position about some point, measured from a line through apogee or aphelion. Anomalies are distinguished according to the points about which they are measured: the mean anomaly about the equant, the eccentric anomaly about the centre of the eccentric, and the true or equated anomaly about the earth (for Ptolemy) or the centre of the earth's orbit (for Copernicus). When Kepler operated with traditional planetary models, he used the conventional definitions. But when he departed from tradition, some changes in the definitions were required. In Kepler's terminology, **mean anomaly** is the measure of the time elapsed since the planet was




[20] 'Summaries of the Individual Chapters', introduction (Below, p. 79).

at aphelion, expressed as an angle where the periodic time is 360°. Kepler uses a variety of means of expressing the mean anomaly geometrically. In its final formulation, however, the mean anomaly is measured by the area QAC in the adjacent figure. This is the sum of the circular sector QBC and the '**triangle of the equation**' AQB. (It is remarkable that in the *New Astronomy* Kepler never uses PAC, the actual area swept out, as a measure of time.) **Eccentric anomaly** is the angle about the centre of the ellipse measured from aphelion to the point the planet would have occupied had it remained on the circle. In other words, if a line HP be erected through the planet P perpendicular to the apsides CD and extended to intersect the eccentric circle (whose diameter CD coincides with the ellipse's major axis) at Q, the eccentric anomaly is the angle QBC. **Equated anomaly** is the angle about the sun between the planet and the apsides: the angle PAC. It is usual to call this the 'true anomaly'. However, the Latin equivalent of 'true anomaly' would be '*anomalia vera*', or something similar, which is not the term used by Kepler. His term is '*anomalia coaequata*', which evidently means 'anomaly with the equation added or subtracted', or, more simply, 'equated anomaly'.

**Kepler's trigonometry** is very much like modern trigonometry, with two exceptions: there are no cofunctions, and no decimals. Where we would use a cosine, Kepler used either the sine of the complement or what he calls the 'secant', which is our exsecant (the secant with the radius subtracted). The lack of decimals is compensated by making the radius equal to 100,000, the standard procedure in those days, which produces five-place tables. Once one grows accustomed to allowing for the presence of a radius that is not unity, the computations become familiar and comfortable.

It may be useful to remark that Kepler used the tables of Philip Lansberg (*Triangulorum geometriae libri IV*, Leiden 1591) for at least some of the computations in this book. See the testimonial at the beginning of chapter 15.

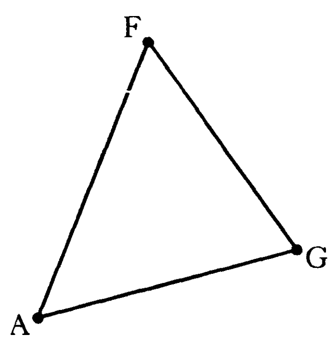
**Kepler's arithmetic** is also not difficult once the short cuts and conventions are understood. Let us take an example from chapter 16 (which is mostly computations).

In the triangle FAG, the sides FA and AG and the included angle FAG are known. According to the law of tangents,

$$\frac{\mathrm{AF} - \mathrm{AG}}{\mathrm{AF} + \mathrm{AG}} \tan \frac{180 - \mathrm{FAG}}{2} = \tan \frac{\mathrm{G} - \mathrm{F}}{2}$$

| | FAG | | GAD | | DAE | | EAF | |
|---|---|---|---|---|---|---|---|---|
| | 44° 31′ 48″ | | 52° 14′ 27″ | | 57° 23′ 4″ | | 25° 50′ 41″ | |
| *Tangents* | 98373 | | 129093 | | 156271 | | 48438 | |
| AF | 59433 | | AG 50703 | | AD 48052 | | AE 52302 | |
| AG | 50703 | | AD 48052 | | AE 52302 | | AF 59433 | |
| *Differences* | 8730 | | 2651 | | 4250 | | 7131 | |
| *Sums* | 110136 | | 98755 | | 100354 | | 111735 | |
| | 770952 | 7 | 197510 | 2 | 401416 | 4 | 670410 | 6 |
| | 102048 | | 67590 | | 23584 | | 42690 | |
| | 99123 | 9 | 59253 | 6 | 20771 | 2 | 33520 | 3 |
| | 2925 | | 8337 | | 3513 | | 9170 | |
| | 2203 | 2 | 7900 | 8 | 3016 | 3 | 8938 | 8 |
| | 722 | 6 | 437 | 4 | 497 | 5 | 232 | 2 |
| *Quotients* | 7926 | | 2684 | | 4235 | | 6382 | |
| *Tangents* | 98373 | | 129093 | | 156271 | | 48438 | |
| | 6886 | 11 | 2581 | 86 | 6250 | 84 | 2906 | 86 |
| | 885 | 33 | 774 | 54 | 312 | 54 | 195 | 34 |
| | 19 | 66 | 103 | 20 | 46 | 86 | 38 | 72 |
| | 5 | 88 | 5 | 16 | 7 | 81 | | 96 |
| *Tangents* | 7797 | | 3465 | | 6618 | | 3142 | |
| *Differen.* F | 4° 27′ 30″ | | D 1° 59′ 4″ | | D 3° 47′ 10″ | | F 1° 47′ 59″ | |

The sides FA and AG are 59433 and 50703, respectively, and the angle FAG is 90° 56′ 23″, whose half is 45° 28′ 12″, complement 44° 31′ 48″. Here you see the computation as Kepler presents it.



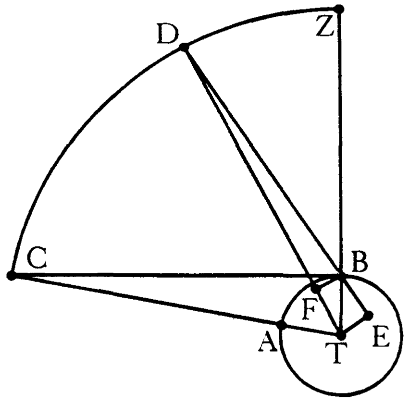
First he finds the tangent (line 2). The third and fourth lines are the two sides, and the fifth and sixth their difference and sum, respectively.

Lines 5–12 show Kepler's version of **long division**. Note that the dividend (line 5) is treated as if it had been multiplied by the radius: this will be divided back out later. Line 7 has two numbers: the first digit of the quotient on the right, and on the left the product of this digit and the divisor (line 6). This left number on line 7 is subtracted from line 5: the difference appears on line 8. On the right side of line 9 is the next digit of the quotient, and on the left, the product of that digit and the divisor (with the last digit struck). This product is subtracted from line 8, and the process is repeated on lines 10–12. So the quotient appears vertically on the right side of the computation, instead of horizontally at the top, as it would appear today (in those few places where the arcane rite of Long Division is still practiced).

Lines 13–19 are an example of **multiplication**. Line 13 is the quotient of the previous division, and line 14 the tangent, brought down from line 2. Note that line 13 is treated as the multiplier, and line 14 as the multiplicand, the inverse of our customary procedure. That is, line 15 is the product of line 14 and the first digit of line 13, line 16 is the product of line 14 and the second digit of line 13, and so on. In each successive partial product a digit is dropped from the multiplicand (line 14); thus, line 16 is the product of 9837 and 9, line 17 the product of 983 and 2, and so on. The vertical line passing through the midst of all the numbers in lines 15–18 serves much the same purpose as a decimal point: note that it has been so placed that the product (line 19) has in effect been divided by the radius (which, in line 5 above, had tacitly been multiplied in). This product is the tangent of the required angle (line 20), which is half the difference of the angles at F and G.

Kepler's **parallax computation** is carried out with the help of the parallactic table in the *Astronomiae pars optica* (Frankfurt, 1604). Since he explained the use of the table in that work, he felt justified in referring the reader to that explanation, for the sake of brevity. But the *Astronomiae pars optica* has not been translated into English, so many readers will find it inconvenient or impossible to take his advice. Here, for their sake, is how Kepler reckons parallax.

The first step is to find the **nonagesimal**. This is the point on the ecliptic 90° from the ecliptic's intersection with the horizon. It is also




necessary to find the altitude of the nonagesimal, or its complement, which is the arc between the nonagesimal and the zenith. This is the same as the angle between the ecliptic and the horizon, or its complement. The importance of the nonagesimal is that a great circle drawn from it to the zenith is always perpendicular to the ecliptic. Therefore, when the planet is at the nonagesimal, the parallax affects only the latitude. Furthermore, the latitudinal part of the parallax is always the same as the parallax at the nonagesimal, regardless of where the planet is. The longitudinal part of the parallax, in turn, is a function of the planet's distance from the nonagesimal.

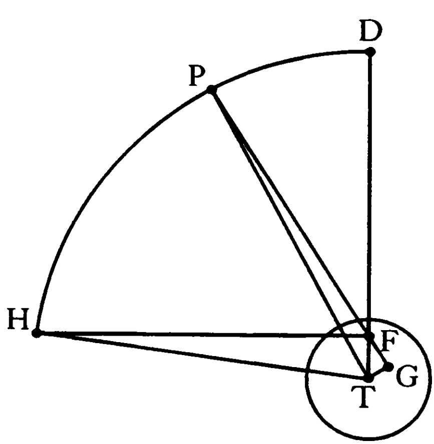
Once the nonagesimal and its zenith distance are known, the rest follows directly. Suppose AB is the earth's surface, T its centre, with an observer at B, Z the zenith, D the nonagesimal, and C the point where the great circle through Z and D intersects the horizon. And let TE be perpendicular to DE extended. The angle at C is the **horizontal parallax**, found by observation or by assuming (or calculating) a magnitude for the distance CT. The sine of this angle is BT/CT, or BT/DT. The sine of the angle at D, the latitudinal parallax, is ET/DT. Therefore, sin D/sin C = ET/BT = sin DBZ, or the sine of the latitudinal parallax is the product of the sines of the horizontal parallax and the zenith distance of the nonagesimal. Or, since the parallax angles are very small, the angles themselves can be used instead of the sines, and thus the latitudinal parallax is the product of the horizontal parallax and the sine of the zenith distance.

The **parallactic table** is designed to perform this computation.

Horizontal parallaxes are set out across the top, and zenith distances at the side, and one can find the corresponding latitudinal parallax by finding the intersection of the appropriate row and column. If the horizontal parallax is not an integral number of minutes, one finds the number of seconds at the top and finds the entry opposite the zenith angle, which will give the extra seconds and 'thirds' of latitudinal parallax.

**Longitudinal parallax** may be a little harder to visualize: it is the part of the parallax that occurs in the plane of the ecliptic, which is perpendicular to the plane of the diagram in the above figure, intersecting it at the line DT. So a new figure is required, showing that plane. But first, in the figure above, let BF be drawn perpendicular to DT, intersecting it at F. Thus F is the point in the plane of the ecliptic corresponding to the position of the observer B, and FT = BE. Now let the new figure be drawn. Here, DH is the ecliptic, with D the nonagesimal as before, and P the planet. The sine of the horizontal longitudinal parallax (the angle at H) is FT/TH or (in the previous figure) BE/TC, which may also be expressed as the product of BE/BT and BT/CT. Therefore, the sine of the horizontal longitudinal parallax is the product of the sines of the altitude of the nonagesimal and the total horizontal parallax. This may also be computed from the table, the total parallax being found at the top, and the horizontal longitudinal parallax being read opposite the altitude of the nonagesimal (note that this is the complement of the zenith distance).

Finally, the actual longitudinal parallax (the angle at P) is found.



The sine of this angle is equal to the product of the sines of the angle at H and the angle PFD, which is the planet's elongation from the nonagesimal. This is exactly analogous to the latitudinal parallax, and the table is used in the same way. That is, after reading the horizontal longitudinal parallax as in the preceding paragraph, one finds that number at the top and reads the actual longitudinal parallax opposite the planet's elongation from the nonagesimal. Odd seconds may be accounted for as before, if desired.

As for the **calendar**, Kepler held to the old Julian dates. The Gregorian calendar, it must be remembered, was a Popish innovation and was not very widely used, particularly in Lutheran countries. Sometimes, when he is using a date obtained from another astronomer, he gives both the Julian and the Gregorian dates. But when a single date is given, it is invariably in the old calendar. The Gregorian date is obtained by adding 10 days to Kepler's dates.

**Time** is measured from noon. Although Kepler usually reports morning observations by giving the number of hours from midnight, in his calculations he refers the time to noon on the previous day. This can be a source of confusion, since '5 am on June 21' becomes '$17^h$ on June 20'. To avoid this confusion, Kepler often reports morning observations by saying (using the same example), 'on the morning following June 20', or more succinctly, 'June 20/21'. If you are checking computations, and find a discrepancy of about 31′ in the mean longitudes, it is probably the result of using the wrong date for a morning observation.

## On the translation

### *General remarks*

Translation is always a more or less criminal act against both author and readers; and, as in other cases, it is incumbent upon the guilty party to excuse his crimes as much as possible by imputing them either to the author or to unavoidable circumstances. Let me then say at the outset that Kepler's Latin is not simple. He was given a thorough classical education of the kind that had evolved during the sixteenth century[21], and sometimes appears to have valued style more than clarity. Often he will choose an unusual word or turn of

[21] For educational trends in the sixteenth century, see R. R. Bolgar, 'Education and Learning', *New Cambridge Modern History* (Cambridge, 1968), vol. III ch. 14. For Kepler's familiarity with classical rhetorical conventions, see N. Jardine, *The Birth of History and Philosophy of Science* (Cambridge, 1984), pp. 74–79.

phrase for no apparent reason, leaving the translator wondering whether to represent it with something awkward or to try to force it to make sense. I am unabashedly on the side of sense. I allow Kepler the courtesy of supposing that what he writes can be understood, and try to put my understanding into reasonably good English. Sometimes, if the Latin is irremediably awkward or odd, the translation will reflect this, but I always give him the benefit of doubt. This approach admittedly involves the risk of a mistranslation based upon a misunderstanding. To this I can only say that any attempt at understanding involves the risk of misunderstanding, and it is a risk I willingly take. It is my fervent hope that the light my efforts shed upon this difficult work will outweigh any errors in translation that remain.

The task of proofreading and searching for blunders has been made much easier by the use of the nearly complete draft translation by Ann Wegner which Owen Gingerich has kindly given me. I have compared my entire work with hers, sentence by sentence, and many errors have thus been caught. In passages where the meaning is doubtful, I have also referred to Caspar's German translation[22].

Mathematical arguments present special problems for the translator. In some respects, mathematical passages require less actual translation than the other parts (symbols and letters, for example, remain unchanged). But it is my experience that no argument can be well translated unless it is well understood. Often the correct translation of a particular preposition cannot be known unless the accompanying operation is known, and the only way to be certain that the operation is known is to repeat it. Therefore, to test my comprehension, I have repeated every computation in the book. Although this may seem an extreme measure, I have time and again found this precaution justified, both in fitting the translation to the argument and in gaining a perspective on the whole that could be acquired in no other way[23].

There is another difficulty in Kepler's language that is not really his fault. As a highly inflected language, Latin possesses an inherent syntactic clarity that far surpasses that of English. As a result, it is

[22] Johannes Kepler, *Neue Astronomie*, Uebersetzt und eingeleitet von Max Caspar (Munich and Berlin, 1929). Caspar's introduction to the translation contains much mathematical material that is not in the notes to the Latin text, including series expansions of many orbital equations.

[23] Recomputation has also resulted in the discovery of many hitherto undetected errors. Where these are significant (that is, where the discrepancy is greater than about 2′ of arc, or about 0.1% in linear magnitudes), corrected figures are given in the notes. Some, but not all, of the data have also been checked.

possible to write a very long Latin sentence which is perfectly clear but which is very cumbersome at best when turned into a single English sentence. Therefore, I have nearly everywhere dissected Kepler's rambling sentences mercilessly in the interest of clarity. The sole exception is the Letter of Dedication, a delightful rhetorical bagatelle, where I have kept the sentence divisions intact. This may give some idea of the flavour of Kepler's language, and will also perhaps inspire in the reader's bosom effusions of gratitude that the rest of the book was treated differently.

As was the custom, the book is prefaced by an assortment of poetry, much of it in praise of Kepler and his work. However, Kepler has also included a substantial poem by Tycho Brahe urging the youth of the day to assist in the reformation of astronomy. He has responded with a poem of his own, depicting his work as arising from Tycho's and answering his call. It is an interesting and informative interchange, and the poetry is not without a degree of elegance. I have accordingly taken the trouble to retain the dactylic hexameter of the original in the translation, in an attempt to capture some of the feeling of the verse.

*The text*

The text used is the excellent modern edition by Max Caspar, volume III of *Johannes Kepler Gesammelte Werke* (Munich, 1937). Since the manuscript is lost, Caspar based his edition entirely upon Kepler's printed edition, correcting typographical errors and noting the computational errors he found. Caspar's emendations have been followed in the translation, usually without comment. Errors that remain in the Caspar edition have been checked in the facsimile of the first edition, where appropriate, and the ensuing corrections have been documented in the footnotes.

In preparing his edition, Caspar added many notes, some of which are most helpful, and some, in my opinion, merely distracting. He was at great pains to translate much of Kepler's mathematics into modern form, presumably so that a modern reader could understand it. This approach seems to me misguided: what the original geometry shows is very different from what a polar equation tells. Both are valuable, but we learn much more about Kepler by trying to see things as he saw them. Moreover, Caspar's analysis has been surpassed, most notably in the article by D. T. Whiteside cited above. I have therefore kept only what seems useful in Caspar's notes,

incorporating it into my own footnotes, giving credit where I have been aided by his insight or am relying upon his information.

Among Caspar's notes are to be found two remarkable long selections from the surviving manuscripts, one a draft table of contents (apparently from 1601–1602[24]), and the other a document of several pages entitled '*Axiomata physica de motu stellarum*'. These are, of course, of great interest. However, to include them in this translation would unduly focus attention upon them at the expense of other important manuscripts and printed letters. I have therefore decided not to include translations of any of the manuscript material, other than a few brief passages in footnotes.

*Format*

In the interest of fidelity to the author's intentions, and of ease of reference to the original, Kepler's layout and typography have been largely retained. Paragraph divisions have been kept even when they seemed odd or illogical. Kepler's marginalia are in the margin. Kepler's idiosyncratic use of italics to distinguish mathematical passages, which reveals much about the distinction then drawn between mathematics and physics, has been followed.

*Notation and abbreviations*

For the sake of clarity and economy of expression, the following notations and abbreviations have been uniformly used in the translation:

**Numerals** (with a very few exceptions) are Arabic. Although Kepler often used Roman numerals, especially for dates and times, I can see no reason to follow his usage, and much to be gained by abandoning it.

**Dates and times** are expressed by stating the year, month, and day, in that order. Times are indicated by superior characters (superscripts); thus, where Kepler writes 'H. VII M. XXXVI' the translation reads '$7^h$ $36^m$'. Where a time interval is greater than one day, the number of days is indicated by a superscript 'd'.

**Angles** are expressed in degrees, minutes, and seconds, denoted by the conventional symbols (°, ′, ″). These are never used to express times. Longitudinal positions and elongations are often expressed in signs, degrees, minutes, and seconds (a sign is 30°). The number of

[24] Gingerich, 'Kepler's Treatment of Redundant Observations', *op. cit.*, note 20, p. 313.

signs is denoted by a superscript 's'. It should be noted that Kepler himself most often used these symbols, but sometimes used the words they stand for (or their abbreviations), and sometimes even the symbols and the words together. No attempt has been made to follow his usage closely in the translation.

**Abbreviations** of frequently cited titles are:

KGW – W. von Dyck, M. Caspar, and F. Hammer, eds., *Johannes Kepler Gesammelte Werke* (Munich, 1937–).

TBOO – J. L. E. Dreyer, ed., *Tychonis Brahe Opera Omnia* (Copenhagen, 1913–1929).

**Numbers in the margins** are page numbers of the 1609 edition. Since these numbers are also in the margin of KGW vol. 3, the translation is conveniently keyed to both of the presently available editions. Where pages in the 1609 edition are not numbered (as in all the prefatory and introductory matter), pages are identified by signature and folium.

**Other material in the margins** is Kepler's.

**Footnotes** are mine.

# Glossary

This glossary serves three purposes: it defines a number of technical or otherwise unusual English words, it discusses the meaning of a number of Latin words which have no exact English equivalents, and it explains a number of instances where context suggests an unusual translation.

**Anomaly**: any of several angular measures of a planet's motion. For an explanation and illustration of the different kinds of anomaly, see 'Technical aspects of Kepler's astronomy and mathematics' in the translator's introduction.

**Anomaly of commutation**: the angular measure of the relative motion of the earth and a planet about the sun or about the centre of the earth's orbit. The former is called the 'true' or 'equated' anomaly of commutation, and the latter, the 'mean' anomaly of commutation.

***Artifex***: 'theorist'. In his translation of Kepler's *Mysterium cosmographicum* (Tubingen 1596; translated as *The Secret of the Universe* (New York 1981)), A. M. Duncan translates this 'practitioner', partly on the basis of a misreading of a passage in chapter 18 of that work. The phrase in question reads, '*Et Physicis quidem sive Cosmographis, qualem hoc libello personam ego sustineo, . . .*', which clearly means, 'And on the other hand, to the physicists or cosmographers, which sort of role I myself am maintaining in this little book, . . .'. Thus it is not correct to say (as Duncan says) that 'Kepler counts himself not in this class' (*op. cit.* p. 13).

An *artifex* is generally the master of some art, but in astronomy it has a sense that is closer to the English 'artificer', in that it denotes one who creates the artifices: a theorist. The aptness of this translation is confirmed by Kepler's use of *artifex* in the present work, and

also in the *Apologia pro Tychone,* although Jardine, following Duncan, has translated it 'practitioner', which does not fit nearly as well. See N. Jardine, *The Birth of History and Philosophy of Science* (Cambridge, 1984), p. 139.

***Aura aetherea***: 'aethereal air'. The term Kepler uses to describe the interplanetary space.

***Complementum***: used to represent both 'complement' and 'supplement'. Usually one must tell which is meant by the context; sometimes, however, Kepler distinguishes them by calling the former '*complementum ad rectum*', and the latter '*complementum ad semicirculum*' or '*ad duos rectos*'. For the 'full circle complement', see below.

***Doctrina***: This word denotes a coherent body of learning, and performs much the same function as the English suffixes '-ology' or its relatives '-ography' and '-ometry'. Therefore, some phrases in which it appears, such as '*doctrina triangulorum*', have been Englished with the appropriate suffix (in this instance, 'trigonometry'). Its use also has close affinity with the use of the word 'theory' in such phrases as 'theory of relativity', and so, when no suitable compound word was found, '*doctrina*' has been translated 'theory'. For example, '*doctrina de gravitate*' becomes 'theory of gravity'.

**Duplicate ratio**: 'When three magnitudes are proportional, the first is said to have to the third the **duplicate ratio** of that which it has to the second.' (Euclid, *Elements*, Book V def. 9). When the magnitudes are expressed in numbers, the ratio of the third to the first is the square of the ratio of the second to the first. As Kepler uses the term, it is practically equivalent to 'square'.

**Eccentric equation**: The amount that must be added to or subtracted from the planet's mean longitude (q.v.) to give its eccentric position (q.v.).

**Eccentric position**: the planet's position on the eccentric, in zodiacal coordinates, measured about the sun.

**Equation**: the angular quantity that must be added to or subtracted from the measure of a planet's position about one centre to give its measure about a different centre. Or, in terms of Kepler's final theory, the angular difference between any two measures of a planet's position.

**Full circle complement**: the difference between a given angle and the full circle (360°).

**Hypothesis**: 'We, however, call "a hypothesis" generically whatever is set out as certain and demonstrated for the purpose of any demonstration whatsoever. [. . .] Specifically, however, when we speak in the

plural of "astronomical hypotheses", we do so in the manner of present-day learned discourses. We thereby designate a certain totality of the views of some notable theorist [*artifex*], from which totality he demonstrates the entire basis of the heavenly motions.' (Kepler, *Apologia pro Tychone*, fol. 266r, in N. Jardine, *The Birth of History and Philosophy of Science* (Cambridge 1984) pp. 138–9.)

***Lex***: 'law'. The preposterous idea that inanimate objects 'obey' the 'laws of motion' much as rational beings obey laws laid down by their ruler has a fascinating history, but this is not the place to go into it. Suffice it to say that the notion of 'laws of motion' was ubiquitous in sixteenth-century astronomical works, and that it was plausible in this context because the motions of the planets were thought to be governed by angels. Kepler is thus apparently only following current usage when he mentions '*leges motuum*'. For more sources, see N. Jardine, *The Birth of History and Philosophy of Science* (Cambridge 1984), p. 240.

**Mean longitude**: a measure of a planet's mean position, in zodiacal coordinates (beginning from the vernal equinox point). The mean longitude is always adjusted so that when the planet's eccentric position (q.v.) is on the line of apsides, the mean longitude is the same as the true longitude. Thus a change in the position of the apsides requires a change in the mean longitudes, an adjustment to which Kepler gives considerable attention.

***Mora***: literally, 'delay'. Kepler uses this word to denote the amount of time a planet takes to traverse a given orbital arc; hence, it has been translated 'elapsed time', or simply 'time'. It is, I believe, of great significance that Kepler treated time as a *dependent* variable, as if more or less of it elapses as the planet traverses uniform arcs. It is at least arguable that he would not have formulated his 'second law' (the law of areas) had he not held this concept of 'time-per-unit-distance', which is the inverse of our Cartesian-Galilean-Newtonian concept of velocity.

***Motus***: although this usually means 'motion', Kepler often uses it where we would say 'position'; for example, '*motus medius Solis est* . . .' is best translated, '*the sun's mean position is* . . .'.

***Mundus***: 'world'. This usually (but not always) denotes the entire visible creation, including the fixed stars (see, for example, chapter 6, p. 156).

**Nonagesimal**: the point on the ecliptic 90° west of its intersection with the eastern horizon. It is an important point in parallax computations, for which see the translator's introduction.

***Orbis***: 'orb'. Although this usually means 'circle', the Latin has the same ambiguity as the English 'orb': it may be a sphere.

***Potentia***: literally, 'power'. In mathematics, it usually means 'second power', or 'square', but can also mean 'sum of the squares'. Usually translated 'square'.

***Potest***: literally, 'is able' or 'has the power'. This is the verb corresponding to the noun '*potentia*' (above), and in mathematical usage means 'is equal in square'.

***Species***: this word, related to the verb '*specio*' (see, observe) has an extraordinarily wide range of meaning. Its root meaning is 'something presented to view', but it also can mean 'appearance', 'surface', 'form', 'semblance', 'mental image', 'sort', 'nature', or 'archetype', to mention only a selection of its most diverse senses. It is in fact the Latin equivalent of the Greek 'ειδος', which is Plato's word for his 'forms' or 'ideas'. The Epicureans used it to denote a 'surface film given off by physical objects' (*Oxford Latin Dictionary*; cf. Lucretius 4-602 and 6-993). Robert Grosseteste and Roger Bacon used what is apparently a Neoplatonized form of this technical meaning, in which the material 'surface film' is transformed into an 'immaterial form' (see A. C. Crombie, *Robert Grosseteste and the Origins of Experimental Science*, second ed. (Oxford, 1962) pp. 104–116 and 144–147). Accordingly, in his translation of Kepler's *Mysterium cosmographicum (The Secret of the Universe*, New York, 1981), A. M. Duncan has translated *species* as 'emanation'. Ann Wegner, in her draft translation of the *Astronomia nova*, independently chose the same word, and Gingerich used it in his published translation of chapter 34 (*The Great Ideas Today, 1983*, Chicago, 1983, pp. 325–329).

C. G. Wallis, on the other hand, stayed closer to the root meaning and the Platonic nuances in translating *species* as 'form' (Kepler's *Epitome*, in *Great Books of the Western World* 16 (Chicago, 1952) p. 897 (translating the first few words of Kepler's p. 517), *et passim*). I began by using 'form', possibly influenced by Wallis's translation, but soon abandoned this as being too far from English usage. I also rejected the nearly suitable 'emanation', for two reasons. First, Kepler also used various Latin cognates of that word which seem to me best translated by their English kin. Hence, if *species* were also translated 'emanation' the reader would have no indication of what word Kepler was using. And second, there are places where 'emanation' does not make sense. Take, for example, this passage from chapter 34, which, in Gingerich's translation (*op. cit.*, p. 326) reads:

> Therefore, . . . when any particle of the solar body moves

> toward some part of the world, the particle of the immaterial emanation which from the beginning of creation corresponded to that particle of the body also always moves toward the same part. *If this were not so, it would not be an emanation*, and would come down from the body in curved rather than straight lines. (Italics supplied.)

The sense of the italicized phrase is much clearer if 'form', or simply '*species*', is substituted for 'emanation'. Kepler's point is that the *species* is a property of the solar body and not a self-subsisting entity, and hence does not behave like a body. This point is lost in the proposed translation, and a puzzled reader would have to consult the original to tell whether the Latin read '*species*' or something like '*emanatio*'.

Having rejected 'form' and 'emanation', I was tempted to use the English word 'species', since it has had a wide range of meanings many of which correspond to the various Latin senses, and since it has also been used to denote 'A supposed emission or emanation from outward things, . . .' (*Oxford English Dictionary*). Unfortunately, all the relevant senses are obsolete, so its use would serve only to confuse matters. I have therefore thrown up my hands, admitted defeat, and declined to translate it at all. It appears as the Latin word '*species*', in italics.

***Speculatio***: 'theory', 'theorizing', 'theoretical argument', 'theoretical consideration'. This is not a classical word. It is derived from the verb '*speculor*', 'spy into, examine', but was later used in a philosophical context by Boethius (*De consolatione philosophiae* 4.1 and elsewhere) to mean something like the English 'speculation'. But the *speculatio* that Kepler has in mind is a reasoned argument, not a wild conjecture. For example, in the *Epitome*, on p. 654, where Kepler supports a mathematical argument 'by the force of physical *speculatio*': he obviously means that the physical theory requires a certain mathematical relation to hold. Therefore, even though 'theory' is also used to translate other words, it seems the best translation for '*speculatio*', a choice that is amply confirmed by Kepler's usage in the *New Astronomy*.

***Theoricae***: 'theories' or 'theories of the planets'. This is a technical term used in several widely read mediaeval books on planetary theory, and denotes the geometrical configurations used to account for planetary motions. It is nearly equivalent to 'hypotheses', although the latter term is not restricted to the geometrical models. However, 'hypotheses' as a translation has the disadvantage of

blurring the subtle distinction between the two. This problem is avoided by the use of 'theories'. Further, despite the use of 'theory' as a translation of other words, it should not be difficult to tell from the context what the Latin is.

**Theory**: this is the English translation of three distinct Latin words: *doctrina, speculatio,* and *theoricae* (the last always plural). The Latin can be distinguished by the usage: if the English is 'theory of . . .', the Latin is '*doctrina*'; if 'physical theory', the Latin is '*speculatio*'; and if 'theories of the planets', the Latin is '*theoricae*'.

***Virtus***: 'power', and

***Vis***: 'force'. In classical usage, *vis* properly refers to physical action, while *virtus* (from *vir*, 'man') denotes those qualities pertaining to manly excellence, both physical and moral. *Vis*, for example, is frequently used classically of violent action, while *virtus* is never used in this way. On the other hand, there appears to be no classical instance of *virtus* where it would not have made sense to use *vis*, and hence, in a physical context, *virtus* would be a particular and more abstract kind of *vis*. With 'power' and 'force', the opposite is true: we think of force as a particular and more concrete form of power. Nevertheless, the two words in most respects adequately represent the meanings of *virtus* and *vis* in classical Latin.

In Kepler's physics, these two words are supremely important, and so the question of their meaning merits serious study. Although in the *New Astronomy* Kepler usually uses '*virtus*' to denote the abstract motive power that emanates from the sun as a *species*, and '*vis*' to denote the actual pushes and pulls on the planet, counter instances can be found. Further, his use of the words in the *Epitome* is different, and it is not clear whether his later usage represents a refinement, a modification, or some combination of the two. In the present translation, no attempt has been made to answer these questions. Instead, it seems most sensible to choose a translation that expresses the classical usage, since it is that usage which Kepler's audience knew, and which therefore formed the context within which Kepler developed his ideas. Also, since the meanings are not perfectly clear, it is desirable to use a consistent translation so that the reader may easily infer the Latin original: hence, '*virtus*' is always 'power, and '*vis*' is always 'force', and these two English words are used only to translate these two Latin words.

**Whole sine**: the sine of a right angle, equal to the radius (100,000 units, in Kepler's trigonometry).

# ASTRONOMIA NOVA
ΑΙΤΙΟΛΟΓΗΤΟΣ,

SEV

# PHYSICA COELESTIS,

tradita commentariis

DE MOTIBVS STELLÆ

# MARTIS,

Ex obſervationibus G. V.

TYCHONIS BRAHE:

Juſſu & ſumptibus

RVDOLPHI II.

ROMANORVM

IMPERATORIS &c:

Plurium annorum pertinaci ſtudio
elaborata Pragæ,

*A S^æ. C^æ. M.^{tis} S^æ. Mathematico*

JOANNE KEPLERO,

*Cum ejusdem C^æ. M.^{tis} privilegio ſpeciali*

ANNO æræ Dionyſianæ cIↃ IↃ c IX.

# NEW ASTRONOMY

BASED UPON CAUSES
OR
CELESTIAL PHYSICS
treated by means of commentaries
ON THE MOTIONS OF THE STAR

# MARS

from the observations of
TYCHO BRAHE, GENT.

BY ORDER AND MUNIFICENCE OF

# RUDOLPH II

EMPEROR OF THE ROMANS, &c.

Worked out at Prague
in a tenacious study lasting many years

*By His Holy Imperial Majesty's Mathematician*

JOANNES KEPLER

*With the same Imperial Majesty's special privilege*

In the Year of the Dionysian era MDCIX

## P. RAMUS, Scholae Mathematicae
## Book II, p. 50

Thus the contrivance of hypotheses is absurd; nevertheless, in Eudoxus, Aristotle, and Callippus[1], the contrivance is simpler, as they supposed the hypotheses to be true – indeed, they have been venerated as if they were the gods of the starless orbs. In later times, on the other hand, the tale is by far the most absurd, the demonstration of the truth of natural phenomena through false causes. For this reason, Logic above all, as well as the Mathematical elements of Arithmetic and Geometry, will provide the greatest assistance in establishing the purity and dignity of the most noble art[2]. Would that Copernicus had been more inclined towards this idea of establishing an astronomy without hypotheses! For it would have been far easier for him to describe an astronomy corresponding to the truth about its stars, than to move the earth, a task like the labour of some giant, so that in consequence of the earth's being moved, we might observe the stars at rest. Why could there not rather arise someone from among the great number of celebrated schools of Germany, a philosopher as well as a mathematician, who would attain the prize of eternal praise that is publically offered? And if any fruit of transitory usefulness can be offered to compare with a prize of such power, I will solemnly promise you the Regius Professorship at Paris as a prize for an astronomy constructed without hypotheses, and will fulfil this promise with the greatest pleasure, even by resigning our professorship.

## The Author to Ramus

Conveniently for you, Ramus, you have abandoned this surety by departing both life and professorship. Had you still held the latter, I would, in my judgement, have won it indeed, inasmuch as, in this work, I have at length succeeded, even by the judgement of your own Logic. As you ask the assistance of Logic and Mathematics for the noblest art, I would only ask you not to exclude the support of Physics, which it can by no means forego. And unless I am mistaken, you readily grant this, seeing that you surround your *Conformator* with Philosophy as well as Mathematics. Thus with the same facility I, too, admit something commonly considered most absurd philosophically, defending[3] it, not with a gigantic effort, but with the best arguments. For when it functions, it effects nothing new, nothing unaccustomed, but only fulfils the function for which it was invented.

It is a most absurd business, I admit, to demonstrate natural phenomena through false causes, but this is not what is happening in Copernicus. For he too considered his hypotheses true, no less than those whom you mention considered their old ones true, but he did not just consider them true, but demonstrates it; as evidence of which I offer this work.

But would you like to know who originated this tale, at which you wax so wroth? 'Andreas Osiander'[4] is written in my copy [sc. of Copernicus's *De revolutionibus*], in the hand of Hieronymus Schreiber of Nurnberg. This Andreas, when he was in charge of publishing Copernicus, thought this preface most prudent which you consider so absurd (as may be gathered from his letters to Copernicus), and placed it upon the frontispiece of the book, Copernicus himself being dead, or certainly unaware of this. Thus Copernicus does not mythologize, but seriously presents paradoxes; that is, he philosophizes. Which is what you wish of the astronomer.

[1] Eudoxus, a pupil of Plato (as was Aristotle), attempted to account for planetary motion through a nest of concentric spheres each of which imparted its motion to the axis of the sphere it immediately contained. Callippus later tried to bring Eudoxus's rather unsatisfactory attempt more nearly into accord with the phenomena by adding more concentric spheres to the schema. Aristotle subsequently gave the Eudoxian-Callippian homocentric system his blessing, adding intelligent beings whose task was to move the spheres, most of which were starless – hence, Ramus's sarcastic remark. See Aristotle, *Metaphysics*, Book XII ch. 8.

[2] Logic, Arithmetic, Geometry, and Astronomy are four of the seven 'Liberal Arts', the other three being Grammar, Rhetoric, and Music. Astronomy was frequently regarded as the noblest of the arts because of the excellence of the objects of its study.

[3] Reading *defendens* instead of *defendentem*.

[4] For more on the theologian Osiander (1498–1552) and his notorious 'instrumentalist' preface to Copernicus's work, see N. Jardine, *The Birth of History and Philosophy of Science* (Cambridge 1984) pp. 150–154.

(**) r

# To Rudolph II
# The Ever August Emperor of the Romans
# King of Germany, Hungary, Bohemia, &c.
# Archduke of Austria, &c.

**Most August Emperor**

In order that Your Holy Imperial Majesty, as well as the entire House of Austria, might be happy and prosperous in most serene renown, I am now at last exhibiting for the view of the public a most Noble Captive, who has been taken for a long time now through a difficult and strenuous war waged by me under the auspices of Your Majesty. I do not think he will object to the name of Captive, since for some time he has been accustomed to dropping his vaulted shield and his
(**) v arms and giving himself over freely and playfully to capture and bondage, whenever custody, prison, or chains are ordered.

The brilliance of this spectacle could not be greater than if I were to write a panegyric upon this most distinguished captive, and shout it out loudly and publicly.

However, one who ventures forth upon this battlefield encounters an astonishing brightness, and averts his squinting eyes, made accustomed to the feeble light of Night, and to scholastic shadows.

I therefore leave it to the writers of history books to describe the greatness of our Stranger, which he acquired in the art of war.

They would certainly say that it is he through whom all armies conquer, all military leaders triumph, and all kings rule, without whose aid no one ever honourably took a single captive. Let them now feast their eyes with looking upon him, captured through my martial efforts.

Those who admire Roman greatness would say that he is the begetter of the Kings Romulus and Remus, the preserver of the City,

protector of the Citizens, Supporter of the Empire, by which favour the Romans discovered military discipline, improved and perfected it, and subjugated the orb of the world. Let them therefore give thanks at his being confined and at his being acquired as a happy omen for the House of Austria.

I, for my part, retreat hence to other ground better suited to my powers. Nor will I set foot upon that part of my profession in which strife arises between me and my fellow soldiers.

(**) 2 r They, for their part, would surely rejoice with a different joy: he has been restrained by the bonds of Calculation, who, so often escaping their hands and eyes, was accustomed to deliver vain prophecies of the greatest moment, concerning War, Victory, Empire, Military Greatness, Civil Authority, Sport, and even the cutting off or calling forth of Life itself. Let them congratulate Your Majesty that the Master of the Horoscope[1] has been brought under control, and even made to be friendly, for by their account Mars rules Scorpio, which contains the Heart of Heaven[2]; in Capricorn, which is rising, he is exalted; in Cancer, into which the moon was entering, he customarily plays the triangular game with knucklebones; in Leo, where the Sun plays host, he is recognized as being one of the family; and finally, he is the ruler of Aries, beneath which Germany is supposed to be, over which he rules in complete harmony with Your Holy Imperial Majesty.

Let them be occupied in this part of the triumph; I do not mind. I shall give them no cause for quarreling on such a festive day: let this impertinence pass as a soldiers' joke. I myself shall occupy myself with Astronomy, and, riding in the triumphal car, will display the remaining glories of our captive that are particularly known to me, as well as all the aspects of the war, both in its waging and in its conclusion.

For he is not to be held without honour among us, whom the eternal Architect of this world, and the Father of Heavens and Humans in common, Jupiter, located in the front lines of the visible

[1] It is undoubtedly the Emperor's horoscope that Kepler means, and the details mentioned (such as the rising sign and the moon sign) are probably those of the time of his birth. The translator has been unable to find information on the '*astragalis lusum trigonicum*', here translated 'the triangular game with knucklebones'. '*Astragalus*' is a Greek word, denoting (among other things) any of a number of different bones, and by derivation, a knucklebone used as a die in gaming.

[2] The Heart of Scorpio is the star Antares. 'Ant-Ares', that is, 'counterpart of Mars', was so named because of its red colour. Astrologers supposed it to have a particular affinity for Mars.

bodies, so that he might serve as a soldier for the glory of his Creator through his perennial course through the ethereal regions, and so that he might raise human minds, lulled to sleep by a deep somnolence, from the slanderous reproach of idleness and ignorance, arouse them
(**) 2 v to venture forth, and provoke them forcefully to carry out investigations in the heavens for the praise of their Creator.

It is he who is the most potent conqueror of human inventions, who, ridiculing all the sallies of the Astronomers, escaping their devices, and striking down the hostile throngs, kept safe the secret of his empire, well guarded throughout all ages past, and performed his rounds in perfect freedom with no restraints: hence, the chief complaint registered by that Priest of Nature's Mysteries and most distinguished of the Latins, C. Pliny, that 'Mars is the untrackable star'[3].

It is said that Georg Joachim Rheticus (a disciple of Copernicus not lacking honour in the memory of our forebears, and who, as the first to dare to yearn for a reconstruction of Astronomy, thereupon strove for it through observations and discoveries that are not to be scorned), when he was brought up short in amazement by the motion of Mars, and did not disentangle himself, fled to the oracle of his familiar Genius, either intending (the gods willing) to explore that being's erudition, or driven by a headstrong desire for the truth, whereupon that stern patron, exasperated, alternately caught the importunate inquirer by the hair and stretched his head towards the low-hanging panelled ceiling, and then threw him down, flattening him on the paved floor, adding the reply: 'This is the motion of Mars.' The story is a bad thing: there is nothing else more injurious to good reputation, for it is as tenacious of deception and distortion as it is informative of the truth. It is nevertheless not unbelievable that Rheticus himself, when his speculations were not succeeding and his spirit was in turmoil, leapt up in fury and pounded his head against
(**) 3 r the wall. For what wonder would it be if the same thing happened to Rheticus, who provoked Mars, as once happened to C. Octavius Augustus Caesar when he lost five legions under the command of Quintilius Varus, surrounded by his enemy Arminius, protege of our Germanic Mars?

Nevertheless, here too, as in other kingdoms, the ruling influence of our enemy has been sustained and supported, more than any other

[3] Pliny, *Natural History*, II, 17.

thing, by the persuasion and confusion of the multitude of people, the defiance of which I have always considered the path to victory. Indeed, when I was but indifferently well versed in this theater of Nature, I formed the opinion, with practice [*usus*] as my teacher, that, just as one human being does not greatly differ from another, neither does one star differ much from another, nor one opponent from another, and hence, no account is to be received easily that says something unusual about a single individual of the same kind.

In this place chief praise is to be given to the diligence of Tycho Brahe, the commander-in-chief in this war, who, under the auspices of Frederic II and Christian, Kings of Denmark, and finally of Your Holy Imperial Majesty as well, explored the habits of this enemy of ours nearly every night for twenty years, observed every aspect of the campaign, detected every stratagem, and left them fully described in books as he was dying.

I, instructed by those books as I succeeded Brahe in this charge, first of all ceased to fear [the enemy] whom I had to some extent come to know, and then, having diligently noted the moments of time at
(**) 3 v which he was accustomed to arrive at his former positions, as if going to bed, I directed the Brahean machines thither, equipped with precise sights, as if aiming at a particular target, and besieged each position with my enquiry as the chariots of the great Mother Earth were driven around in their circuit.

The campaign did not, however, succeed without sweat, since it frequently happened that machines were lacking where they were most needed, or that they were transported over muddy roads by inexperienced charioteers at great expense of time and material, or that the launching of some of them, where I had not yet investigated the matter, occurred in other directions than I had had in mind. Often the brightness of the sun or of the moon, and often an overcast sky, cheated the commander's eyes; and more often the interposition of vapourous air deflected the globe, forcing it from the straight path. Also not infrequently, the walls, where they were presented most obliquely, received ineffectual blows, however numerous they might be. Add to this the enemy's enterprise in making sallies, and his vigilance for ambuscades, while we were frequently asleep. Also, his constancy in defence: whenever he was driven or fled from one castle, he repaired to another, all of which required different means to be conquered, and none of which was connected to the rest by an easy path – either rivers lay in the way, or brambles impeded the attack,

but most of the time the route was unknown. Each of these things is thoroughly described in its own place in this commentary.

Meanwhile, in my camp, is there any sort of defeat, any kind of disaster that has not occurred? The overthrow of the Most Distinguished Leader, rebellion, plague, pestilences, domestic matters both good and bad, destined in either case to take time; a new, unforeseen, and terrifying rear attack by the enemy, as I have recounted in the book *On the New Star*[4]; at another time, an enormous Dragon with a very long tail, vomiting fire and attacking
(**) 4 r my camp; desertion and poverty of the soldiers; the inexperience of novices; and, at the head of all, the extreme deficiency of provisions.

At last, when he saw that I held fast to my goal, while there was no place in the circuit of his kingdom where he was safe or secure, the enemy turned his attention to plans for peace: sending off his parent Nature, he offered to allow me the victory; and, having bargained for liberty within limits subject to negotiation, he shortly thereafter moved over most agreeably into my camp with Arithmetic and Geometry pressing closely at his sides.

However, from the time when, after surrendering, he abode by our house's fair laws of friendship, he, through hidden illusions (being unaccustomed to rest), did not cease to incite among us I know not what further fears of war, and if we happened to become terrified, we would give him much to laugh at. But, seeing us strong in spirit, he agreed to live with us in earnest, and, dropping the appearance of hostility, confirmed his faith with us.

This one thing he begs of Your Majesty: since his alliance in the ethereal regions is great (for indeed, his father is Jupiter, his grandfather Saturn, Venus is his sister as well as his mistress, and from now on the chief alleviation of his chains, Mercury his brother and faithful herald); and since he is possessed by desire of them, and they of him, owing to their similar ways, he wishes that they too might live among humanity, becoming partakers of the honour with which he is bestowed; and that Your Majesty might give them to him as soon as possible, since the remnants of this expedition are severely dimin-
(**) 4 v ished, and, as they have surrendered themselves, no longer pose any threat. To this end, I readily offer Your Majesty a work that is not

[4] In the autumn of 1604 a nova appeared in Ophiuchus, about which Kepler wrote the pioneering work *De stella nova in pede Serpentarii*, (Prague 1606). According to Caspar, the 'Dragon' is the comet that appeared in the winter of 1607, about which Kepler wrote *Ausführlicher Bericht von dem . . . Cometen . . .*, (Hall/Sachsen 1608).

without usefulness (it being trained in the most combative circumstances, and well acquainted with the terrain) and no less trustworthy than its predecessor[5]. I pray and beseech you for this one favour (seeing that throughout these nine years, conversation in this hall, packed with soldiers, centurions, and commanders, has supplied me with the word 'beseech', as well as the rest of the oration): that Your Imperial Majesty command the chiefs of the treasury to take thought for the sinews of war and supply me with new funds to enlist the army. I pray thus, seeing that I both know that these things already are approved by Your Majesty, and consider that they promote the glory of God and the immortality of the Name of Your August Majesty, to Whom I have devoted all my work for a long time, and to Him I now most humbly commend myself.

March the 28th, in the year of the Dionysian[6] era 1609.

Your Holy Imperial Majesty's

Most Humble Mathematician
Joannes Keppler[7]

[5] A reference to the *Tabulae Rudolphinae* (not published until 1627), which applied the conclusions of the present work to the other planets.

[6] Here and on the title page, Kepler refers to the Roman Abbot Dionysius Exiguus, who established the commonly used Christian chronology in the early sixth century.

[7] Spelled as in the original.

(**) 5 r

# Epigrams[1] On these commentaries on the motions of Mars

**Urania to Kepler**[2]

'Cease, O Keplerides, in a war against Mars to do battle:
Mars submits to none, but it be he to himself.
Vainly therefore you strive to make him submit to your bondage
Who has lived quite free over numerous ages.'
Thus speaks the Muse. But contrariwise he thus answers: 'What then?
Have the accounts of Pallas Minerva escaped your mind?
Pallas was able to prostrate horrific Mars with a boulder –
True this is, if the songs sung by Homer are yours.
Why, therefore, with the help of mighty Minerva, could not
Mars now, however fierce, also submit to the yoke?
Look at the book that we've published, under the Rudolphine aegis:
"Surely," you will say, "Mars now suffers great hardship."'

[1] An attempt has been made in the translations to retain approximately the same metre as in the originals, although the time values in Latin verse cannot be reproduced in English. That is, where the English has an accented syllable, the Latin has a long syllable; thus, the rhythm is quite different.

[2] This poem and the two following are written in elegiac couplets. Each such couplet consists of two dactylic hexameters the second of which lacks an arsis in the third and sixth feet. It is a form of verse peculiarly suited to Greek and Latin, and is not easily rendered in English. Furthermore, the verses here are sometimes defective. So although the elegiac form is retained in the translation, it is not adhered to strictly.

### Another

A Liparaean[3] once captured Mars, ensnared in some netting:
He was on his way, Venus, to your embrace.
Now once again the War god is chained in the same bonds and fetters,
Nor is Venus to blame: it's your fault, O Pallas.
You, O Minerva, gave the netting to Tycho, and Tycho
Gave it to Kepler, who threw it around Mars's ankles.
Vulcan the great smith, and with him another, fashioned this wonder;
Kepler yet surpasses both the one and the other.
Short is the time over which the Vulcanian chains have endured.
Contrast them with Kepler's, which will stay strong for aye.

Saxirupius wrote this at Prague
in the year 1609.

### Another

Keplerius, an alumnus of earth, makes assault on the heavens.
Seek not for ladders: the earth itself takes flight.

J. Seussius wr. at Dresden.

(**) 5 v

### A Hortatory Ode[4]
### of
### Thycho Brahe

The Highest Astronomer, to Those Who Cultivate Astronomy,
Appended to the Restitution of the Fixed Stars,
in the *Progymnasmata* Vol. I page 295.

Smooth is the road now, that formerly no one could travel for ages:
Though it exhausted the watchman, and needed the greatest of labours,
Now it allows one to climb the peaks of the unapproached heavens,
And to go through to the highest of houses, Divinities' dwellings.
Now one can fully describe heaven's fires, whether fixed or traversing

[3] This is of course Vulcan, associated with the Aeolian island of Lipara; cf. Juvenal, *Satires*, 13. 45.

[4] Latin *Paraeneticum*, which is a transliteration of the Greek παραινετικον, meaning 'hortatory'. Tycho's name is given here as it is spelled in the text. There is a copy of the first ten lines of this poem in Kepler's hand in the Leningrad Kepler manuscripts, vol. XIV f. 372, in which Tycho's name (in the genitive case) is spelled 'Trehenis Brahe'. Elsewhere, Kepler consistently has 'Tycho', genitive 'Tychonis'. The poetry, which is heroic verse (dactylic hexameter), is much more elegant than the previous examples.

Various paths, and prove their celestial course and position.
Thus can the wonders of Jupiter, highest of gods, be established.
So bestir yourselves, youths, with spirited vigour, and lofty
Favour of Wit and of Genius, by which, from her birth, the renowned
Goddess Urania has inspired a divine love of heaven.
Letting the earth and all things earthly come second to heaven's
Goods. For you do not care for the thoughtless beliefs that are held by
Common louts, nor give heed to the gloomy talk of the lazy.
You dismiss those moles to dig in their dingy dark caverns,
Sightless that they may remain, for that is their dearest desire.
Hither bring your glad spirits, and leave the many behind you;
Hither turn, and let not the mind, a divine part of heaven,
Lose the good of its homeland. Your studies as well as your labours
Hither bring with one spirit, to let help come to the weary
King Alfonso[5], who, as successor to Atlas his neighbour,
Bore the weight in like manner, with forces not up to the effort;
Likewise, to let great Copernicus sense the help that is ready,
Lest, as he gives himself to the Herculean labour, approaching
Trustingly, he might succumb to the burden exceedingly heavy,
Causing the poles, wanting Atlas and Hercules, pillars so mighty,
Already starting to nod, to bring on tremendous disasters,
Even so far as to move at the same time the earth from its station*,
Throwing disorder on simplistic tolerance (stupid because it
Comes from dull ignorance of the heavens), and, in a double
Downfall, on all of humanity, with all the wild beasts and cattle,
Mingling the courts of the world with blind darkness and primeval
Chaos.
Do not accept this obscenity: battle such decadent ruin,
Come with me to ascend Olympus with forces redoubled,
Let us now hasten to close the cracks that have lately been broken,
Firm up the coffered ceiling of heaven with sturdy new crossbeams:
Now is the time to act, ere the whole machine tumbles to pieces.
(**) 6 r Is there then anyone here who would wish to acquire this crown,
Beautiful for its pure gold, and striking with ivory, gems, and
Red-glowing bronze, but stronger, enduring throughout all the ages,

* Meaning, that the poles go to ruin. For this passage indicts the imperfection of astronomy, and ignorance of it, but not the hypothesis of Copernicus, which makes the earth mobile.

[5] Alfonso X, 'the Wise', (1221–1284), King of Castile and Leon, for whom the Alphonsine Tables were prepared. These were the standard planetary tables of the Middle Ages and the early Renaissance, and were still widely used in Kepler's time, although superseded by Erasmus Reinhold's *Prutenicae tabulae coelestium motuum* (Tubingen, 1551). According to legend, Atlas was supposed to have been turned to stone in North Africa, becoming the Atlas Mountains; hence, he was Alfonso's 'neighbour'.

Furthermore wishing to mingle his soul with the souls of the highest?
Will there be any among the numberless earth-dwellers that the
Orb holds, who has accepted things so sublime in his heart?
Is there any who strives to acknowledge the Author of All, for
So many marvellous spectacles set in the measureless heavens?
Do you not all as one keep silent about such great questions?
What use is this silence? The hand must be set to the labour,
Bringing to light at last the heavens' recondite secrets.
Should you be stayed by ambition, by ignorance, laxness, or lucre,
From such lofty attempts, driven down to much lower endeavours,
Do not be harsh with the others: withhold not your utmost assistance.
I myself, if divine wills give me their favouring glances,
Letting me (as hitherto) surmount any hindrance whatever,
Alas. Lofty and constant in spirit, – I will exert myself further,
Stretching each sinew to open great heaven's innermost secrets
Unto the natives of earth, revealing its roofed-in recesses.
Nod in wondrous way thine assent, O Olympus's sapient
Founder, and grant thine aid to one heeding thy deeds so astounding.

*The Author of the Work replies:*

O hero refulgent in lineage, brilliant in highness of birth, to
Whom the undoubted celestial source of your spirit has granted
Precedence over the rest in achievement, and effort in song and
Exhortation, granting new life to those who are dying:
Why are you scourging with wind and flame the soul, that so lately
Fed such great conflagrations, aroused by the things it had wished for?
For although such great undertakings, surpassing my forces,
No other masters demand than those that thy Muse bears; and
Nature according to law gave me in my generation[6]
Less intellect than spirit, than intellect less strength of arm:
Nevertheless, the Ninth Sister 'inspired a divine love of heaven'[7]
Ominous love, to what does it not drive the spirits of mortals?[8]
Intellect it gave me, and vigourous arms it gave me,
Vivifying with hope disproportionate. Yet you and I are
Sundered by unfair Juno's unequal countenance into

[6] Reading '*Nascendo*' for '*Nascendi*'.
[7] Cf. Tycho's 'Hortatory Ode', line 10. The 'Ninth Sister' is of course Urania, muse of astronomy.
[8] Cf. Virgil, *Aeneid*, III 56 and IV 412.

Contrary aims: to you she gave the means of acquiring
Power; to me the harsh one denies it: the cunning reverts to
Her; she keeps me from places aethereal, also more closely
Watching the holy Fires, stung by Prometheus' thieving.
Thus she indulgently heaps you with riches, bedazzling the eyes with
Lustre of metal, to make them more slothful in seeking the Caelian
Lights, and to make them prefer to cleave to the purple-clad pomp that
Flattering whispered praise of the popular whim ever follow.
(**) 6 v Fortune, when spurned, would threaten to bring on unspeakable sorrow.
Blessed be for the strength of your spirit, O victor of Gods and
Humans, and of your own character, you who approve of things that
Are to be striven for by Reason's eyelet: having pursued this,
Ceaselessly daring, you could spurn patrimonial riches.
Summon no longer your comrades to utter these praises in private,
Write not your words on the waters. Virtue and treasure do not make
Friendly companions: the earth and the Pole are enormously far from
One another, and one is easily viewed from the other.
Scornful of me, the potent goddess begrudged me honour
Measureless. Binding me closely by means of her almighty wishes,
She has bestowed on me nothing that I could scornfully treat as
Second in worth to the Muses, or to block astronomical studies.
Loathing might have won out, might have hindered feats of great daring;
Envious Nemesis, too, on the ground, might have weighed down the mental
Faculties, with their potential to fly through heaven's high reaches;
But that the love of singing heaven's mysteries, in which
I had followed your footsteps, forestalled me at life's first crossroads.
Therefore, surveying in thought the well-worn paths of the planets,
Portents immense, and the walls of the world bound for ruin because of
Great gaping spaces that never were braced by the placement of columns,
Causes meanwhile hid by Stygian night, while, sure that they're right, the
Crowd of sages slumbers, not knowing the Prussian Master[9],

[9] Copernicus

Trustingly I approach to take up such a great weight, and
'Firm up the coffered ceiling of heaven with sturdy new cross-beams:'[10]
Famous timbers from Samos, the five regular solids:
Euclid provided the measure, and Pallas renowned the mind.
Cheers and applause redoubled, from more than one commentator
Sang to Urania the brilliant triumph, pleased it's successful.
  Wonderful is your daring, O Brahe, and sweet your labour,
Even though you preferred not to stray from received opinion,
Doubting much above earth, and many things in the heavens.
Nevertheless I was pleased to be numbered among your disciples,
Spreading your nights out before me, and secrets you've found in your searching,
Over long years, and to shine a clear light on your great undertakings.
  Would that you had lived on, that the Fates had never snatched from you
Prizes matching such deeds, and such well-merited triumphs.
No other orbits spread themselves out for your vision or subtle
Instruments, other than those that are buttressing my new cross-beams:
You would have been the expert on 'great heaven's innermost secrets'[11].
  Now, when the Goddesses have carried off our too-hasty Master,
Taken away our patron, whom they were to have gladdened,
Who has upset the festive day, the ornate celebration,
What can I do but in veneration call you again to
Life, O manifest Hero, in the image of Mind the maker.
You will stand near, a mighty Priest in star-studded vestments,
(***) r
Here where the offerings heaped up to God the author in the
Temple cerulean rise, while the six curved paths move around in
Order; the same number moved by the lamp's most rapid gyration;
And, in the middle, the Hearth[12], and flames of eternal Light.

The opinion of Aristarchus and Copernicus.

[10] Cf. 'Hortatory Ode', line 35.
[11] 'Hortatory Ode', line 54.
[12] Latin, *Focus*. It is fascinating that, although Kepler had already applied this term to the ellipse in the *Astronomiae pars optica* of 1604, this is the sole appearance of the word in the *New Astronomy*. Even as he constructed the elliptical orbit (in chapter 59) he was apparently unaware that the sun was at the point that had the focal property. Could it be that, before writing this poem (which was probably written after the rest of the work), Kepler realized the connection? Or, still more fascinating, could the use of the word here in a poetical context have reminded him of the technical meaning he had given it earlier, leading him to discover the connection?

Suppliant, I approach the parent of things, with these my
Labours, inscribed in a learned book, 'tis frankincense sweetest,
Sweated from your trees, under my care collected, while you
Patiently waited. To you, with hands raised on high, I give it.
Heia! O offering pure, behold! I follow, with mighty
Cry, and join in chaste prayer: 'May Olympus's sapient
Founder grant his kind aid to one heeding his deeds stupefying'[13].

*The same author's Elegy written in the friendship book*[14]
*next to the hand and Motto of Brahe:*
*'In looking up I look down.'*

Generously make room, and do not disdain one who follows:
  All that I am is your gift, and all that I will be.
You who have marvelled at all of humanity's empty worries,
  Only in you so far have I not played the Satyr.
All your monuments give me rest from my worries: without you
  I was a shadow; with you now as father, a body.
Though for me the earth swims starlike in airy gyrations,
  You see the same earth stand fixed in the place that is central:
I am inclined to ascribe these beliefs to the Ancient Masters.
  They were not mine: they displeased you while you were living.
Nevertheless, I, weak as I am, only set out these lights* to
  Mars's rutilent fires by your nights' labours.
Only by 'looking up' could you rightly adjust your dioptra,
  I would 'look down' upon earth's course from there.
I would measure swift paces, Libra confronting the Goat, and,
  Phoebe[15], the ratio giving to you the path's centre,
That by a similar step it might seek and flee from the sun, while
  Nonetheless not being spun with a spin like the Lord's, but
Gathering forces as it approaches its source, and in turn,
  Languishing as it retires to more distant places;
Whence the seven Globes are borne by the sevenfold Minds in
  Sequence, and by the eighth soul, from Father Sun.
Nature thus is exempted from numberless twistings and turnings:

*As if in the diagram on p. 149 [of the first edition] of this work, an eye were depicted at the letter η (the star Mars).

[13] Cf. 'Hortatory Ode', lines 56-7. The Latin, like the English, is slightly different in the two passages.

[14] Latin, '*Philothesio*'. This puzzling word does not appear in classical Latin, nor (despite its obvious Greek origin) in any ancient Greek text. It seems to be a compound formed from '*philos*' and '*thesis*'. If so, it would most likely mean something like 'setting down the friend'. Caspar translates this '*Freundschaftsbuch*', literally, 'friendship book', a reading which is at least plausible.

[15] The moon.

*Arist. *Metaph.* XII. 8.

Nine times five* thus depart from the family of God.
Cheat against reason in tens, O Tycho; cheat it by minutes,
Which none but you would number: the whole would collapse.
O human cares – the amount of vanity in our affairs! To
Think that the stars can't be reached by a different road.

(***) v

*The same author's epigram on the studies of Tycho Brahe*

Tycho has described the course of the sun and the fixed stars,
To this he joined the Moon's circuit, ere he died.
Phaeton grieves when ascending his four-horse, light-bearing chariot.
This artful care does you no harm, O Tycho:
Loving Diana, Endymion slept with a slumber eternal;
Love of the Goddess gives you too a sleep everlasting.

**To the reader**[16]

**GR.**

I had decided to say many things to you (reader), but the heap of political business, by which I am detained more than usual these days, and the over-hasty departure of our Kepler, at this very moment about to go to Frankfurt, has hardly left me even this little scrap of an occasion to write. And so I thought I should give you just three words' warning, lest you be moved by anything of Kepler's, but especially his liberty in disagreeing with Brahe in physical arguments, groundlessly complicating the work on the *Rudolphine Tables*, and familiar to all philosophers from the creation of the universe to the present. As for the rest, you will come to know from the work itself that it has been constructed upon Brahe's foundation, that is, upon his restitution of the fixed stars and the sun, and that all the heaped up materials (observations, that is) were the works of Brahe. Meanwhile, by this important work of Kepler's, appearing among these tumults of rebellion and wars sprouting again from time to time, while the

16 The author of this preface was Tycho's son-in-law, who, through his position as Tycho's heir and his influence with the Emperor, delayed the publication of the *New Astronomy* for several years. Although for some time he succeeded in preventing Kepler from publishing anything without his consent, he finally agreed to allow Kepler's work to appear on condition that he be allowed to write a preface stating what he saw as the correct Tychonic position. Clearly, he postponed the writing until the last possible moment – the incoherence and jumbled syntax, preserved in the translation, make this plain even without Tengnagel's lame excuses. The result is a comical ending to a profoundly aggravating situation. As Arthur Koestler put it, '. . . in Tengnagel's preface to the *New Astronomy*, we hear the braying of a pompous ass echoing down the centuries.'

Republic of letters shows compassion for it all, it is as if we were by this fact blessed with a precursor of the Tables, and after them the observations (seeing the light of day later); and, more zealous for the further progress of the work so fervently desired, also pray with us to God for more felicitous times.

*Franz Gansneb Tengnagel von Campp.*
His Imperial Majesty's Councillor

(***) 2 r

# Introduction to this work

On the difficulty of reading and writing astronomical books.

It is extremely hard these days to write mathematical books, especially astronomical ones. For unless one maintains the truly rigourous sequence of proposition, construction, demonstration, and conclusion[1], the book will not be mathematical; but maintaining that sequence makes the reading most tiresome, especially in Latin, which lacks the articles and that gracefulness possessed by Greek when it is expressed in written symbols. Moreover, there are very few suitably prepared readers these days: the rest generally reject such works. How many mathematicians are there who put up with the trouble of working through the *Conics* of Apollonius of Perga? And yet that subject matter is the sort of thing which can be expressed much more easily in diagrams and lines than can astronomy.

I myself, who am known as a mathematician, find my mental forces wearying when, upon rereading my own work, I recall from the

[1] Kepler here refers to the formal procedure of Euclidean geometry. First, the theorem is stated in its general form; for example (in Euclid I. 17), 'In any triangle two angles taken together in any manner are less than two right angles'. Then, the appropriate construction is performed and the theorem is restated as expressed in the construction. For example, 'Let ABC be a triangle; I say that two angles of the triangle ABC taken together in any manner are less than two right angles.' Next, the theorem is demonstrated. And finally, it is restated exactly as originally proposed, with the word 'therefore' inserted at the beginning, and a phrase or abbreviation at the end indicating that this is what was to be proven (English translations use the abbreviation 'Q. E. D.', which stands for '*quod erat demonstrandum*', the Latin equivalent of Euclid's phrase, 'ὅπερ ἔδει δεῖχαι'). Since it is understood that the conclusion is a restatement of the proposition, it is usual to omit the full conclusion, representing it briefly with the words, 'Therefore etc. Q. E. D.'. Although Kepler occasionally reverts to this formality (as in chapter 60 proposition 4), he usually omits it, for the reason given here. However, he distinguishes the mathematical arguments from the rest of the text by having the mathematics set in italics, as he explains in the opening section of the 'Summaries of the Individual chapters'.

diagrams the sense of the proofs, which I myself had originally introduced from my own mind into the diagrams and the text. But then when I remedy the obscurity of the subject matter by inserting explanations, it seems to me that I commit the opposite fault, of waxing verbose in a mathematical context.

Furthermore, prolixity of phrases has its own obscurity, no less than terse brevity. The latter evades the mind's eye while the former distracts it; the one lacks light while the other overwhelms with superfluous glitter; the latter does not arouse the sight while the former quite dazzles it.

These considerations led me to the idea of including a kind of elucidating introduction to this work, to assist the reader's comprehension as much as possible.

I conceived this introduction as having two parts. In the first I present a synoptic table of all the chapters in the book. I think this is going to be useful, because the subject matter is unfamiliar to most people, and the various terms and various procedures used here are very much alike, and are closely related, both in general and in specific details. So when all the terms and all the procedures are juxtaposed and presented in a single display, they will be mutually explanatory. For example, I discuss the natural causes which led the ancients, though ignorant of them, to suppose an equant circle or equalizing point. However, I do this in two places, namely, in parts three and four. A reader who encounters this subject in part three might think I am dealing here with the first inequality, which is a property of the motions of each of the planets individually. And indeed, this is the case in part four. However, in the third part, as the summary indicates, I am discussing that equant which, under the name of the second inequality, varies the motion of all the planets in common, and primarily governs the theory of the sun. Thus the synoptic table will serve to make this distinction clear.

Nevertheless, the synopsis will not be of equal assistance to all. There will be those to whom this table (which I present as a thread leading through the labyrinth of the work) will appear more tangled than the Gordian Knot. For their sake, therefore, there are many points that should be brought together here at the beginning which are presented bit by bit throughout the work, and are therefore not so easy to attend to in passing. Furthermore, I shall reveal, especially for the sake of those professors of the physical sciences who are irate with me, as well as with Copernicus and even with the remotest antiquity,

The introduction to this work is aimed at those who study the physical sciences.

on account of our having shaken the foundations of the sciences with the motion of the earth – I shall, I say, reveal faithfully the intent of the principal chapters which deal with this subject, and to propose for inspection all the principles of the proofs upon which my conclusions, so repugnant to them, are based.

For when they see that this is done faithfully, they will then have the free choice either of reading through and understanding the proofs themselves with much exertion, or of trusting me, a professional mathematician, concerning the sound and geometrical method presented. Meanwhile, they, for their part, will turn to the principles of the proofs thus gathered for their inspection, and will examine them thoroughly, knowing that unless they are refuted the proof erected upon them will not topple. I shall also do the same where, as is customary in the physical sciences, I mingle the probable with the necessary and draw a plausible conclusion from the mixture. For since I have mingled celestial physics with astronomy in this work, no one should be surprised at a certain amount of conjecture. This is the nature of physics, of medicine, and of all the sciences which make use of other axioms besides the most certain evidence of the eyes.

On the schools of thought in astronomy.

The reader should be aware that there are two schools of thought among astronomers, one distinguished by its chief, Ptolemy, and by the assent of the large majority of the ancients, and the other attributed to more recent proponents, although it is the most ancient. The former treats the individual planets separately and assigns causes to the motions of each in its own orb, while the latter relates the planets to one another, and deduces from a single common cause those characteristics which are found to be common to their motions. The latter school is again subdivided. Copernicus, with Aristarchus of remotest antiquity, ascribes to the translational motion of our home the earth the cause of the planets' appearing stationary and
(***) 2 v retrograde. Tycho Brahe, on the other hand, ascribes this cause to the sun, in whose vicinity he says the eccentric circles of all five planets are connected as if by a kind of knot (not physical, of course, but only quantitative). Further, he says that this knot, as it were, revolves about the motionless earth, along with the solar body.

For each of these three opinions concerning the world[2] there are several other peculiarities which themselves also serve to distinguish

[2] *Mundus*, in Latin. This comprises the entire corporeal universe, including the fixed stars.

these schools, but these peculiarities can each be easily altered and amended in such a way that, so far as astronomy, or the celestial appearances, are concerned, the three opinions are for practical purposes equivalent to a hair's breadth, and produce the same results.

The twofold aim of the work.

My aim in the present work is chiefly to reform astronomical theory (especially of the motion of Mars) in all three forms of hypotheses, so that our computations from the tables correspond to the celestial phenomena. Hitherto, it has not been possible to do this with sufficient certainty. In fact, in August of 1608, Mars was a little less than four degrees beyond the position given by calculation from the Prutenic tables. In August and September of 1593 this error was a little less than five degrees, while in my new calculation the error is entirely suppressed.

On the physical causes of the motions.

Meanwhile, although I place this goal first and pursue it cheerfully, I also make an excursion into Aristotle's *Metaphysics*, or rather, I inquire into celestial physics and the natural causes of the motions. The eventual result of this consideration is the formulation of very clear arguments showing that only Copernicus's opinion concerning the world (with a few small changes) is true, that the other two are false, and so on.

Indeed, all things are so interconnected, involved, and intertwined with one another that after trying many different approaches to the reform of astronomical calculations, some well trodden by the ancients and others constructed in emulation of them and by their example, none other could succeed than the one founded upon the motions' physical causes themselves, which I establish in this work.

The first step towards those causes is taken. The planes of all six eccentrics intersect at a single point, namely, the centre of the solar body.

Now my first step in investigating the physical causes of the motions was to demonstrate that [the planes of] all the eccentrics intersect in no other place than the very centre of the solar body (not some nearby point), contrary to what Copernicus and Brahe thought. If this correction of mine is carried over into the Ptolemaic theory, Ptolemy will have to investigate not the motion of the centre of the epicycle, about which the epicycle proceeds uniformly[3], but the motion of some point whose distance from that centre bears the same

[3] In the Ptolemaic theory of Mars, the epicycle represents the sun's motion, which is carried out uniformly about the epicycle's centre. Although this would not have to change, the line of intersection of the planes of the epicycle and the eccentric would in general not pass through the centre of the epicycle, but through the other nearby point described by Kepler. Thus when he says 'about which the epicycle proceeds uniformly', he means that the planet is proceeding uniformly on the epicycle about that point.

ratio to the diameter [of the eccentric] as does the distance of the centre of the solar orb from the earth for Ptolemy, which point is also on the same line, or one parallel to it.

Here the Braheans could have raised the objection against me that I am a rash innovator, for they, while holding to the opinion received from the ancients and placing the intersection of the [planes of the] eccentrics not in the sun but near the sun, nevertheless construct on this basis a calculation that corresponds to the heavens. And in translating the Brahean numbers into the Ptolemaic form, Ptolemy could have said to me that as long as he upheld and expressed the phenomena, he would not consider any eccentric other than the one described by the centre of the epicycle, about which the epicycle proceeds uniformly. Therefore I have to look again and again at what I am doing, so as to avoid setting up a new method which would not do what was already done by the old method.

So to counter this objection, I have demonstrated in the first part of the work that exactly the same things can result or be presented by this new method as are presented by their ancient method.

In the second part of the work I take up the main subject, and describe the positions of Mars at apparent opposition to the sun, not worse, but indeed much better, with my method than they expressed the positions of Mars at mean opposition to the sun with the old method.

Meanwhile, throughout the entire second part (as far as concerns geometrical demonstrations from the observations) I leave in suspense the question of whose procedure is better, theirs or mine, seeing that we both match a number of observations (this is, indeed, a basic requirement for our theorizing). However, my method is in agreement with physical causes, and their old one is in disagreement, as I have partly shown in the first part, especially chapter 6.

But finally in the fourth part of the work, in chapter 52, I consider certain other observations, no less trustworthy than the previous ones were, which their old method could not match, but which mine matches most beautifully. I thereby demonstrate most soundly that Mars's eccentric is so situated that the centre of the solar body lies upon its line of apsides, and not any nearby point, and hence, that all the [planes of the] eccentrics intersect in the sun itself.

This should, however, hold not just for the longitude, but for the latitude as well. Therefore, in the fifth part I have demonstrated the same from the observed latitudes, in chapter 67.

(***) 3r

The second step towards the physical causes of the motions is taken: there is also an equant in the theory of the sun or the earth, and thus the solar eccentricity must be bisected.

This could not have been demonstrated earlier in the work, because one of the constituents of these astronomical demonstrations is an exact knowledge of the causes of the second inequality in the planet's motion, for which some other new thing had likewise to be discovered in the third part, unknown to our predecessors, and so on.

For I have demonstrated in the third part that whether the old method, which depends upon the sun's mean motion, is valid, or my new one, which uses the apparent motion, nevertheless, in either case there is something from the causes of the first inequality that is mixed in with the second, which pertains to all planets in common. Thus for Ptolemy I have demonstrated that his epicycles do not have as centres those points about which their motion is uniform. Similarly for Copernicus I have demonstrated that the circle in which the earth is moved around the sun does not have as its centre that point about which its motion is regular and uniform. Similarly for Tycho Brahe I have demonstrated that the circle on which the common point or knot of the eccentrics mentioned above moves does not have as its centre that point about which its motion is regular and uniform. For if I concede to Brahe that the common point of the eccentrics may be different from the centre of the sun, he must grant that the circuit of that common point, which in magnitude and period exactly equals the orbit of the sun, is eccentric and tends towards Capricorn, while the sun's eccentric circuit tends towards Cancer. The same thing befalls Ptolemy's epicycles.

However, if I place the common point or knot of the eccentrics in the centre of the solar body, then the common circuit of both the knot and the sun is indeed eccentric with respect to the earth, and tends towards Cancer, but by only half the eccentricity shown by the point about which the sun's motion is regular and uniform.

And in Copernicus, the earth's eccentric still tends towards Capricorn, but by only half the eccentricity of the point about which the earth's motion is uniform, also in the direction of Capricorn.

Likewise, in Ptolemy, on each of the diameters of the epicycles that run from Capricorn to Cancer, there are three points, the outer two of which are at the same distance from the middle ones; and their distances from one another have the same ratio to the diameters as the whole eccentricity of the sun has to the diameter of its circuit. And of these three points, the middle ones are the centres of their epicycles, those that lie toward Cancer are the points about which the

motions on the epicycles are uniform, and finally those that lie toward Capricorn are the ones whose eccentrics (described by them) we would be tracing out *if instead of the sun's mean motion we follow the apparent motion*, just as if those were the points at which the epicycles were attached to the eccentric. The result of this is that each planetary epicycle contains the theory of the sun in its entirety, with all the properties of its motions and circles.

The earth is moved and the sun stands still. Physico-astronomical arguments.

With these things thus demonstrated by a reliable method, the previous step towards the physical causes is now confirmed, and a new step is taken towards them, most clearly in the theories of Copernicus and Brahe, and more obscurely but at least plausibly in the Ptolemaic theory.

For whether it is the earth or the sun that is moved, it has certainly been demonstrated that the body that is moved is moved in a nonuniform manner, that is, slowly when it is farther from the body at rest, and more swiftly when it has approached this body.

Thus the physical difference is now immediately apparent, by way of conjecture, it is true, but yielding nothing in certainty to conjectures of doctors on physiology or to any other natural science.

First, Ptolemy is certainly exploded. For who would believe that there are as many theories of the sun (so closely resembling one another that they are in fact equal) as there are planets, when he sees that for Brahe a single solar theory suffices for the same task, and it is the most widely accepted axiom in the natural sciences that Nature makes use of the fewest possible means?

That Copernicus is better able than Brahe* to deal with celestial physics is proven in many ways.

First, although Brahe did indeed take up those five solar theories from the theories of the planets, bringing them down to the centres of the eccentrics, hiding them there, and conflating them into one, he nevertheless left in the world the effects produced by those theories. For Brahe no less than for Ptolemy, besides that motion which is proper to it, each planet is still actually moved with the sun's motion, the two being mixed into one, the result being a spiral. That it results from this that there are no solid orbs, Brahe has demonstrated most firmly. Copernicus, on the other hand, entirely removed this extrinsic motion from the five planets, assigning its cause to a deception arising from the circumstances of observation. Thus the motions are still multiplied to no purpose by Brahe, as they were before by Ptolemy.

*Of whom, in all fairness, most honest and grateful mention is made, and recognition given, since I build this entire structure from the bottom up upon his work, all the materials being borrowed from him.

II. Second, if there are no orbs[4], the conditions under which the intelligences and moving souls must operate are made very difficult, since they have to attend to so many things to introduce to the planet two intermingled motions. They would at least have to attend at one and the same time to the principles, centres, and periods of the two motions. But if the earth is moved, I show that most of this can be done with physical rather than animate faculties[5], namely, magnetic ones. But these are more general points. There follow others arising specifically from demonstrations, upon which we now begin.

(***) 3 v

III. For if the earth is moved, it has been demonstrated that the increases and decreases of its velocity are governed by its approaching towards and receding from the sun. And in fact the same happens with the rest of the planets: they are urged on or held back according to the approach toward or recession from the sun. So far, the demonstration is geometrical.

And now, from this very reliable demonstration, the conclusion is drawn, using a physical conjecture, that the source of the five planets' motion is in the sun itself. It is therefore very likely that the source of the earth's motion is in the same place as the source of the other five planets' motion, namely, in the sun as well. It is therefore likely that the earth is moved, since a likely cause of its motion is apparent.

IV. That, on the other hand, the sun remains in place in the centre of the world, is most probably shown by (among other things) its being the source of motion for at least five planets. For whether you follow Copernicus or Brahe, the source of motion for five of the planets is in the sun, and in Copernicus, for a sixth as well, namely, the earth. And it is more likely that the source of all motion should remain in place rather than move.

[4] It is telling that Kepler omits the word 'solid' here. He implies that if there are any orbs, they must be solid (that is, hard and impenetrable). Earlier natural philosophers would have been less likely to have jumped to this conclusion. Instead it was commonly held that the orbs might be made of some substance so utterly unlike terrestrial matter that they could not be characterized as either solid or not solid. Indeed, there was considerable doubt whether the heavens were made of any material at all. For further discussion of the orbs in astronomy and natural philosophy, see Nicholas Jardine, *The Birth of History and Philosophy of Science*, Cambridge University Press, Cambridge, 1984, and 'The Significance of the Copernican Orbs', *Journal for the History of Astronomy* 13 (1982), pp. 168–194, as well as the translator's essay, 'The Solid Planetary Spheres in Post-Copernican Natural Philosophy', in Robert S. Westman, editor, *The Copernican Achievement*, University of California Press, Berkeley, 1975.

[5] Latin, *facultas animalis*, a term borrowed from physiology and psychology, whose meaning varies from author to author. It is clear from chapter 57 of this work that Kepler held to a threefold division of faculties: natural, animal, and mental. These appear to correspond to Aristotle's three faculties of soul: vegetative, sensitive, and rational. If so, then the animate faculty would govern both perception and motion from place to place. See chapter 57 footnote 9, and Aristotle's *On the Soul*, book I chapter 2. Also, footnotes 32 and 33 below.

V. But if we follow Brahe's theory and say that the sun moves, this first conclusion still remains valid, that the sun moves slowly when it is more distant from the earth and swiftly when it approaches, and this not only in appearance, but in fact. For this is the effect of the circle of the equant, which, by an inescapable demonstration, I have introduced into the theory of the sun.

Upon this most valid conclusion, making use of the physical conjecture introduced above, might be based the following theorem of natural philosophy: the sun, and with it the whole huge burden (to speak coarsely) of the five eccentrics, is moved by the earth; or the source of the motion of the sun and the five eccentrics attached to the sun is in the earth.

Now let us consider the bodies of the sun and the earth, and decide which is better suited to being the source of motion for the other body. Does the sun, which moves the rest of the planets, move the earth, or does the earth move the sun, which moves the rest, and which is so many times greater? Unless we are to be forced to admit the absurd conclusion that the sun is moved by the earth, we must allow the sun to be fixed and the earth to move.

VI. What shall I say of the motion's periodic time of 365 days, intermediate in quantity between the periodic time of Mars of 687 days and that of Venus of 225 days? Does not the nature of things cry out with a great voice that the circuit in which these 365 days are used up[6] also occupies a place intermediate between those of Mars and Venus about the sun, and thus itself also encircles the sun, and hence, that this circuit is a circuit of the earth about the sun, and not of the sun about the earth? These points are, however, more appropriate to my *Mysterium cosmographicum,* and arguments that are not going to be repeated in this work should not be introduced here.

VII. For other metaphysical arguments that favour the sun's position in the centre of the world, derived from its dignity or its illumination, see my little book just mentioned, or look in Copernicus. There is also something in Aristotle's *De coelo*, Book II, in the passage on the Pythagoreans, who used the name 'fire' to signify the sun. I have touched upon a few points in the *Astronomiae pars optica* ch. 1 p. 7, and also ch. 6, especially p. 225.

[6] Behind this odd phrase lies Kepler's peculiar treatment of time as a dependent variable: he makes consistent use of the amount of time to traverse a unit of distance, rather than considering the distance traversed in a unit of time (as Galileo and his successors did). It is quite likely that this different viewpoint was of importance in developing the 'area law' which later became known as Kepler's Second Law. See especially the beginning of chapter 40.

VIII. But on the earth's being suited to a circular motion in some place other than the centre of the world, you will find a metaphysical argument in chapter 9 p. 322 of that book.

Objections to the earth's motion.

I trust the reader's indulgence if I take this opportunity to present a few brief replies to a number of objections which, capturing men's minds, use the following arguments to shed darkness. For these replies are by no means irrelevant to matters that concern the physical causes of the planets' motion, which I discuss chiefly in parts three and four of the present work.

I. On the motion of heavy bodies.

Many are prevented by the motion of heavy bodies from believing that the earth is moved by an animate motion, or better, by a magnetic one. They should ponder the following propositions.

The theory of gravity is in error.

A mathematical point, whether or not it is the centre of the world, can neither effect the motion of heavy bodies nor act as an object towards which they tend. Let the physicists prove that this force is in a point which neither is a body nor is grasped otherwise than through mere relation.

It is impossible that, in moving its body, the form of a stone seek out a mathematical point (in this instance, the centre of the world), without respect to the body in which this point is located. Let the physicists prove that natural things have a sympathy for that which is nothing.

It is likewise impossible for heavy bodies to tend towards the centre of the world simply because they are seeking to avoid its spherical extremities. For, compared with their distance from the extremities of the world, the proportional part by which they are removed from the world's centre is imperceptible and of no account. Also, what would be the cause of such antipathy? With how much force and wisdom would heavy bodies have to be endowed in order to be able to flee so precisely an enemy surrounding them on all sides? Or what ingenuity would the extremities of the world have to possess in order to pursue their enemy with such exactitude?

Nor are heavy bodies driven in towards the middle by the rapid whirling of the *primum mobile*, as objects in whirlpools are. That motion (if we suppose it to exist) does not carry all the way down to these lower regions. If it did, we would feel it, and would be caught up by it along with the very earth itself; indeed, we would be carried ahead, and the earth would follow. All these absurdities are consequences of our opponents' view, and it therefore appears that the common theory of gravity is in error.

(***) 4 r

True theory of gravity.

The true theory of gravity rests upon the following axioms[7].

Every corporeal[8] substance, to the extent that it is corporeal, has been so made as to be suited to rest in every place in which it is put by itself, outside the sphere of influence of a kindred[9] body.

Gravity is a mutual corporeal disposition among kindred bodies to unite or join together; thus, the earth attracts a stone much more than the stone seeks the earth. (The magnetic faculty is another example of this sort).

Heavy bodies (most of all if we establish the earth in the centre of the world) are not drawn towards the centre of the world *qua* centre of the world, but *qua* centre of a kindred spherical body, namely, the earth. Consequently, wherever the earth be established, or whithersoever it be carried by its animate faculty, heavy bodies are drawn towards it.

If the earth were not round, heavy bodies would not everywhere be drawn in straight lines towards the middle point of the earth, but would be drawn towards different points from different sides.

If two stones were set near one another in some place in the world outside the sphere of influence of a third kindred body, these stones, like two magnetic bodies, would come together in an intermediate place, each approaching the other by an interval proportional to the bulk [*moles*] of the other.

If the moon and the earth were not each held back in its own circuit by an animate force or something else equivalent to it, the earth would ascend towards the moon by one fifty-fourth part of the interval, and the moon would descend towards the earth about fifty-three parts of the interval, and there they would be joined together; provided, that is, that the substance of each is of the same density.

[7] As Max Caspar notes in his edition of the *Astronomia Nova*, the theory presented in these terse statements constitutes a complete rejection of the Aristotelian view of gravity and plays a fundamental role in Kepler's physical thought. In later works Kepler refers back to them, especially in Book I part 4 of the *Epitome of Copernican Astronomy*, where he develops them further. Especially important in providing an insight into Kepler's early thoughts on the subject are his letters to David Fabricius of 11 October 1605 and 10 November 1608 (KGW 15 p. 240 and 16 p. 194). In the former, he likens gravity to magnetism, and says, '. . . not only does a stone approach the earth, but the earth also approaches the stone, and they divide the space between them in the inverse ratio of their weights.' Also illuminating is Kepler's letter to Herwart of January 1607 (KGW 15 p. 386).

[8] Latin, *corporea*. There is a close relation in Kepler's thought here between *corpus* (body) and *corporea* which is not made entirely clear in this translation. Other possible renderings would be 'physical' or 'bodily'. The former would not adequately represent Kepler's meaning in the first axiom, while the latter is sufficiently at odds with correct usage to lead the translator to reject it, though with some regret.

[9] Latin, *cognata*, 'of the same origin'.

If the earth should cease to attract its waters to itself, all the sea water would be lifted up, and would flow onto the body of the moon.

Reason for the ebb and flow of the sea.

The sphere of influence of the attractive power in the moon is extended all the way to the earth, and in the torrid zone calls the waters forth, particularly when it comes to be overhead in one or another of its passages. This is imperceptible in enclosed seas, but noticeable where the beds of the ocean are widest and there is much free space for the waters' reciprocation. It thus happens that the shores of the temperate latitudes are laid bare, and to some extent even in the torrid regions the neighbouring oceans diminish the size of the bays. And thus when the waters rise in the wider ocean beds, the moon being present, it can happen that in the narrower bays, if they are not too closely surrounded, the water might even seem to be fleeing the moon, though in fact they are subsiding because a quantity of water is being carried off elsewhere.

But the moon passes the zenith swiftly, and the waters are unable to follow so swiftly. Therefore, a current arises in the ocean of the torrid zone, which, when it strikes upon the far shores, is thereby deflected. But when the moon departs, this congress of the waters, or army on the march towards the torrid zone, now abandoned by the traction that had called it forth, is dissolved. But since it has acquired impetus, it flows back (as in a water vessel) and assaults its own shore, inundating it. In the moon's absence, this impetus gives rise to another impetus until the moon returns and the impetus is restrained, moderated, and carried along with the moon's motion. So all shores that are equally accessible are flooded at the same time, while those more remote are flooded later, some in different ways because of their various degrees of accessibility to the ocean.

Effects of the sea's ebb and flow.

I will point out in passing that the sand dunes of the Syrtes[10] are heaped up in this way; that thus are created or destroyed countless islands in bays full of eddies (such as the Gulf of Mexico); that it seems that the soft, fertile, and friable earth of the Indies was thus at length broached and penetrated by this current, this perpetual inundation, with help from a certain all-pervading motion of the earth. For it is said that India was once continuous from the Golden Chersonnese towards the east and south, but now the ocean, which was once farther back between China and America, has flowed in, and the shores of the Moluccas and of other neighbouring islands,

[10] Shoals off North Africa.

which are now raised on high because of the subsidence of the surface of the sea, bear witness[11] to this event.

The Taprobane of the ancients is lost today.

Taprobane[12], too, seems to have been submerged through this cause (as is consistent with the account of the Calcuttans that several localities there were once submerged), when the China Sea burst in through breaches into the Indian Ocean, with the result that nowadays nothing of Taprobane remains but the peaks of the mountains, which take the form of the innumerable islands known as the Maldives. For it is easy to prove, from the geographers and Diodorus Siculus, that that was once the site of Taprobane, namely, to the south opposite the mouths of the Indus and the promontory of Corium. Moreover, in ecclesiastical history one individual is said to have been bishop of Arabia and Taprobane together, and so the latter must surely have been nearby and not five hundred German miles to the east (indeed, more than a thousand, following the roundabout routes used in those days). The island of Sumatra, nowadays considered to be Taprobane, I think was once the Golden Chersonnese, joined to the Indian isthmus at the city of Malacca. For Chersonnesus[13], which nowadays we believe to be the Golden, seems to have no more right than Italy to the name 'Chersonnese'.

Although these things are appropriate to a different place, I wanted to present them all in one context in order to make more credible the ocean tide and through it the moon's attractive power.

For it follows that if the moon's power of attraction extends to the earth, the earth's power of attraction will be much more likely to extend to the moon and far beyond, and accordingly, that nothing that consists to any extent whatever of terrestrial material, carried up on high, ever escapes the grasp of this mighty power of attraction.

(***) 4 v

True theory of levity.

Nothing that consists of corporeal material is absolutely light. It is only comparatively lighter, because it is less dense, either by its own nature or through an influx of heat. By 'less dense' I do not just mean that which is porous and divided into many cavities, but in general that which, while occupying a place of the same magnitude as that occupied by some heavier body, contains a lesser quantity of corporeal material.

The motion of light things also follows from their definition. For it should not be thought that they flee all the way to the surface of the

11 Reading, with Caspar, 'approbant' for 'opprimunt'.

12 The island now known as Sri Lanka.

13 The Thracian Chersonnese (Gallipoli).

world when they are carried upwards, or that they are not attracted by the earth. Rather, they are less attracted than heavy bodies, and are thus displaced by heavy bodies, whereupon they come to rest and are kept in their place by the earth.

To the objection that objects projected vertically fall back to their places.

But even if the earth's power of attraction is extended very far upwards, as was said, nevertheless, if a stone were at a distance that was perceptible in relation to the earth's diameter, it is true that, the earth being moved, such a stone would not simply follow, but its forces of resistance would mingle with the earth's forces of attraction, and it would thus detach itself somewhat from the earth's grasp. In just the same way, violent motion detaches projectiles somewhat from the earth's grasp, so that they either run on ahead if they are shot eastwards, or are left behind if shot westwards, thus leaving the place from which they are shot, under the compulsion of force. Nor can the earth's revolving effect impede this violent motion all at once, as long as the violent motion is at its full strength.

But no projectile is separated from the surface of the earth by even a hundred thousandth part of the earth's diameter, and not even the clouds themselves, or smoke, which partake of earthy matter to the very least extent, achieve an altitude of a thousandth part of the semidiameter. Therefore, none of the clouds, smokes, or objects shot vertically upwards can make any resistance, nor, I say, can the natural inclination to rest do anything to impede this grasp of the earth's, at least where this resistance is negligible in proportion to that grasp. Consequently, anything shot vertically upwards falls back to its place, the motion of the earth notwithstanding. For the earth cannot be pulled out from under it, since the earth carries with it anything sailing through the air, linked to it by the magnetic force no less firmly than if those bodies were actually in contact with it.

When these propositions have been grasped by the understanding and pondered carefully, not only do the absurdity and falsely conceived physical impossibility of the earth's motion vanish, but it also becomes clear how to reply to the physical objections, however they are framed.

The opinion of Copernicus.

Copernicus preferred to think that the earth and all terrestrial bodies (even those cast away from the earth) are informed by one and the same motive soul, which, while rotating its body the earth, also rotates those particles cast away from it. He thus held it to be this soul, spread throughout the particles, that acquires force through

violent[14] motions, while I hold that it is a corporeal faculty (which we call gravity, or the magnetic faculty), that acquires the force in the same way, namely, through violent motions.

Nevertheless, this corporeal faculty is sufficient for anything removed from the earth: the animate faculty is superfluous.

II. To objections concerning the swiftness of the earth's motion.

Although many people fear the worst for themselves and for all earth's creatures on account of the extreme rapidity of this motion, they have no cause for alarm. On this point see my book, *De stella nova*, chapters 15 and 16, pp. 82 and 84[15]

III. To objections concerning the immensity of the heavens.

In the same place, you will find that full-sail voyage along the world's immense orbit, which is usually held to be unnatural, in objection to Copernicus. There it is demonstrated to be well-proportioned, and that, on the contrary, the speed of the heavens would become ill-proportioned and unnatural were the earth ordered to remain quite motionless in its place.

IV. To objections concerning the dissent of holy scripture, and its authority.

There are, however, many more people who are moved by piety to withhold assent from Copernicus, fearing that falsehood might be charged against the Holy Spirit speaking in the scriptures if we say that the earth is moved and the sun stands still[16].

But let them consider that since we acquire most of our information, both in quality and quantity, through the sense of sight, it is impossible for us to abstract our speech from this ocular sense. Thus, many times each day we speak in accordance with the sense of sight, although we are quite certain that the truth of the matter is otherwise. This verse of Virgil furnishes an example:

We are carried from the port, and the land and cities recede.[17]

Thus, when we emerge from the narrow part of some valley, we say that a great plain is opening itself out before us.

[14] A technical Aristotelian term, for which there is no satisfactory modern equivalent. Aristotle categorized all motions as being either 'natural' or 'violent', depending upon whether they are carried out in accordance with some inner principle or are caused by something external. Here, the 'natural' motion is the coming together of all terrestrial bodies, while the 'violent' motion is the separation of those bodies. Kepler is arguing that when a body is separated from kindred bodies, there is some faculty that brings into action a force tending to bring the bodies back together.

[15] In KGW I.

[16] The following arguments on the interpretation of scripture were to become the most widely read of Kepler's writings. They were often reprinted from the seventeenth century on, and translated into modern languages. Indeed, this part of the Introduction was the only work of Kepler's to appear in English before 1700.

[17] *Aeneid* III. 72. This line was also quoted by Copernicus, *De revolutionibus* I. 8.

Thus Christ said to Peter, 'Lead forth on high'[18], as if the sea were higher than the shores. It does seem so to the eyes, but optics shows the cause of this fallacy. Christ was only making use of the common idiom, which nonetheless arose from this visual deception.

Thus, we call the rising and setting of the stars 'ascent' and 'descent', though at the same time that we say the sun ascends, others say it descends. See the *Astronomiae pars optica* Ch. 10 p. 327[19].

Thus, the Ptolemaic astronomers even now say that the planets are stationary when they are seen to stay near the same fixed stars for several days, even though they think the planets are then really moving downwards in a straight line, or upwards away from the earth.

Thus writers of all nations use the word 'solstice', even though they in fact deny that the sun stands still.

Thus there has not yet been anyone so doggedly Copernican as to avoid saying that the sun is entering Cancer or Leo, even though he wishes to signify that the earth is entering Capricorn or Aquarius. And there are other like examples.

(***) 5 r Now the holy scriptures, too, when treating common things (concerning which it is not their purpose to instruct humanity), speak with humans in the human manner, in order to be understood by them. They make use of what is generally acknowledged, in order to weave in other things more lofty and divine.

No wonder, then, if scripture also speaks in accordance with human perception when the truth of things is at odds with the senses, whether or not humans are aware of this. Who is unaware that the allusion in Psalm 19 is poetical? Here, under the image of the sun, are sung the spreading of the Gospel and even the sojourn of Christ the Lord in this world on our behalf, and in the singing the sun is said to emerge from the tabernacle of the horizon like a bridegroom from his marriage bed, exuberant as a strong man for the race. Which Virgil imitates thus:

Aurora leaving Tithonus's saffron-coloured bed[20]

(The Hebrew poetry was, of course, earlier.)

The psalmodist was aware that the sun does not go forth from the

[18] Luke 5: 4. The Latin *altum* can mean either 'high' or 'deep'. However, Kepler cannot have been unaware that the original Greek verse unambiguously has the latter meaning, and hence must be charged with making a rather silly distortion in order to prove a point.

[19] In KGW 2 p. 281.

[20] *Aeneid* IV. 585.

horizon as from a tabernacle (even though it may appear so to the eyes). On the other hand, he considered the sun to move for the precise reason that it appears so to the eyes. In either case, he expressed it so because in either case it appeared so to the eyes. He should not be judged to have spoken falsely in either case, for the perception of the eyes also has its truth, well suited to the psalmodist's more hidden aim, the adumbration of the Gospel and also of the Son of God. Likewise, Joshua makes mention of the valleys against which the sun and moon moved[21], because when he was at the Jordan it appeared so to him. Yet each writer was in perfect control of his meaning. David was describing the magnificence of God made manifest (and Syracides with him), which he expressed so as to exhibit them to the eyes, and possibly also for the sake of a mystical sense spelled out through these visible things. Joshua meant that the sun should be held back in its place in the middle of the sky for an entire day with respect to the sense of his eyes, since for other people during the same interval of time it would remain beneath the earth.

But thoughtless persons pay attention only to the verbal contradiction, 'the sun stood still' versus 'the earth stood still', not considering that this contradiction can only arise in an optical and astronomical context, and does not carry over into common usage. Nor are these thoughtless ones willing to see that Joshua was simply praying that the mountains not remove the sunlight from him, which prayer he expressed in words conforming to the sense of sight, as it would be quite inappropriate to think, at that moment, of astronomy and of visual errors. For if someone had admonished him that the sun doesn't really move against the valley of Ajalon, but only appears to do so, wouldn't Joshua have exclaimed that he only asked for the day to be lengthened, however that might be done? He would therefore have replied in the same way if anyone had begun to present him with arguments for the sun's perpetual rest and the earth's motion.

Now God easily understood from Joshua's words what he meant, and responded by stopping the motion of the earth, so that the sun might appear to him to stop. For the gist of Joshua's petition comes to this, that it might appear so to him, whatever the reality might meanwhile be. Indeed, that this appearance should come about was not vain and purposeless, but quite conjoined with the desired effect.

[21] Joshua 10: 12 ff.

But see Chapter 10 of the *Astronomiae pars optica*, where you will find reasons why, to absolutely all men, the sun appears to move and not the earth: it is because the sun appears small and the earth large, and also because, owing to its apparent slowness, the sun's motion is perceived, not by sight, but by reasoning alone, through its change of distance from the mountains over a period of time. It is therefore impossible for a previously uninformed reason to imagine anything but that the earth, along with the arch of heaven set over it, is like a great house, immobile, in which the sun, so small in stature, travels from one side to the other like a bird flying in the air.

What absolutely all men imagine, the first line of holy scripture presents. 'In the beginning,' says Moses, 'God created the heaven and the earth,' because it is these two parts that chiefly present themselves to the sense of sight. It is as though Moses were to say to man, 'This whole worldly edifice that you see, light above and dark and widely spread out below, upon which you are standing and by which you are roofed over, has been created by God.'

In another passage, Man is asked whether he has learned how to seek out the height of heaven above, or the depths of the earth below[22], because to the ordinary man both appear to extend through equally infinite spaces. Nevertheless, there is no one in his right mind who, upon hearing these words, would use them to limit astronomers' diligence either in showing the contemptible smallness of the earth in comparison with the heavens, or in investigating astronomical distances. For these words do not concern measurements arrived at by reasoning. Rather, they concern real exploration, which is utterly impossible for the human body, fixed upon the land and drawing upon the free air. Read all of Chapter 38 of Job, and compare it with matters discussed in astronomy and in physics.

Suppose someone were to assert, from Psalm 24, that the earth is founded upon rivers, in order to support the novel and absurd philosophical conclusion that the earth floats upon rivers. Would it not be correct to say to him that he should regard the Holy Spirit as a divine messenger, and refrain from wantonly dragging Him into physics class? For in that passage the psalmodist intends nothing but what men already know and experience daily, namely, that the land, raised on high after the separation of the waters, has great rivers flowing through it and seas surrounding it. Not surprisingly, the same

[22] Jeremiah 31: 37.

figure of speech is adopted in another passage, where the Israelites sing that they were seated upon the waters of Babylon[23], that is, by the riverside, or on the banks of the Euphrates and Tigris.

If this be easily accepted, why can it not also be accepted that in other passages usually cited in opposition to the earth's motion we should likewise turn our eyes from physics to the aims of scripture?

(***) 5 v A generation passes away (says Ecclesiastes)[24], and a generation comes, but the earth stands forever. Does it seem here as if Solomon wanted to argue with the astronomers? No; rather, he wanted to warn men of their own mutability, while the earth, home of the human race, remains always the same, the motion of the sun perpetually returns to the same place, the wind blows in a circle and returns to its starting point, rivers flow from their sources into the sea, and from the sea return to the sources, and finally, as these men perish, others are born. Life's tale is ever the same; there is nothing new under the sun.

You do not hear any physical dogma here. The message is a moral one, concerning something self-evident and seen by all eyes but seldom pondered. Solomon therefore urges us to ponder. Who is unaware that the earth is always the same? Who does not see the sun return daily to its place of rising, rivers perennially flowing towards the sea, the winds returning in regular alternation, and men succeeding one another? But who really considers that the same drama of life is always being played, only with different characters, and that not a single thing in human affairs is new? So Solomon, by mentioning what is evident to all, warns of that which almost everyone wrongly neglects.

It is said, however, that psalm 104, in its entirety, is a physical discussion, since the whole of it is concerned with physical matters. And in it, God is said to have 'founded the earth upon its stability, that it not be laid low unto the ages of ages[25].' But in fact, nothing could be farther from the psalmodist's intention than speculation about physical causes. For the whole thing is an exultation upon the greatness of God, who made all these things: the author has composed a hymn to God the creator, in which he treats the world in order, as it appears to the eyes.

If you consider carefully, you will see that it is a commentary upon

[23] Psalm 137.
[24] Ecclestiastes 1: 4.
[25] The Latin of the Vulgate, quoted by Kepler, differs markedly from the Greek (and hence from most English translations) here.

the six days of creation in Genesis. For in the latter, the first three days are given to the separation of the regions: first, the region of light from the exterior darkness; second, the waters from the waters by the interposition of an extended region; and third, the land from the seas, where the earth is clothed with plants and shrubs. The last three days, on the other hand, are devoted to the filling of the regions so distinguished: the fourth, of the heavens; the fifth, of the seas and the air; and the sixth, of the land. And in this psalm there are likewise the same number of distinct parts, analogous to the works of the six days.

In the second verse, he enfolds the Creator with the vestment of light, first of created things, and the work of the first day.

The second part begins with the third verse, and concerns the waters above the heavens, the extended region of the heavens, and atmospheric phenomena that the psalmodist ascribes to the waters above the heavens, namely, clouds, winds, tornadoes, and lightning.

The third part begins with the sixth verse, and celebrates the earth as the foundation of the things being considered. The psalmodist relates everything to the earth and to the things that live on it, because, in the judgement of sight, the chief parts of the world are two: heaven and earth. He therefore considers that for so many ages now the earth has neither sunk nor cracked apart nor tumbled down, yet no one has certain knowledge of what it is founded upon.

He does not wish to teach things of which men are ignorant, but to recall to mind something they neglect, namely, God's greatness and potency in a creation of such magnitude[26], so solid and stable. If an astronomer teaches that the earth is carried through the heavens, he is not spurning what the psalmodist says here, nor does he contradict human experience. For it is still true that the land, the work of God the architect, has not toppled as our buildings usually do, consumed by age and rot; that it has not slumped to one side; that the dwelling places of living thing have not been set in disarray; that the mountains and coasts have stood firm, unmoved against the blast of wind and wave, as they were from the beginning. And then the psalmodist adds a beautiful sketch of the separation of the waters by the continents, and adorns his account by adding springs and the amenities that springs and crags provide for bird and beast. He also does not fail to mention the adorning of the earth's surface, included by Moses among the works of the third day, although the psalmodist derives it

[26] Cf. Virgil, *Aeneid*, I. 33

from its prior cause, namely, a humidification arising in the heavens, and embellishes his account by bringing to mind the benefits accruing from that adornment for the nurture and pleasure of humans and for the lairs of the beasts.

The fourth part begins with verse 20, and celebrates the work of the fourth day, the sun and the moon, but chiefly the benefit that the division of times brings to humans and other living things. It is this benefit that is his subject matter: it is clear that he is not writing as an astronomer here.

If he were, he would not fail to mention the five planets, than whose motion nothing is more admirable, nothing more beautiful, and nothing a better witness to the Creator's wisdom, for those who take note of it.

The fifth part, in verse 26, concerns the work of the fifth day, where he fills the sea with fish and ornaments it with sea voyages.

The sixth is added, though obscurely, in verse 28, and concerns the animals living on land, created on the sixth day. At the end, in conclusion, he declares the general goodness of God in sustaining all things and creating new things. So everything the psalmodist said of the world relates to living things. He tells nothing that is not generally acknowledged, because his purpose was to praise things that are known, not to seek out the unknown. It was his wish to invite men to consider the benefits accruing to them from each of these works of the six days.

Advice to astronomers. (***) 6 r

I, too, implore my reader, when he departs from the temple and enters astronomical studies, not to forget the divine goodness conferred upon men, to the consideration of which the psalmodist chiefly invites. I hope that, with me, he will praise and celebrate the Creator's wisdom and greatness, which I unfold for him in the more perspicacious explanation of the world's form, the investigation of causes, and the detection of errors of vision. Let him not only extol the Creator's divine beneficence in His concern for the well-being of all living things, expressed in the firmness and stability of the earth, but also acknowledge His wisdom expressed in its motion, at once so well hidden and so admirable.

Advice for idiots.

But whoever is too stupid to understand astronomical science, or too weak to believe Copernicus without affecting his faith, I would advise him that, having dismissed astronomical studies and having damned whatever philosophical opinions he pleases, he mind his own business and betake himself home to scratch in his own dirt patch,

abandoning this wandering about the world. He should raise his eyes (his only means of vision) to this visible heaven and with his whole heart burst forth in giving thanks and praising God the Creator. He can be sure that he worships God no less than the astronomer, to whom God has granted the more penetrating vision of the mind's eye, and an ability and desire to celebrate his God above those things he has discovered.

Commendation of the Brahean hypothesis.

At this point, a modest (though not too modest) commendation to the learned should be made on behalf of Brahe's opinion of the form of the world, since in a way it follows a middle path. On the one hand, it frees the astronomers as much as possible from the useless apparatus of so many epicycles and, with Copernicus, it includes the causes of motion, unknown to Ptolemy, giving some place to physical theory in accepting the sun as the centre of the planetary system. And on the other hand, it serves the mob of literalists and eliminates the motion of the earth, so hard to believe, although many difficulties are thereby insinuated into the theories of the planets in astronomical discussions and demonstrations, and the physics of the heavens is no less disturbed.

V. To objections concerning the authority of the pious.

So much for the authority of holy scripture. As for the opinions of the pious[27] on these matters of nature, I have just one thing to say: while in theology it is authority that carries the most weight, in philosophy it is reason. Therefore, Lactantius is pious, who denied that the earth is round[28], Augustine is pious, who, though admitting the roundness, denied the antipodes, and the Inquisition[29] nowadays is pious, which, though allowing the earth's smallness, denies its motion. To me, however, the truth is more pious still, and (with all due respect for the Doctors of the Church) I prove philosophically not only that the earth is round, not only that it is inhabited all the way around at the antipodes, not only that it is contemptibly small, but also that it is carried along among the stars.

But enough about the truth of the Copernican hypothesis. Let us return to the plan I proposed at the beginning of this introduction.

[27] Latin, *Sancti*, literally, 'the Saints', or 'the Holy'. Context shows, however, that in modern usage 'pious' or 'saintly' fits Kepler's meaning better, even though it does miss his verbal play on *Sanctum Officium* (see note 29, below).

[28] In *De Revolutionibus*, in his dedicatory letter to Pope Paul III, Copernicus also mentions Lactantius as a revered theologian whose cosmological opinions are acknowledged as false. See Lactantius, *Institut. Divin.*, III. 24, and Augustine, *The City of God*, XVI. 9.

[29] Latin, *Officium*, the so-called 'Holy Office', by which name the Inquisition was officially known. Kepler literally says, 'the Office is Holy', referring to its name and implying approval.

I had begun to say that in this work I treat all of astronomy by means of physical causes rather than fictitious hypotheses, and that I had taken two steps in my effort to reach this central goal: first, that I had discovered that the planetary eccentrics all intersect in the body of the sun, and second, that I had understood that in the theory of the earth there is an equant circle, and that its eccentricity is to be bisected.

The third step towards the physical hypotheses of the motions. The eccentricity of Mars's equant is to be precisely bisected.

Now we come to the third step, namely, that it has been demonstrated with certainty, by a comparison of the conclusions of parts 2 and 4, that the eccentricity of Mars's equant is also to be precisely bisected, a fact long held in doubt by Brahe and Copernicus.

Therefore, by induction extending to all the planets (carried out in part 3 by way of anticipation), since there are (of course) no solid orbs, as Brahe demonstrated from the paths of comets, the body of the sun is the source of the power that drives all the planets around. Moreover, I have specified the manner [in which this occurs] as follows: that the sun, although it stays in one place, rotates as if on a lathe, and out of itself sends into the space of the world an immaterial *species*[30] of its body, analogous to the immaterial *species* of its light. This *species* itself, as a consequence of the rotation of the solar body, also rotates like a very rapid whirlpool throughout the whole breadth of the world, and carries the bodies of the planets along with itself in a gyre, its grasp stronger or weaker according to the greater density or rarity it acquires through the law governing its diffusion.

Once this common power was proposed, by which all the planets, each in its own circle, are driven around the sun, the next step in my argument was to give each of the planets its own mover, seated in the planet's globe (you will recall that, following Brahe's opinion, I had already rejected solid orbs). And this, too, I have accomplished in part 3.

By this train of argument, the existence of the movers was established. The amount of work they occasioned me in part 4 is incredible, when, in producing the planet-sun distances and the eccentric equations[31] that were required, the results came out full of flaws and in disagreement with the observations. This is not because they should not have been introduced, but because I had bound them to the millstones (as it were) of circularity, under the spell of common

[30] For the meaning of this untranslatable term, see the Glossary.
[31] For an explanation of this and related terms, see the Glossary.

opinion. Restrained by such fetters, the movers could not do their work.

Fourth step to the physical hypotheses. The course that a planet describes in the heavens, is an oval path. (***) 6 v

But my exhausting task was not complete: I had a fourth step yet to make towards the physical hypotheses. By most labourious proofs and by computations on a very large number of observations, I discovered that the course of a planet in the heavens is not a circle, but an oval path, perfectly elliptical.

Geometry gave assent to this, and taught that such a path will result if we assign to the planet's own movers the task of making the planet's body reciprocate along a straight line extended towards the sun. Not only this, but also the correct eccentric equations, agreeing with the observations, resulted from such a reciprocation.

Finally, the pediment was added to the structure, and proven geometrically: that it is in the order of things for such a reciprocation to be the result of a magnetic corporeal faculty. Consequently, these movers belonging to the planets individually are shown with great probability to be nothing but properties of the planetary bodies themselves, like the magnet's property of seeking the pole and catching up iron. As a result, every detail of the celestial motions is caused and regulated by faculties of a purely corporeal nature, that is, magnetic, with the sole exception of the whirling of the solar body as it remains fixed in its space. For this, a vital faculty[32] seems required.

Next, in part 5, it was demonstrated that the physical hypotheses we just introduced also give a satisfactory account of the latitudes.

There are some, however, who are put off by a few extraneous and seemingly valid objections and do not wish to put such great trust in the nature of bodies. Therefore, in parts 3 and 4, some room was left for Mind, so that the planet's proper mover could attach the faculty of Reason to the animate faculty of moving its globe[33]. These people would have to allow the mind to make use of the sun's apparent diameter as a measure of the reciprocation, and to be able to sense the angles that astronomers require.

All this has been said for the sake of the physicists. The astro-

[32] *Facultas vitalis*. This faculty does not fit into the threefold division of physiological and psychological faculties mentioned in footnote 5 above. It comes from a different tradition, Galenic rather than Aristotelian. Although it is rather like the animate faculty, it does not include the power of perception.

[33] Kepler has clearly returned to the Aristotelian schema. Aristotle argued that the faculties of perception and locomotion are always conjoined, and consequently placed them under a single aspect of soul, the sensitive. His highest faculty was the rational. Clearly, Kepler follows Aristotle in considering the animal and vital faculties to be the same or similar, and subordinate to reason.

nomers and geometers will find the rest in the following summaries of the individual chapters, each in its proper place. I intentionally made this rather detailed, partly so that it might serve as an index, and partly so that a reader who gets stuck here and there in the synoptic table, whether because of the obscurity of the material or of the style, might seek some additional light from these summaries. If the reasons for the order and the coherence of topics lumped together in the same chapter turn out to be hard to see in the text itself, the reader might perceive them more readily among the summaries, which are divided into paragraphs. I therefore ask the reader to consider it well.

The Synoptic Table.

# SYNOPSIS OF THE WHOLE WORK

- In this work, I put forward an Astronomical Edifice; and of this edifice there may be seen:
  - The ground plan, which consists of the equivalence of hypotheses. This is taught by... To "A", p. 71
  - The structure, which proceeds
    - In imitation of the ancients, and consists of
      - Preparation of observations, which has... To "B", p. 71
      - The work itself, where the hypothesis of the first inequality is... To "C", p. 72
    - From my own opinion. It concerns, moreover, the motion in
      - Longitude, in which there are two inequalities.
        - The second, of the epicycle or annual orb. But that it also participates in the first inequality, that is, the accumulated eccentricity,
          - Is established, whether it is referred to the sun's... To "D", p. 72
          - Is put to use, in establishing... To "E", p. 73
        - The first, proper to the planet, and specifically to its eccentric, which is established by physical causes
          - With a preconceived opinion of the orbit (which the planet describes), whether
            - Circular, in imitation of the ancients... To "F", p. 74
            - Or a narrower oval, seized upon on the occasion of the circle's being apprehended as too wide, through a certain manner of physical causes. In which the theory
              - Is constructed... To "G", p. 74
              - Is rejected as too narrow... To "H", p. 75
          - It being granted that the opinions are false, one exceeding and the other falling short, where the theory... To "K", p. 76
      - Latitude, consideration of which is related to times... To "L", p. 77

# FIRST PART

From "A", p. 70

- The ground plan, which consists of the equivalence of hypotheses. This is taught by
  - Distinction of the inequalities into first and second, in Chapter 1
  - Deduction through the first inequality
    - In itself, supposing an eccentricity that is
      - Simple, with the observer
        - Remaining in place. Ch. 2.
        - Moving from place to place. Ch. 3.
      - Compound, where the observer and orbit
        - Remain in place. Ch. 4.
        - Move from place to place. Ch. 5.
    - Mixed with the second inequality, which we suppose to be measured from the sun's {mean / apparent} position, in the {Ptolemaic / Copernican / Brahean} form of the second inequality. Ch. 6.

# SECOND PART

From "B", p. 70

- Preparation of observations, which has
  - An appraisal, by which the matter
    - Is placed before the eyes by explaining
      - The occasions of its compilation. Ch. 7.
      - The table, ch. 8.
    - Is examined with respect to
      - Adjustment. Ch. 9.
      - The origin, or the forest from which it was cut. Ch. 10.
  - An adjustment, accomplished by
    - Legitimate reduction to the orbit
      - By removing the obstacle of parallax. Ch. 11.
      - By investigating
        - The nodes. Ch. 12.
        - The orbs' inclinations'
          - Quantity. Ch. 13.
          - Constancy. Ch. 14.
    - Assumption from observations. Ch. 15.

- From "C", p. 70
  - The work itself, where the hypothesis of the first inequality is
    - Geometrically established, by seeking out
      - The position and quantity of the eccentricity. Ch. 16.
      - The motion of the aphelion and nodes. Ch. 17.
    - Examined through observations in which participate
      - The first inequality alone, where it is confirmed. Ch. 18.
      - The second inequality also, where it is destroyed again by
        - Arguments
          - Of latitude. Ch. 19.
          - Of longitude. Ch. 20.
        - Dissolution or destruction of the previous confirmation, by equivalence. Ch. 21.

## THIRD PART

- From "D", p. 70
  - Is established, whether it is referred to the sun's
    - Mean motion, where the apogee
      - Is presupposed, using two carefully sought observations through which is demonstrated
        - The thing itself in general. Ch. 22.
        - The measure of the thing. Ch. 23.
      - Is demonstrated simultaneously, using any three observations, through which is demonstrated
        - The thing itself. Ch. 24.
        - The measure of the thing. Ch. 25.
    - Apparent motion. And the measure of it and the apogee are obtained simultaneously, the planet's eccentric position being
      - Presupposed, using three observations. Ch. 26.
      - Demonstrated at the same time, with a
        - Direct argument, using four observations. Ch. 27.
        - Convertible argument, from what is to be demonstrated to the principles, five observations. Ch. 28.

From "E", p. 70

Is put to use, in establishing

- The distances, of which [there is]
  - The method of computing. Ch. 29.
  - A table. Ch. 30.
- The equations, which are computed from suppositions
  - Geometrical. Ch. 31.
  - Physical, in which are explained
    - The motive causes, of which one
      - Drives around in an orb, which is considered
        - In itself, as
          - It emanates among the movable bodies. Ch. 32.
          - It is in its source, and this is confirmed
            - By arguments as to
              - What it is. Ch. 33.
              - What its qualities are. Ch. 34.
            - By solving objections concerning the outpouring
              - In an impeded state. Ch. 35.
              - In a free state. Ch. 36.
        - In comparison with the moon's motion. Ch. 37.
      - Causes the eccentricity. Further, it is shown
        - In what it inheres. Ch. 38.
        - In what manner it inheres. Ch. 39.
    - The procedure for computing from these physical causes. Ch. 40.

# FOURTH PART

- From "F", p. 70 Circular, in imitation of the ancients, where I presuppose the manner in which the physical causes act, as in ch. 39. Upon this, the theory is
  - Constructed. Ch. 41.
  - Rejected, as too wide. And the refutation has
    - Principles, which are
      - The distances, which are investigated. Ch. 42.
      - The equations, which are shown to be in disagreement. Ch. 43.
    - A demonstration from the principles. Ch. 44.
- From "G", p. 70 Is constructed
  - By description of
    - The motive causes. Ch. 45.
    - The geometrical effect, or figure traced out. Ch. 46.
  - By investigation of the equations, which are sought by a method that is
    - Direct, by measurement of
      - The effects, and of the traversed
        - Area within the orbit. Ch. 47.
        - Orbit around that area. Ch. 48.
      - The principles bringing them about. Ch. 49.
    - Indirect, by an [exhaustive] induction of the six possible ways [of generating them]. Ch. 50.

- From "H", p. 70
  - Is rejected as too narrow. The refutation has
    - Principles, which are
      - The equations, for which see the five preceding chapters.
      - The distances,
        - Of which many are sought at more select positions that are
          - Matched pairs on the eccentric,
            - For present use. Ch. 51.
            - For confirming the sun's apparent motion. Ch. 52.
          - On the epicycle, or annual orb. Ch. 53.
        - Which are compared with the diameter of the eccentric. Ch. 54.
    - Demonstration from the principles. Ch. 55.

From "K", p. 70

- It being granted that the opinions are false, one exceeding and the other falling short, where the theory
  - Is constructed by demonstrating
    - The correct distances
      - Geometrically, in their nature, quality, and quantity. Ch. 56.
      - Physically, from the corrected motive causes, the form of the motion being shown. Ch. 57.
    - The truly [conceived] oval orbit, the mean between the previous oval and the circle,
      - Through the correct equations. Ch. 58.
      - Through the concurrence of the distances and equations in a single hypothesis. Ch. 59.
  - Is made fit for use, the true method for the equations being demonstrated. Ch. 60.

# FIFTH PART

From "L", p. 70

- Latitude, consideration of which is related to times
  - Present only, and simple. This is done
    - For its own sake, in a consideration of inequalities
      - Individually and separately:
        - The first; and of it,
          - The geometrical measurement
            - Of its beginning, or the position of the nodes. Ch. 61.
            - Of its boundary, or the maximum inclination of the limits. Ch. 62.
          - The physical causes. Ch. 63.
        - The second, or the maximum latitude at conjunction with and opposition to the sun. Ch. 65.
      - Mixed with each other; that is, where the maximum latitude happens to fall. Ch. 66.
    - For confirming the hitherto constucted theory's
      - Form, that its foundation is the sun itself, and not some substitute point. Ch. 67.
      - Ratio to the earth's diameter, or examination of parallax. Ch. 64.
  - Past also, and mingled with several other matters. For enquiry is made in the
    - First inequality concerning the
      - Inclination of the planes, as well as the ecliptic's change of position and the nonuniform motion of the nodes and equinoxes. Ch. 68.
      - Mean motion of the nodes, as well as that of the aphelion and of the eccentric. Ch. 69.
    - Second inequality concerning the greatest latitude of all. And also, the ratio of the orbs, and the change of the sun's eccentricity. Ch. 70.

# Summaries of the individual chapters

Since there is one method which the nature of a subject teaches, and another which our understanding requires, and both are subject to the rules of art, the reader should expect neither from me undiluted. The scope of this work is not chiefly to explain the celestial motions, for this is done in the books on Spherics and on the theories of the planets. Nor yet is it to teach the reader, to lead him from self-evident beginnings to conclusions, as Ptolemy did as much as he could. There is a third way, which I hold in common with the orators, which, since I present many new things, I am constrained to make plain in order to deserve and obtain the reader's assent, and to dispel any suspicion of cultivating novelty.

No wonder, therefore, if along with the former methods I mingle the third, familiar to the orators; that is, an historical presentation of my discoveries. Here it is a question not only of leading the reader to an understanding of the subject matter in the easiest way, but also, chiefly, of the arguments, meanderings, or even chance occurrences by which I the author first came upon that understanding. Thus, in telling of Christopher Columbus, Magellan, and of the Portuguese, we do not simply ignore the errors by which the first opened up America, the second, the China Sea, and the last, the coast of Africa; rather, we would not wish them omitted, which would indeed be to deprive ourselves of an enormous pleasure in reading. So likewise, I would not have it ascribed to me as a fault that with the same concern

for the reader I have followed this same course in the present work[1]. For although we by no means become Argonauts by reading of their exploits, the difficulties and thorns of *my* discoveries infest the very reading – a fate common to all mathematical books. Nevertheless, since we are human beings who take delight in various things, there will appear some who, having overcome the difficulties of perception, and having placed before their eyes all at once this entire sequence of discoveries, will be inundated with a very great sense of pleasure.

That the whole work is well ordered through this method will now become apparent through the summaries of the individual chapters.

I have so arranged the book that whenever the text presents a geometrical demonstration, construction, or lemma, it is printed in 'running letters'[2], as the print shops call them. If this is not so everywhere, you should attribute it to the material, which will mingle physics with geometrical matters, or to the typesetters, who might not always have noticed my signs.

## PART I

### Chapter 1

Explains the manner in which astronomers perceived a difference between the first motion and the second or planetary motions, and also, the manner in which two inequalities, called the first and the second, were discovered in the planets' proper motions.

The occasion of this chapter, as well of the whole first part, is this. When I first had come to Brahe, I became aware that in company with Ptolemy and Copernicus he reckoned the second inequality of a planet in relation to the mean motion of the sun. Now four years previously, it seemed to me that for physical reasons it ought to be measured by the apparent motion of the sun, as is stated in the *Mysterium cosmographicum*. So when this point came up in discussion between us, Brahe said in opposition to me that when he used the mean sun he accounted for all the appearances of the first inequality. I replied that this would not prevent my accounting for the

[1] The reader must beware of supposing that Kepler is presenting a historical account of his enquiry. Although he does lead us through much of his erroneous reasoning, he has also spared us many thorny byways. Nor does he claim to present such a history: he says only that he is mingling some history with the theoretical and didactic matter of the book. What is remarkable, however, is his sense that mistakes are important, that we approach truth by first being wrong.

[2] *Litera cursoria*: that is, italics.

same observations of the first inequality using the sun's apparent motion, and thus it would be in the second inequality that it would be seen which was more nearly correct.

What I answered then is what has been set out to be proved in the first part of the book.

## Chapter 2

Next, since what is proposed is the perplexing and difficult matter of the equivalence of hypotheses, I have begun with the first and simplest equivalence: when a concentric with an epicycle is changed to an eccentric.

And lest the geometrical distinction be thought trivial, I have discussed the causes, both physical and rational or mental, by which it would be reasonable to suppose either of the equivalent hypotheses to be administered and to carry out its motions. This is done in one way if solid orbs be granted, and in another if they be denied. (Brahe has in fact demonstrated from the trajectories of comets that there are no solid orbs).

## Chapter 3

This simple eccentric, or the concentric with a single epicycle equivalent to it, being supposed, it is shown what difference it makes to the eye or in the natural causes of motions, if the mean motion of the sun is exchanged for the apparent; that is, if the observer, or rather if the source of power, be moved in imagination to another place.

## Chapter 4

1. Transition is made from the unconditioned and simple eccentric to an eccentric with an equant having twice the eccentric's eccentricity, which Ptolemy had assigned to the first inequality of the five planets. 2. This is shown to be an absurd device on the assumption of solid orbs, but elegant and physically supportable if such orbs be denied. 3. It is next shown how Copernicus changed this eccentric with an equant into a concentric with two epicycles. 4. This hypothesis of Copernicus's is shown to be rather poor physics on the assumption of solid orbs, but absurd if they be denied. 5. But it is also proved that it results in a planetary path that is deficient in geometrical beauty. 6. Nor is it everywhere equivalent to the Ptolemaic eccentric. The discrepancy is small in the first inequality, but greater in the second. 7. In the same place there is a demonstration of a method for easily

computing the equation in either form of hypothesis. 8. A way of eliminating the discrepancy between the two hypotheses. 9. Finally, another form of the Copernican hypothesis, using a concentric with an epicycle.

### Chapter 5

Chapter 5 is to 4 as 3 is to 2. For the business at hand is more weighty, and concerns: 1. The changes in the hypothesis that would result from moving the observer or seat of power from its original position to another, by substituting the motion of the apparent sun for the mean. This is all done in the form of Copernican hypothesis presented at the end of chapter 4. 2. Which things in turn would be changed in the physical causes of the motions [resulting] from the same hypothesis. 3. This transposition is outlined and constructed in the Ptolemaic form of the first inequality. 4. It is demonstrated that, if one allows two lines of apsides (the original one and another arising from the
(****) v transposition) and thus changes the form of hypothesis, there will follow appearances of two kinds, though the planet's path in the heavens remains the same. 5. But if a single line of apsides be set up, passing through the original centre of the eccentric, it is demonstrated that the same planetary orbit does not give rise to the original required appearances, even though the path remains the same, nor is even the same form of hypothesis retained. 6. Finally, assuming a new line of apsides passing through the centre of the equant, and keeping the same form of hypothesis, it is demonstrated that the orbit changes position in the heavens. 7. The position and size of the circle of greatest difference or aberration from the appearances caused by this transposition is demonstrated geometrically from what is supposed. 8. It is demonstrated that all these phenomena occur if, the centre of vision remaining fixed, the centre of the equant be moved an equal amount in the opposite direction. 9. All things said concerning an eccentric with an equant, which Ptolemy adopted, are applied to the Copernico-Brahean concentric with two epicycles (which, by chapter 4, is equivalent).

### Chapter 6

Here things demonstrated in chapter 5, especially numbers 6, 7, and 8, are to some extent put into practice. And hitherto, respecting these hypotheses, only those things concerning the first inequality, different for different authors, have been in question. Now, however,

those things that are attributed to the second inequality are added, which, as the principal hypotheses (ahead of those discussed previously), are named for their authors Ptolemy, Copernicus, and Tycho Brahe. Usually when we say 'Copernican Hypothesis' we mean that of the second inequality. 1. Therefore, to begin, I compare these.

2. In the Copernican, I show how the hypothesis of the first inequality was derived from the sun's mean motion, and how the eccentricity arises from the point taken in place of the sun. 3. I argue from physics that this is not done correctly, and that the eccentricity ought to be reckoned from the centre of the sun's body itself. 4. If we judge the second inequality to take place according to the apparent motion of the sun, why physical arguments support this. 5. It is demonstrated that in this case positions in longitude in the first inequality do not vary much, but distances of the body of a planet from the body of the sun vary considerably. 6. The position on the earth's orbit at which an observer would encounter the greatest difference in distances as well as the greatest error is demonstrated geometrically. 7. The maximum value of the error is shown by arithmetic operations to be one degree and about 20 minutes.

8. In the Ptolemaic hypothesis I show how the hypothesis of the first inequality was derived from the sun's mean motion. 9. In a general manner, on physical or metaphysical grounds, many objections are raised against this hypothesis as well as the sun's mean motion. 10. But in particular some objections are raised upon the same basis against the sun's mean motion specifically. 11. If we reckon the second inequality from the sun's apparent motion, the physical objections are satisfied. 12. The position, size, and shape of the new hypothesis is shown by transposing the point of uniform motion. 13. The discrepancy of the appearances of the first inequality, the place on the epicycle where the error in appearances of the second inequality is greatest, and the amount of this error, are found by applying the above considerations.

14. In the Brahean hypothesis I show how the hypothesis of the first inequality was derived from the sun's mean motion, and how the centre of Mars's orbit was thus attached to the solar orbit, not at the centre of the sun's body, but nearby. 15. A few general objections are raised against the Brahean hypothesis, but in particular, on physical grounds, I raise many objections against this type of attachment, maintaining (on the supposition of this form of hypothesis) that it

ought to occur at the very centre of the sun's body. 16. The position, size, and shape of the new hypothesis is shown by transposition of the point to which the planetary orbits are attached, and by applying the above considerations, the places on the eccentric as well as on the great orb bearing the eccentric (or concentric with epicycles) are found at which the error is greatest.

And that is the extent of part I.

## PART II

### Chapter 7

I explain in greater detail the occasions both of my having taken up the theory of Mars, and of my having been induced to follow the sun's apparent motion and to begin the work with the first part, just completed, as it stands here. You have a synopsis of this chapter in the summary of chapter 1.

### Chapter 8

Displays the hypothesis of Mars's first inequality, as constructed by Brahe. This is done by means of a table, which has foundations (namely, acronychal observations), a result (namely, computed positions compared with those observed), and an examination of these, aimed at revealing whether this hypothesis might agree with the observations in minute detail.

### Chapter 9

Concerns the corrected adoption of observed positions. 1. The need is shown of constructing the position on the ecliptic corresponding to the planet instead of its position in its own circle. 2. The notion, followed in the table, that the arc from a node to the planet's apparent position is equal to the arc from a node to the corresponding position on the ecliptic, is refuted. 3. The notion of equality is likewise refuted where one of the arcs ends at the true position on the orbit instead of the apparent position. 4. The method of reduction by means of the angle of apparent latitude is also refuted, and a method of reduction by means of the angle of inclination of planes is established.

### Chapter 10

Pertains to the same thing, and concerning the table's adopted positions inquires whether they were correctly and aptly deduced as

being at opposition to the mean sun from observations close to opposition. Also, gentle admonitions on other subtle matters, chiefly parallax, are added. And this concludes the examination of the table.

### Chapter 11

For the sake of beginning my accommodation of the data to the sun's apparent [position] with a legitimate reduction and deduction, so as not to err in any way, I first investigate the diurnal parallax of Mars. 1. I tell what Brahe thought about it. 2. I prove from Brahe's observations, through hourly and daily motions, that it is almost imperceptible, less than what we consider the solar parallax to be. 3. For fun, I bring in my own observations of Mars's parallax, in the course of which I explain my own method of investigating the diurnal parallax by means of a stationary latitude.

### Chapter 12

1. Brahe's own way of finding the nodes of Mars from nearby observations, and a criticism of it. 2. Another way, which presupposes knowledge of the eccentric equations from the *Prutenics,*
(****) 2 r Ptolemy, or Brahe. By which it is simultaneously demonstrated that the descending node (found by four observations) and the ascending node (by two) are at opposite positions on the ecliptic.

### Chapter 13

1. The reckoning of the inclination of the planes is shown to be somewhat more intricate, in all three forms of hypotheses. 2. One way, presupposing known eccentric equations, when Mars, setting in the evening or rising in the morning, is at the limits by the first inequality, for then the apparent latitude is equal to the true inclination of the limit from the ecliptic. 3. The size of the arc of elongation from the sun for which this is so is shown, in the Copernican as well as the Ptolemaic hypotheses, and this is carried out using several observations about either limit. 4. A second way, requiring nothing besides select and rare observations, in which the sun is at the nodes and Mars is 90° from the sun. This is carried out by means of a few observations. 5. This is extended, as Mars, other things being the same, can be at some angle other than 90° from the sun, and hence an inclination at a certain place, other than that of the limit but

nonetheless fixed, may be found. 6. This way is also applied to the Ptolemaic hypothesis, involving some difficulty. 7. A third way is advanced, using latitudes observed at a position opposite the sun's, with the help of the previously known ratio of the orbs. This is treated in all three forms of hypotheses.

### Chapter 14

Next, from what has been demonstrated in chapter 13, the opinion of the ancients, that the planes of the eccentrics are subject to oscillation, is refuted. For it is demonstrated that the inclination remains constant, at least within the limits of one or two centuries.

### Chapter 15

From nearby observations, arithmetically, the positions Mars has at the moments of opposition to the apparent motion of the sun are sought out, and are corrected by precautions already considered. Finally, these are presented in a table, as a foundation for new operations.

### Chapter 16

Now, in imitation of the ancients, physical causes aside, it is posited that the course of the planet is a circle, that within this circle there exists some point about which the planet traverses equal angles in equal times, and that between this point and the centre of the sun lies the centre of the planetary circle, at some unknown distance. With these assumptions, four acronychal observations with zodiacal positions and their time intervals are used to seek out, by a most labourious method, the zodiacal positions of those centres, their distances from the centre of the sun, and the ratios of the two eccentricities, both to one another and to the radius of the circle.

### Chapter 17

By comparing the positions of the aphelion and nodes of Ptolemy's time with those which they are found to have in our time, their motion, required in the next chapter, is found.

### Chapter 18

Finally it is shown that this hypothesis, constructed in the way described and depending upon the sun's apparent motion, accounts for all observed motions in longitude near opposition to the sun, and

does so much more accurately than before, since the Brahean hypothesis was founded upon the sun's mean motion.

### Chapter 19

1. Although so far the hypothesis that has been constructed has worked well for motion in longitude near opposition to the sun, it is nonetheless demonstrated that it does not work well for motions in latitude near opposition to the sun. 2. It is further demonstrated that the Brahean hypothesis is similarly deficient. In both instances, this is done in the Copernican form. 3. The same, in the Ptolemaic and Brahean form of hypothesis. 4. It is shown that the error in latitude arises from the eccentricity's not having been bisected. 5. But if the eccentricity be bisected, then the hypotheses are in error in motions of longitude. Which makes it clear why I was compelled to forsake the ancients and to search more diligently into these matters.

### Chapter 20

Just as in the preceding chapter my hypothesis was convicted of error in motions of latitude near opposition to the sun, it is now so convicted in motions of longitude at other places. 2. So also the Brahean hypothesis, founded upon the sun's mean motion. 3. The demonstration is also applied to the Ptolemaic and Brahean form of the motions. 4. A finger is pointed at the sources of the errors and at the way to correct them. 5. A protheorem is inserted that states the sort of line in the plane of the ecliptic that may be substituted for the lines of the planet's distance from the sun in the plane of the planet's eccentric, when the planet has some latitude.

### Chapter 21

Causes are sought from geometry that would result in the truth's proceeding from a false hypothesis; and it is shown to what extent this can happen.

And this is the end of the second part, in which I have imitated the ancients.

## PART III

### Chapter 22

Now, following my method, I begin the whole inquiry anew, not with the first inequality but with the second. And 1. the circumstances are

recounted that led me to suspect the operation of an equant circle in the theory of the sun. 2. I demonstrate in the three forms of hypotheses that on the supposition of an equant (which I have been favouring), the earth's orbit (or epicycle, for Ptolemy) appears to increase and decrease, as Brahe used to assert. 3. A method is
(****) 2 v presented for finding observations from which this equant may be proved. 4. The proof is carried out, using two select observations, even supposing the Brahean rendition, which depends upon the sun's mean motion.

### Chapter 23

Using the sun's distance from the earth at two zodiacal positions (found in the preceding chapter) together with the position of the sun's apogee (or earth's aphelion), a geometrical demonstration is used to find out the eccentricity of the sun's (or earth's) circle. (It is presupposed that this is a perfect circle).

### Chapter 24

The same as in chapter 22 is demonstrated, but by means of four observations less discriminately selected, which nonetheless have Mars in the same eccentric position. That is, it is shown that some part of the sun's or the earth's eccentricity must be given to the equant circle. This is also shown in the three forms of hypotheses compared with one another, and likewise even on the supposition of the Brahean rendition of Mars's motion, which is founded upon the sun's mean motion.

### Chapter 25

Using the distances of the zodiacal positions of the sun from the earth found in the preceding chapter, three by three, through a geometrical demonstration that presupposes nothing but a perfectly circular path, I discover not only the eccentricity of the sun's or earth's circle (as in ch. 23), but also the position of the sun's apogee or the earth's aphelion opposite it, and this is nearly the same as that found by Brahe using observations of the sun, while here there are only observations of Mars.

### Chapter 26

These four observations of chapter 24 are transferred from the sun's mean motion to the true motion, from the Brahean rendition to mine,

and the same thing is concluded as in ch. 25. And the demonstration is presented in all three forms of hypotheses.

### Chapter 27

By an even more daring method, assuming no rendition of Mars whatever, and using no less than four observations of Mars, different from the preceding ones and compared among themselves as before, I demonstrate not only the eccentricity of the sun or earth, and at the same time the aphelion as before, as well as the ratio of the orbs at this eccentric position, but also the actual eccentric position of Mars beneath the fixed stars, which before was presupposed as known from the rendition.

### Chapter 28

By nearly the same form of demonstration, but taking as given the eccentricity of the sun or earth, and the aphelion (now so many times established), and adding a number of observation, namely, the five that are here compared with one another as before, it is shown that one and the same eccentric position of Mars always results, nearly as in chapter 27. You may recall, however, that in all preceding chapters of Part III I have presupposed that the earth's path is perfectly circular, as it indeed is to the senses. For because of the small eccentricity of its ellipse it can only barely depart from circularity.

### Chapter 29

A perfectly circular eccentric with known eccentricity and a point of uniform motion with doubled eccentricity are posited. Then from these are sought by geometry, first, the distances at apogee and perigee, second, those at 90° of equated anomaly, and third, those elsewhere. In the same place a short cut for finding four distances in one operation is shown. Further, the point at which the circle is distant from the sun's centre by the magnitude of the semidiameter, is found by demonstration. And finally, the point on the circle, different from the other, at which one part of the equation is a maximum.

### Chapter 30

Distances of the sun from the earth are set out in a table, and a way of making selections is shown which, though overstepping the bounds of the principles by making the star's orbit oval, justifiably referring ahead to chapters 31, 40, 44, and 55, where doubts about this are

quelled, is nevertheless shown to depart imperceptibly from what has been previously demonstrated.

### Chapter 31

Brahe feared that if I bisected the sun's eccentricity I would vitiate his equations of the sun's motion. Therefore, this fear is shown to be groundless by a demonstration that, whether there be a simple eccentricity or a bisected one or one formed by doubling the halved eccentricity, the equation of the sun's motion remains ever the same. Thus the doubt raised here in chapter 31 is different from that raised in chapter 30. There, the concern was about distance, while here it is about the Brahean equations; there the cause of concern was the shape of the orbit, while here it is the ratio of the eccentricity. In the one, the consideration was anticipated, while in the other it finds its true place.

### Chapter 32

First comes an inductive argument that all planets, without exception, have an equant circle, with the eccentricity of the point of uniform motion bisected.

Upon this principle is constructed, by a geometrical demonstration, the following universal proposition: the elapsed times[3] of a planet over equal arcs of the eccentric are proportional to the planet's distances from the centre whence the eccentricity originates. Physicists, prick up your ears! For here is raised a deliberation involving an inroad to be made into your province.

### Chapter 33

Now, from the conclusion of the preceding demonstration, with the help of a few generally accepted and purely physical axioms, it is evinced that the distances of a planet from the centre whence the eccentricity is calculated are the administrative causes[4] of the planet's elapsed times over equal arcs of the eccentric.

Second, it is argued that these administrative causes reside in the one terminus of the distances common to them all; that is, in the centre of the planetary system.

Third, in relation to what was thus demonstrated, it is assumed –

[3] *Moras*, literally, 'delays'. In his kinematics, Kepler usually kept the distances constant and considered the times required to traverse them, a procedure that resulted in cumbersome proofs. The arguments of this chapter are a good example of this. On Kepler's use of time as a dependent variable, see footnote 6 to the Introduction.

[4] *Causas dispensatrices*.

partly on the basis of Part I, as proved to be likely, partly on the basis of Parts IV and V, as proved necessarily and geometrically, and also partly made probable in this very place and in Part II – that the sun's body is itself at the centre of the planetary system.

Fourth, it is shown to be consistent with the preceding that the motive power or the power that administers the elapsed times is in the body of the sun. Physical arguments are added.

(****) 3 r Then, in passing, this also is inferred: that the sun is at rest in the centre of the world, and that the earth is moved around the centre of the world. Here the physicist should observe that these physical theories are supported by the earth's motion, but are deduced from elsewhere, and are valid in the Brahean system as well as in the Copernican. In fact, on the contrary, it is upon these very theories that the earth's motion and the sun's rest are now being founded.

Fifth, it is demonstrated that the motive power is quantifiable exactly as light is, and is more tenuous in a greater circle, and more concentrated in a lesser.

Sixth, it is demonstrated from all the preceding that that which moves the planets from place to place is an immaterial *species* of the power in the sun's body, similar to the immaterial *species* of light.

## Chapter 34

The physical theory is brought to completion: it is demonstrated from the foregoing that the *species* of the power that moves the planets moves around through the extent of the world like a river or whirlpool[5], more swiftly than do the planets.

Second, it is shown as a consequence that the sun's body also rotates upon its axis. An enquiry is made into the probable periodic time of this rotation, and at the same time the question is raised what moves the earth and what the moon.

Third, the sun's body is proved to be a sort of magnet; and, by the example of the earth, it is shown that there are magnets in the heavens.

## Chapter 35

A reply is made to the objection that if the motion of the heavenly bodies comes from the sun, it would be impeded by the interposition of bodies, as light is. At the same time, many points from the

[5] *Vortex.*

preceding chapter are illustrated, namely, how the motive power and light are akin, and how each accompanies the other.

### Chapter 36

Other objections are dispelled. The first, which actually has a geometrical foundation, argues from a point on the sun's body to a line, from this to its surface (to all appearances a plane), and so on to its volume, in order to establish that the ratio in which the density of light spreads could not allow it to be placed on an equal footing with the motive power. The reply to this is from the principles of optics: the argument cannot begin with a point or a line; it must start with the surface itself. Further, it is denied that the apparent magnitude of the sun's disc needs to be considered in the physical effect – this could have been shown on many grounds. For it could not possibly be an indication of this physical effect, since it varies in a different ratio. (It does, however, become an indication of something else, below.) And thus it is affirmed that the manner in which light[6] spreads is wholly commensurate with the administration of the planetary motions.

Another objection argues, to the contrary, that light is not fitted for association with motion, since light is also spread towards the poles. This is dispelled by means of the principles already adopted (that is, physical principles), but in a purely geometrical manner, in order that the solution show the natural causes of the zodiac, and why the planets are never outside it.

### Chapter 37

From the physical principles laid down, the causes are sought for the anomaly in the moon that Brahe called the 'variation', which makes the new and full moon swifter than it is otherwise. Here, two false opinions on the subject are rejected. Next, from the same principles, the causes are sought by which the moon's equation is made greater at the quadratures than at opposition and conjunction with the sun. Other remarks are added pertaining to the explanation of the particular power by which the moon is moved.

### Chapter 38

Apart from the common motive force coming from the sun, the individual planets are shown to administer their own motions by

[6] Reading '*lucis*' instead of '*luci*'.

other, individual motive causes. This is done through two arguments: one drawn from the motion in longitude, the other from the motion in latitude.

## Chapter 39

To begin, there are adopted as premises six physical axioms necessary for investigating the power that is attributed solely to the individual planets.

At the same time, however, there are these two preconceived opinions that hold for this chapter as a whole: first, that the planet's orbit is arranged in a perfect circle; second, that this orbit is administered by a Mind. The question thus naturally arises how this mind would make a circle of the planet's path. And it is first demonstrated that could happen if the planet's own power were to endeavour to move its body in a perfect epicycle whilst the body is being carried by the solar power. In opposition to this way, five physical absurdities are raised. Second, it is demonstrated that this could happen if the planet were to observe a fixed point outside the sun from which it would maintain a constant distance in its entire circuit about the sun. But this keeping track of a certain incorporeal point is also shown absurd by three refutations.

Third, it is demonstrated that a perfect circle could result if the planet's own power were to make it reciprocate upon the diameter of the epicycle directed towards the sun, doing so by a prescribed law as if it were running around the epicycle's circumference. But at the same time it is shown that the exact reciprocations cannot be described by the planet if it moves on the diameter of the epicycle, nor do they correspond to eccentric arcs traversed, nor to the time, nor to the equated anomaly – on the assumption, that is, that a perfect circle ought to result from the planet's composite path.

Fourth, this too is denied: that by means of a mind the force proper to the planet somehow conceives an imaginary eccentric or epicycle, and by using it as a rule sets up the distances required for a perfectly circular orbit.

So, as long as we consider the planet's orbit to be a perfect circle, it remains in doubt by what standard the planet's own mind might measure out these reciprocations of its body.

Now that I have aired out the question of the standard of this reciprocation, I proceed further to consider the means by which the

planet's mind might grasp this standard and the reciprocation defined by it. Whether the epicycle be taken as the standard, or its diameter, or the centre of the eccentric, all must be rejected as ill suited for comprehension: they all need a commensurate means, suited to being comprehended, by which the reciprocations might be comprehended by the mind. Here it is added that the planet's mind observes the increase and decrease of the sun's diameter, and uses this to work out the distance of its body from the sun (this is shown by a probable argument drawn from the latitudes). A reply is also made to objections based upon the narrow angle under which the sun appears, and the planet's lack of senses. And at the end it is proposed that the opinion of a governing mind is nevertheless not entirely to be despised.

Finally, there is a discussion of the difficulty involved in having the local motion of the planet's body result from an inherent animate force. And thus, since so many difficulties have been encountered on all points, there is but one thing to be done, namely, to call into question on physical grounds the preconceived notion of the perfectly circular planetary orbit (and also, to some extent, that of the control of the reciprocation by a mind). Shortly hereafter, in chapter 44, this notion will be torn to shreds on geometrical grounds.

### Chapter 40

1. A method by which the physical part of the equation, that is, the elapsed time of a planet over any arc of the eccentric, may be found
(****) 3 v from the distances of the points of its arc from the sun. 2. Here there is a geometrical proof of how the distances from the sun of the infinity of points on an arc are approximately contained in the area enclosed by the arc and the lines drawn to the sun from the ends of the arc. Also, how the one triangle defined by the sun, the centre of the eccentric, and the end of the arc, displays both parts of the equation: the angle at the end of the arc is the optical part, and the area is the physical. 3. A demonstration that in the [theory of the] sun the optical and physical parts of the equation are equal to the senses. 4. As a preliminary, it is proved that triangles on equal bases are to one another as their altitudes. 5. By this theorem it is demonstrated that the area of the triangle of the equation[7] increases with the sine of the eccentric anomaly; whence there is a short cut for computing this

[7] *Triangulum aequatorium.*

area. At the same time it is shown by a numerical example that aliquot parts of the equation do not differ perceptibly. This is done first at 90°, then at 45°. 6. There follows a minor exception, showing that the area is somewhat less than the sum of the distances at all degrees of the eccentric, and somewhat greater than the sum of the distances at all degrees of equated anomaly. 7. A geometrical construction of a quadrilateral conchoid equivalent to the distances from the sun at all degrees of the eccentric. Here, geometers are challenged to find the quadrature of this area. 8. The area between the two conchoids is shown not to be of the same width at places equidistant from the middle. (Of this, more in chapter 43.)

## PART IV

### Chapter 41

On the assumption that the planet's path is a perfect circle, by means of the distances from the sun's body of three points on Mars's eccentric (demonstrated with great certainty in Part III), a geometrical proof is used to conjure up a false position for the apogee, a false eccentricity, and a false ratio of eccentricity.

### Chapter 42

In a new approach, the distances of two points on the eccentric near aphelion are sought by means of five observations, and near perihelion, by three. Then, by halving the periodic time and the circle of the zodiac, the position of the aphelion is investigated very precisely, and is found to be the same as in Parts I and II. The mean longitude of Mars is thereby corrected. Then, by comparing the two distances, the true eccentricity is found, and the ratio of Mars's orb to the earth's. With the eccentricity of the eccentric found very precisely (if not delicately) by observations of the sun, it becomes clear at the same time that it is half the eccentricity of the equant, found elsewhere. Therefore the preliminary theories of chapter 32 are valid for Mars.

### Chapter 43

A fundamental principle is laid down, which was demonstrated previously in chapter 42: eccentricities are to one another in a double ratio. Second, it is supposed that the planet's orbit is arranged in a perfect circle. Third, it is supposed (as demonstrated in chapter 33) that the elapsed times of the planet over equal arcs of the orbit are

proportional to the distances of those arcs from the sun. On these suppositions, faulty equations, disagreeing with experience, are conjured up. Then attention is drawn to one place where that falsehood could not lie hidden. 2. For solving this problem, it is necessary to have a measure of the area between the two conchoids of chapter 40, to the solution of which geometers' attention is drawn, as it involves many contrivances.

Thus it is manifest that some one of the premises of this false conclusion is itself false.

## Chapter 44

It is demonstrated by two arguments that the planet's orbit is not a circle, but an oval figure.

In the first place, those things demonstrated in chapters 41 and 42 are presupposed. Now a perfect circle results in certain distances, and the diameter of this circle was found in chapter 42. But the observations which were reintroduced in chapter 41 require different distances, namely, ones that are shorter at the sides. But an oval figure admits such distances. Therefore, the orbit is an oval.

In the second argument, the same things are presupposed as in chapter 43. The elapsed times shown to us by experience are not admitted by a circular figure, but are admitted by an oval. Therefore, the orbit of the planet is an oval.

## Chapter 45

In what follows, the reader should overlook my credulity, since I am judging everything by my own wits. Indeed, the occasions by which people come to understand celestial things seem to me not much less marvellous than the nature of the celestial things itself. I therefore display these occasions scrupulously, with, no doubt, some attendant difficulty for the reader. Nonetheless, that victory is sweeter that was born in danger, and the sun emerges from the clouds with redoubled splendour. Therefore, O reader, pay heed to the dangers of our army, and contemplate the clouds horrifying in their darkness. Contemplate, I say; for beyond these clouds the sun of truth truly lies hidden, and shortly will emerge. Therefore, an account is given of the occasions that enticed me to suppose what turned out false: that the planet, by its inherent force[8], endeavours to describe a perfect

[8] *Vi insita.*

epicycle, traversing equal parts in equal times; but that the same planet is swept around by the extrinsic force[9] of the sun, through unequal arcs in equal times, as before. From this it is demonstrated that the orbit or path shaped by the two causes comes out to be an oval figure.

### Chapter 46

1. First, this physical hypothesis, which is properly epicyclic, is transformed into an eccentric. 2. Next is taught one way of describing the line of the planet's motion in accord with this opinion. 3. Four obstacles to calculation associated with this method are recounted. It is shown here that the mean of the sums of the terms is not the same as the mean of the terms themselves. 4. A second way of describing this curve is proposed, and the obstacles to calculation in this method are likewise shown. Both methods are meanwhile useful in numerical operations. 5. A third way of describing the orbit of the planet is proposed, through the conjunction of the two hypotheses. 6. A fourth way which one might propound is rejected. 7. It is demonstrated that the line thus created is truly oval, not elliptical.

### Chapter 47

Now, on the assumption that the line of the planet's path is perfectly elliptical, it is demonstrated that the area of the ellipse is less than the area of the circle by the small area of the epicycle or of the circle described by the eccentricity of the eccentric, very nearly. 2. The area of this circle is sought, and so also the area of the oval-shaped plane. 3. It is shown also to be necessary to find a geometrical means of cutting this oval-shaped area in a given ratio – a problem to which geometers' attention is drawn. 4. The lunule by which the oval area differs from a circle, is stretched out straight, in a manner as nearly geometrical as possible. 5. It is proposed to geometers for study, whether this figure thus extended is double the true lunule. 6. Although there is no ready means of dividing an ellipse or oval without assistance, it is demonstrated that an ellipse can be easily divided by means of a circle. 7. Therefore, the ellipse being supposed, and being divided by means of the circle, a way is shown of computing both the distance and the equation, 8. The equation is computed at an anomaly of 90°, with the area expressed numerically in terms of the

[9] *Vi extranea.*

square on the diameter. 9. A way of correcting the eccentricity by (****) 4 r means of the physical equation. 10. The equation is computed at the octants of anomaly, where the area of the triangle of the equation is expressed in numbers significant in seconds. 11. As these equations are also seen to be false, no less than those above in ch. 43, the causes of the error are sought.

## Chapter 48

I have tried to eliminate all the inconveniences and geometrical imperfections of chapter 46 by abandoning areas in favour of numerically defined divisions of the ovoid's circumference.

1. A way is taught of using geometry to find the corresponding portion of the oval path, using the method of distances found at equal intervals of time, from what has been demonstrated in chapter 33, as well as the supposition that the total length of the oval is known. 2. Geometrical support is given for a contrivance whereby the single distance of the midpoint of an arc is substituted for the two distances of the beginning and end of that arc. 3. By another contrivance, which nonetheless proceeds by a geometrical path, the approach of the ends of the segments of the oval towards the centre of the eccentric, and hence the angle that a segment of the oval subtends at the centre, and thus finally the angle that the same segment of the oval subtends at the centre of the sun, are demonstrated. 4. Another contrivance for seeking the length of the oval path, which is, however, accompanied by other geometrical theories. For two circles are given, and two means between them, one arithmetical and the other geometrical, by the former of which a greater circle is constructed, while by the latter, a lesser. Then, by two arguments, the ellipse is shown to be equal to the arithmetic mean; the one, more general, depending upon a contraction of the extremes, and the other, purely geometrical, showing that the ellipse certainly exceeds the lesser mean and therefore probably equals the greater. 5. One procedure for seeking the equations, which ignores what has been said under numbers 3 and 4, exactly as if, as in the whole, so also in the parts, the two cancelled each other. 6. It is demonstrated geometrically that the visual lengthening in the parts caused by the approach, mentioned in no. 3, and the contrary shortening of the elliptical arcs of no. 4, are not equal. 7. A genuine process is described that is in agreement with all that is demonstrated in this chapter, and the equations found thereby are still convicted of error.

## Chapter 49

1. The above method is shown to beg the question, and to offend against what it was proposed to do. 2. The areas of chapter 46 and 47, as well as the oval circumferences of chapter 48, are dismissed, and a return is made to the causes by which the oval is formed. And because hitherto the epicycle has been transformed into an eccentric, thus confounding the planet's own power with the power coming from the sun, the epicycle on a concentric is taken up again, and the physical causes treated in ch. 45 are applied in order that by this means a proper foundation for seeking the equations may be obtained. 3. The actual method of constructing the equations is reviewed, and the equations are convicted, by experience, of the same error that occurred above in ch. 47. 4. Therefore, the suspicions of error in the computations carried out above in ch. 47 are dismissed, and it is concluded that the hypothesis in ch. 45 is itself at fault.

## Chapter 50

Has six attempts at finding the equation by the distances themselves; that is, at finding the elapsed time of the planet on a certain arc of the eccentric, the distances having been taken before I knew that their sum is contained in the plane surface. For it is most certain from ch. 33 that the elapsed times are to be obtained from the distances. But since there are three anomalies: one, which is the measure of the time; a second, of the arc of the eccentric; and a third, of the angle that this arc subtends around the sun; I have given individual distances to the 360 equal parts of all three anomalies. So by this token the consideration of distances has been made threefold. Now, from the same ch. 33 it is clear that the diurnal path of a planet at aphelion to the diurnal path at perihelion, as if seen from the centre of the sun, is inversely in the duplicate ratio of the aphelial and perihelial distances of the planet from the sun. I have therefore squared all the distances and divided by the standard 100,000, so that the result, compared to the standard 100,000, might represent that duplicate ratio that holds between the apparent diurnal motions as seen from the centre of the sun. So to the three kinds of distances the same number of kinds of third proportionals are added. With these things diligently sought out, I hoped I had left out nothing pertaining to the effects of natural causes (which tell us to seek the eccentric position of the planet by means of the distances). Thus were the six methods developed.

In the first and second, which have the distances of the second anomaly (that is, the eccentric anomaly), there occurs a geometrical matter worthy of consideration. For the sum of the 360 third proportional lines has come out equal to the sum of the 360 radii, or first proportional lines. This is proposed to geometers for demonstration.

Aside from this, here is how the six methods compare. Two (the fourth and the fifth) result in absurdities, and double the errors of the equations. The remaining four coincide with the methods of the preceding chapters. Two (the second and the third) suppose the planet's path to be a circle, and two (the first and the sixth) transfer the distances and set up the oval path as in the theory in chapter 45. And the latter err in defect by the same amount as the former in excess: they have the truth in the middle.

## Chapter 51

Now that it is clear that the oval of chapter 45 produces false equations, the question is raised whether it is also in error concerning the distances.

Consequently, in this chapter, first the observations and second the distances of the sun from the earth are adopted, such as were established with great certainty in Part III. Aside from these, nothing is assumed or admitted as a principle of demonstration. From these, in turn, are demonstrated the distances of Mars from the sun in many places on the eccentric throughout the entire orbit, and especially in those places so chosen that individual pairs on their respective ascending and descending semicircles, are equidistant from the position of the aphelion, as determined above by more than one route. Whence the position of the aphelion is corroborated, and at the same time the trustworthiness of the vicarious hypothesis is investigated.

## Chapter 52

From what was demonstrated in the preceding chapter, it is demonstrated further, that parts equally removed from the established aphelion, and equidistant from the sun, are not equidistant from any other point not on the line joining the sun and the aphelion. Therefore, the line of Mars's apsides passes right through the body of the sun, since all other lines divide Mars's eccentric in an absurd manner, namely, into two unequal segments. It is added in antici-

pation of anyone who would construct that eccentric upon another point, such that it would be cut into two equal parts by some line other than that which passes through the sun, that he is refuted by the observations. In the same way, it is demonstrated that since the sun is on the longer diameter of the oval eccentric, the point that Copernicus used in place of the sun in constructing the eccentric is not on that longer diameter. And it is by no means probable that the line of apsides of an oval eccentric be other than the longer diameter of the oval; hence, the line of apsides does not bypass the sun, and so the lines of apsides of all planets come together in the very centre of the sun, and not in some point of the sun's mean position.

## Chapter 53

A unique method of finding the distances of Mars from the sun near its opposition to the sun, and at the same time a demonstration of the
(****) 4 v point on the earth's orb[10] at which an error in the distance appears greatest. Here it is presupposed that the difference of two eccentric positions and the difference of their distances from the sun are known approximately. At the same time, by this means, the trustworthiness of the vicarious hypothesis is investigated, as in ch. 51 above.

## Chapter 54

By gathering together things that were demonstrated in various places, the ratio of the eccentricity and of the orbs is very carefully established and adjusted.

## Chapter 55

Finally we return to the course from which we had departed in chapter 45. For, by a complete induction[11], it is demonstrated that, just as the circle in chapter 44 was too broad at the sides, so the oval of chapter 45 is too narrow. The arguments are two. One is drawn from the distances: those observed and presented in ch. 51 and 53 are compared with the distances computed from the hypothesis, using the ratio of the orbs given in chapter 54 and the form of the motions given in chapters 45, 46, and 49. And it is shown that the observed

[10] *Orbis magnus.*

[11] *Inductio omnium. Inductio*, in the signification given it by logicians of the sixteenth century, usually denoted enumeration of the next instance in a well-defined class or sequence: hence, *inductio omnium* means an enumeration of all instances, or a 'complete induction'. See N. Jardine, *The Birth of History and Philosophy of Science*, Cambridge University Press, Cambridge, 1984, p. 77.

distances are greater. The other argument is taken from the equations. For the equations computed from the circle in ch. 43 were in error in one direction, while those of the oval of chapter 45, computed in chapters 46, 47, 48, 49, and 50, were in error by the same amount in the opposite direction.

### Chapter 56

It is now demonstrated from the preceding that the distances are not to be derived from the circumference of the epicycle, whether the planet moves uniformly upon it, as in ch. 45, or retains the ratio of the eccentric motion, as in ch. 41, but are to be taken from the diameter of the epicycle. The premises are the same as before.

### Chapter 57

Since the physical arguments of chapter 45 necessarily have some admixture of falsehood, on account of their false conclusion, while now the genuine conclusions are made clear, those physical arguments are reformed, and the theoretical argument of chapter 39 is continued.

First it is shown that the reciprocation on the diameter of the epicycle (which supplies distances in agreement with the observations) follows the natural laws of bodies. 2. Since reciprocation is a translation from place to place, it is shown that this translation of the body of the planet is caused and carried through by the sun, no less than the motion of revolution in Part III, but in such a manner that the reins of the reciprocation are in the hands of the planet itself. This is made clear by two examples, one imperfect (using oars), the other more perfect (using a magnet). 3. In applying the magnetic example, I suppose two faculties in both the planet and the magnet, one of direction and the other of appetency. The magnet is directed towards the pole, and seeks out iron. Just so, the globe of the planet is directed with respect to the fixed stars, and seeks out the sun. Now the function of direction, upon which depend the motion and position of the aphelion, I at first leave in doubt, whether it belong to Mind or Nature. The function of appetency, upon which depends the eccentricity, I ascribe to Nature, and I show approximately that the measure of the reciprocation obtained by observing is in agreement with the physical causes by parts. 4. Afterwards I treat this more accurately, initially considering the faculty of direction, and conceding that a certain amount of declination is removed from it arising from the

appetency of the sun, just as a magnet directed towards the pole is nonetheless somewhat deflected by iron and nearby mountains. I then demonstrate that the position and the very slow eastward motion of the aphelion can be accounted for by a natural and corporeal faculty, with no assistance from a mind. 5. The measure of appetency I show to be analogous to the balance; or, more particularly, the strength of appetency at any given point of time is measured by the sine of the equated anomaly. 6. Concerning the distance traversed in the reciprocation at any given time, hear, reader, what I shall demonstrate. Its measure is clear from chapter 56, namely the versed sine of the eccentric anomaly, not of the equated anomaly. This measure is founded upon the observations. I therefore had to work out here a demonstration that would furnish this distance traversed in the reciprocation (that is, the versed sine of the eccentric anomaly), given the above mentioned measure of the strength of the reciprocation at any place (that is, the sine of the equated anomaly). To obtain this, I had to show that, with a quadrant divided into any number of equal parts, the versed sine of any arc has an imperceptibly smaller ratio to the versed sine of the whole quadrant, than the sum of the sines on the arc has to the sum of the sines in the quadrant. 7. Here, there seemed to be two considerations tending to make the latter premise inconsistent with the former conclusion. First, the eccentric anomaly, which gives the measure of the reciprocation, was greater in the upper semicircle, and displayed a greater number of sines, than the equated anomaly, which gives the measure of the strength. But the answer is that this is exactly what should happen, since in the equated anomaly the planet also takes more time, and consequently also pours out more forces. 8. The other obstacle: that the sines of the equated anomaly are less than the sines of the eccentric anomaly in the upper semicircle. It has therefore been shown that the versed sine as well is somewhat less than the sum of the sines of its arc, and thus is equivalent to the sum of the smaller sines. 9. Objections which may be raised against the example of the magnet are in part dispelled, and in part give occasion for calling Nature into question and reintroducing Mind, in order to see whether and by what means Mind might cause the eccentricity through reciprocation. 10. And so, positing that which is demonstrated most certainly in ch. 56, that the versed sine of the eccentric anomaly measures the reciprocation, it is now demonstrated that the versed sine of the equated anomaly measures the increment in the apparent diameter of

the sun. That is, the sun's apparent diameter not only begins to increase as does the versed sine of the equated anomaly, and reaches a maximum when the latter does so, but also stands at a mean between the extremes when the versed sine of the equated anomaly is the semidiameter, the versed sine of the eccentric anomaly then being greater. 11. On the contrary, if the versed sine of the eccentric anomaly be the semidiameter, it is demonstrated that the apparent diameter of the sun is smaller yet, since it is a mean between extremes. 12. In order to show that this measure is appropriate and comprehensible to the planet's mind, a comparison is first of all made between the eccentric anomaly and the equated anomaly, and it is denied that the angle of the eccentric anomaly (if it be proposed as a measure) could be comprehended by the planet's mind. 13. But it turns out to be arguable that the angle of equated anomaly, whose versed sine is proportional to the increase of the sun's diameter, is comprehended by the planet's mind. 14. But since it is not this angle but its versed sine that measures the increment of the sun's diameter, it is shown by reasoning, by physical hypotheses, and by examples of natural phenomena, to be arguable that the planet's mind can comprehend the sine (the strength, in physical terms) of this angle. 15. A comparison is made of the two ways so far considered by which the motions proper to the planetary bodies (that is, the reciprocations) may be carried out: the former placed under the care of Nature, and the other, of Mind. A conclusion is finally made in favour of Nature, repudiating Mind. 16. Among the arguments for this conclusion, the chief is the geometrical uncertainty admitted by that form of motion administered by Mind. An explanation of this is given. 17. It is shown that this uncertainty might provide occasion for the progressive motion of the aphelion. But since in ch. 35 above another cause for the progression of the aphelion was suggested, the two are compared here, and it is shown that a single interposed body, if it is left with any efficacy, does not cause the progression of the aphelia whether they are moved by Nature or by Mind. 18. And so the physical positions are given limits, lest an interposition affect anything else. 19. But in order that the progression of the aphelion
(****) 5 r might arise hence, it is shown that a peculiar mental function must be associated with any interposition, which was rejected in no. 17 as absurd. That we might be freed from this, a conclusion is made in favour of the opinion which, in no. 4, ascribed the motion of the aphelion to Nature.

## Chapter 58

Now that the true proportions of the planetary reciprocation have been found, it is shown how, on that basis, an orbit can be made for the planet (by compounding the two motions of revolution and reciprocation) in a 'puff-cheeked' form as well. It is also shown how, through an error that seemed like the truth, I happened upon this puff-cheeked orbit.

2. This orbit is convicted of error by the equations, though the distances are correct. This is contrary to what happened previously, when both the equations and the distances were simultaneously in error.

3. I show how, in ostensibly doing something else, and bringing the ellipse back in, I unwittingly corrected the error.

4. It is demonstrated that the orbit is made puff-cheeked by the erroneous hypothesis I used.

5. It is shown that since the elliptical orbit provided the correct equations, the reciprocation that was deformed into the puff-cheeked orbit is called into question.

## Chapter 59

1. The geometry of the ellipse in 10 propositions, by which 2. it is demonstrated in proposition 11 that the distances constructed by the reciprocation and supported by the observations are contained in the perfect ellipse no less than in the puff-cheeked orbit introduced in ch. 58 and convicted of falsity. And so, since the ellipse furnishes both the equations and the distances, the orbit of the planet is therefore elliptical.

3. On the same basis, it is demonstrated, in proposition 12, that the area of the ellipse is a most perfect measure of the distances of the unequal arcs on an ellipse corresponding to equal arcs on a circle.

4. By solving the objection to using unequal arcs on the ellipse, it is shown, in proposition 13, that this ellipse is precisely in accord with the physical principles of Part III.

5. It is demonstrated, in proposition 14, that the arcs of the ellipse are to be bounded by lines drawn ordinatewise through the individual degrees of the circle. At the beginning and end of a quadrant, this is done with two perfect demonstrations, but for intermediate motion it is done less perfectly, though by an indication that is clear enough – and to this problem geometers' attention is directed.

6. With these conclusions, and especially those appearing under

no. 3, with the help of those under no. 1, it is demonstrated in addition to what was already mentioned, in proposition 15, that the area of the circle itself is also a very perfect measure of the distances that are assigned to unequal arcs of the ellipse (which arcs must be defined by lines drawn ordinatewise through equal arcs of the circle). This is also confirmed by numerical computation, by which the observations are satisfied in both of these methods.

### Chapter 60

1. From what was demonstrated in chapter 59, a method of finding the equations is established.

2. Demonstration of a precept telling how the mean anomaly and the equated anomaly may be found from a given eccentric anomaly.

3. One way of finding the eccentric anomaly from a given equated anomaly and eccentricity. This is founded upon a most beautiful and purely geometrical theoretical consideration of the short lines of the planet's incursion from the circumference of the circle towards the line of apsides. The consideration comprises five problems, and is carried out by means of the rectangle on the quadrant.

4. Another approach to this problem, using analytical rules.

5. A clumsy method, using a kind of iteration, for finding the eccentric anomaly and the equated anomaly, given the mean anomaly or the time; and the reason why a geometrical method cannot be given.

## PART V

### Chapter 61

Now that the hypothesis of the longitudes has been found, the position of the two nodes is investigated more accurately, from the observations.

### Chapter 62

Now that the distances have been found, the inclination of the planes is investigated more accurately, from acronychal observations in both semicircles. 2. The ratio of the apparent latitude to the inclination at any point is demonstrated to be the inverse of the ratio of the distances of the sun and the earth from the planet. 3. A small table of observed latitudes at opposition, compared with the computations from our hypothesis.

## Chapter 63

1. The physical cause of the latitudinal deviation is presented. 2. It is demonstrated geometrically that a plane is swept out in this deviation. 3. The question is raised, whether this is the work of corporeal Nature or of Mind, and a conclusion is made in favour of Nature. 4. The question is raised, whether the axis of the latitudes is the same as, or different from, the axis causing the eccentricity, and it is shown what form a body must have if its Nature alone is to do everything. 5. On the supposition of solid orbs, a simple and easy hypothesis for the latitudes is presented.

## Chapter 64

The theory of latitudes has been finished; now, the diurnal parallax is examined more accurately, and is shown to be quite imperceptible by two arguments, one from the position of the nodes and the other from the inclination of the planes.

## Chapter 65

The value of the maximum latitudes at both opposition and conjunction is determined, assuming knowledge of the cycles of all the motions under all circumstances, and of the exact space of elapsed time. 2. The same value is determined for our time.

## Chapter 66

The value of the maximum latitudes elsewhere than at syzygy is investigated, and their positions are determined. 2. The cause of a
(****) 5 v paradox concerning the latitude at opposition to the sun is revealed. 3. An accurate method for computing the latitude elsewhere than at acronychal position.

## Chapter 67

The same thing as in chapter 52 is demonstrated: that the eccentricities have their common origin in the very middle of the sun, not in some other point substituting for the sun. This is shown by two arguments, the first from the position of the nodes, and the other from the inclination of the planes.

## Chapter 68

1. A theory of the change in latitude of the fixed stars, which proceeds through physical causes and the mean ecliptic, or rather, through the

introduction of the royal circle (as we call the *via regia*). 2. It is shown that the northern limit of the ecliptic is in $5\frac{1}{2}°$ Aries, and this lends plausibility to the notion that the mean or constant path passes through the positions of the planets' apsides. 3. The mean ecliptic (or rather, royal circle) is added, by changing the obliquity of the common or true ecliptic. In the margin is the theory of the precession of the equinoxes, done by means of a cylindrical annual translation of the earth's axis and poles, and a very slow inclination deflecting them into a cone. 4. Hence it is shown that the inclination of the planes of Mars and the ecliptic does not stay the same over the ages. 5. The same is concluded less clearly from a comparison of the Ptolemaic observations with our own.

## Chapter 69

1. The observations the ancients made of Mars, and left in writing. 2. On the nonuniformity of the precession of the equinoxes, pro and con. 3. On the inconvenient number of spheres in recent theories. 4. Was the sun's eccentricity once greater? Or, on the length of summer and winter, at the time of Ptolemy. 5. The sun's apogee at the time of Hipparchus is uncertain; and the means he used to investigate it. 6. That the positions of the fixed stars at the time of Ptolemy are somewhat uncertain; and his means of investigating them. 7. Side effects of errors in the positions of the fixed stars upon the theory of Mars. 8. From three of Ptolemy's acronychal observations, adjusted to the modern equations, the correction of the motions for Ptolemy's time is found. This is done eight times, as one or another of Ptolemy's preconceptions, already discussed, is changed. 9. Next, in order to clear up this confusion, it is shown that, when refraction and the defect of the solar eccentricity, which cancel each other, are neglected, the fixed stars retain the zodiacal positions assigned by Ptolemy. 10. Upon this foundation, the epoch of the mean motion of Mars is established for the times of Ptolemy and of Christ. 11. The epoch of the sun's mean sidereal motion, at the times of Ptolemy and of Christ, is added.

## Chapter 70

Through two ancient and untrustworthy observations, the values in ancient times for the ratio of the orbs of Mars and the sun, the latitude of Mars, and the sun's eccentricity, are examined.

(****) 6r

# Index of a great many terms in the margins, as well as of authors of whom mention is made[1]

A.



[1] Page numbers are from the first edition (included in the margins of the present translation). Entries are rearranged so as to be in alphabetical order in English. Names are as given by Kepler (Latin endings removed where appropriate), and entered where he chose to enter them (thus, David Fabricius is under D, while Philip Lansberg is under L). Where the placement does not square with modern usage, a cross reference in provided in square brackets.

## E.



## F.



## G.



## H.



## I.

## L.



## M.



## N.



## O.



## P.

R.



S.



T.



V.



W.



Z.

IN THE NAME OF THE LORD

THE COMMENTARIES
ON THE MOTION OF THE STAR MARS

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PART I

# On the relationships of hypotheses

# 1

## On the distinction between the first motion and the second or proper motions; and in the proper motions, between the first and the second inequality

Terms.
1. The first motion is that of the whole heaven and of all its stars from the east past the meridian to the west, and from the west through the lowest part of the heavens to the east, in the period of 24 hours; in the present diagram, ABCD.
2. The second motions are those of the individual planets from the west to the east, from A to E, from F to G, in longer periods.
3. The smaller circles are those which are nearer to one of the poles, as HLK which is closer to pole Q than to pole R.

2

4. The greatest circle of the sphere is that

The testimony of the ages confirms that the motions of the planets are orbicular. It is an immediate presumption of reason, reflected in experience, that their gyrations are perfect circles. For among figures it is circles, and among bodies the heavens, that are considered the most perfect. However, when experience is seen to teach something different to those who pay careful attention, namely, that the planets deviate from a simple circular path, it gives rise to a powerful sense of wonder, which at length drives men to look into causes.

It is just this from which astronomy arose among men. Astronomy's aim is considered to be to show why the stars' motions appear to be irregular on earth, despite their being exceedingly well ordered in heaven, and to investigate the circles wherein the stars may be moved, that their positions and appearances at any given time may thereby be predicted.

Before the distinction between the first motion[1] and the second motions[2] was established, people noted (in contemplating the sun, moon and stars) that their diurnal paths were visually very nearly equivalent to circles. These were, however, seen to be entwined one upon another like yarn on a ball, and the circles were for the most part smaller[3] circles of the sphere, rarely the greatest[4] (*such as here* ABCE, FMNG *cutting the equator* AB *in* CN), part of them north and part south of the greatest circle. They also saw that the stars have different speeds in this diurnal and apparent motion. The fixed stars are fastest of all, since those that are in conjunction with any of the planets on the preceding day (*such as* H, *with* A *and* F) come to their

which is equidistant from the two poles.

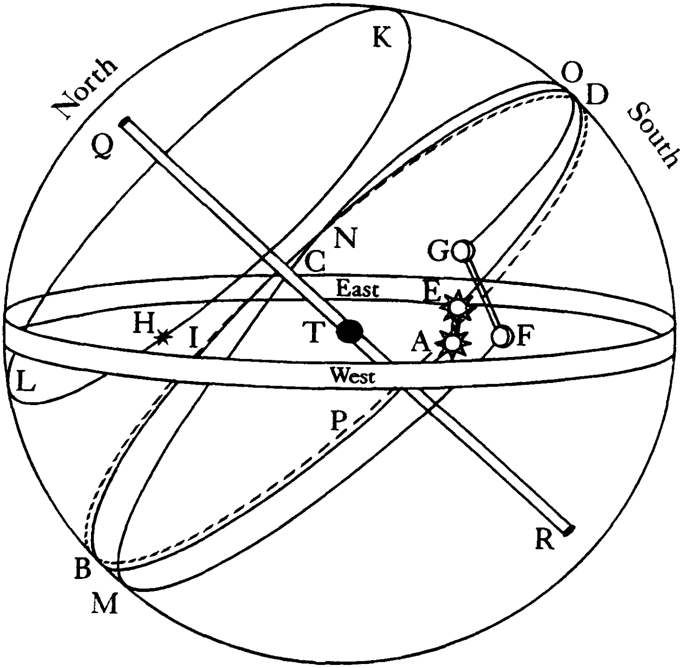



Fig. 5

setting first (*such as* H, *moving along* LK *to* I). The sun (*on* ABE) is slower, as *on the following day it stands at* E and so its setting follows that of the fixed stars *at* I with which on the previous day it was conjoined *on* HA. Slower still than this, slowest of all the stars, is the moon, since after setting with the sun today (*the moon being at* F), it lags by an appreciable interval (EG) tomorrow when the sun sets (*at* E, *the moon having made a circuit of the whole heaven and of the earth along* FMNOG). Hence the Pythagoreans, when they shared out musical tones among the stars, gave the lowest (the '**hypate**' among the strings of the lyre)[1] to the moon, because the motions of both are slowest. Hence have originated the words ***proegoumenos*** and ***hypoleiptikos***[2]. The former of these terms originally corresponded to a star which, on the next day, comes to its setting before another (*the sun* E *is said to be* ***proegoumenos*** *with respect to the moon* G). The

[1] The Greek word **hypate** actually means 'highest'; however, the Greek convention regarding 'high' and 'low' in music is the opposite of our own.

[2] 'Leading' and 'left behind', respectively.

latter term corresponded to a star that is slower in the first motion (*such as the moon here*), which is, as it were, abandoned and left behind (*at* G) by the swifter ones (E *and* I). For more on this subject see our *Optics*, ch. X.[3]

This first adumbration of astronomy explains no causes, but consists solely of the experience of the eyes, extremely slowly acquired. It cannot be explained in figures or numbers, nor can it be extrapolated into the future, since it is always different from itself, to the extent that no spiral is equal to any other in elapsed time, and none carries over into the next with a curvature of the same quantity. Nevertheless, there are some people today who, riding roughshod over two thousand years' work, care, erudition, and knowledge, are trying to revive this, obtruding admiration of themselves upon the mob (an attempt which has not been fruitless among the ignorant). Those with more experience consider them with good reason to be incompetent, or (if, like that man Patricius[4], they want to be known as philosophers) to act mad with reasoning.

Term.
*'In consequence' means 'according to the sequence of signs' (Aries, Taurus, and so on) which series run from the west through the meridian to eastern parts, then down towards the nadir and again to the west: from F to G, from A to E.

For it was very helpful to astronomers to understand that two simple motions, the first one and the second ones, the common and the proper, are mixed together, and that from this confusion there necessarily follows that continuous sequence of conglomerated motions. Thus, when that common and extrinsically derived diurnal revolving effect is removed, the fixed stars are suddenly no longer the swiftest and the moon slowest, but quite the opposite, the latter being the swiftest in itself *and in its proper motion* FG while the former are clearly very slow or immobile. When a planet (*such as the moon at* G) is 'left behind' (*by the sun at* E *or the fixed stars at* I), it is carried in consequence* *through* FG more swiftly than the sun (*through* AE) or the fixed stars (*through* HI). If, however, it appears to be 'leading' with respect to the fixed stars, it goes along with a retrograde motion. *For example, if the sun at A along with a fixed star at H were released*
3 *from the same starting line* AH *on the previous day, so as to traverse* BCDE and arrive at P *while the fixed star traversed* HLK *and arrived at* I, *the sun, in the space of one day, would retrogress through the interval* AP.

This turned out to be enormously helpful in astronomy in grasping

[3] *Astronomiae pars optica*, Frankfurt, 1604, ch. 10, esp. pp. 333–4, in KGW 2 pp. 286–7.
[4] Francesco Patrizi (1529–1597), professor of philosophy at Ferrara and at the *Sapienza* in Rome. His chief work, *Nova de universis philosophia*, Venice 1593, delineates in an engaging manner an innovative Hermetic-Platonic cosmology. Patrizi made no secret of his dislike for astronomers.

the simplicity of the motions. Instead of unending spirals, *an entirely new one always being added to the end of the earlier one at* E *or* G, there remained little but the solitary circles FG and AE, and a single common motion, either of all the planets and the whole world as well in a direction opposite to the proper motions, or (following Aristarchus, making the world stand still) of the earth's globe T around the axis QR in the same direction as the proper motions.

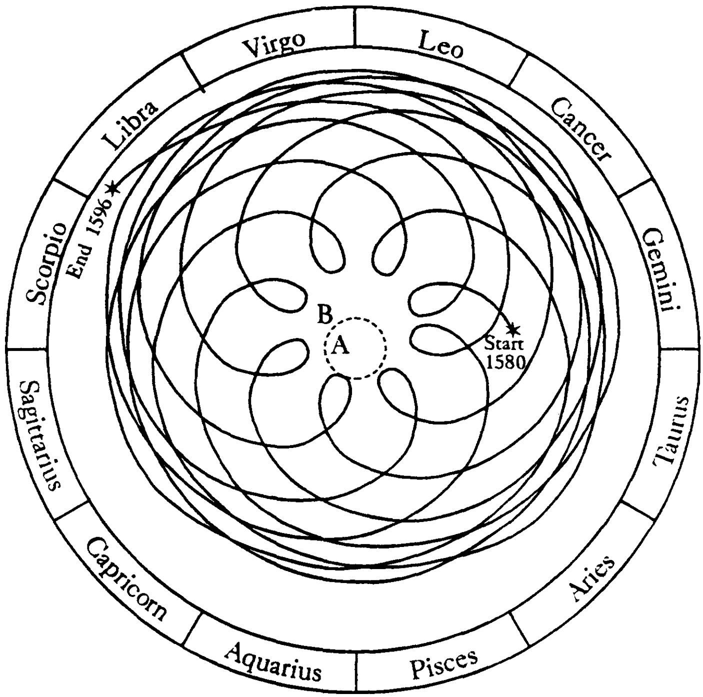
Now that the first and diurnal motion had thus been set aside, and those motions that are apprehended by comparison over a period of days, and that belong to the planets individually, had been considered in themselves, there appeared in these motions a much greater confusion than before, when the diurnal and common motion was still mixed in. For although this residual confusion was there before, it was less observed, less striking to the eyes, because the diurnal motion was very swift, so that this residual motion was divided into minute parts and spread out over several days and several diurnal spirals. But now, that minute division and distribution of the stars' proper motions over so many days was removed by the removal of the diurnal motion, and so all the proper motions of the planets, as many as they are, and all the confusion of this multitude shone forth more obviously.

First, it was apparent that the three superior planets, Saturn, Jupiter, and Mars, attune their motions to their proximity to the sun. For when the sun approaches them they move forward and are swifter than usual, and when the sun comes to the sign opposite the planets they retrace with crablike steps the road they have just covered. Between these two times they become stationary. These things always occur, no matter what the sign in which the planets are seen. At the same time, it was clear to the eye that the planets appear large when retrograde, and small when anticipating the coming of the sun with a swift and direct motion. From this, the conclusion was easily reached that when the sun approaches they are raised up and recede from the earth, and when the sun departs towards the opposite sign they descend again towards the earth. And finally, it was observed that this phenomenon of retrogression and increase of luminosity, just described, moves through the signs of the zodiac in order, tending from west through the meridian eastward, so that whatever has happened at one time in Pisces soon comes to pass similarly in Aries, then in Taurus, and so on in consequence.

If one were to put all this together, and were at the same time to

believe that the sun really moves through the zodiac in the space of a year, as Ptolemy and Tycho Brahe believed, he would then have to grant that the circuits of the three superior planets through the ethereal space, composed as they are of several motions, are real spirals, not (as before) in the manner of balled up yarn, with spirals set side by side, but more like the shape of pretzels[5], as in the following diagram.

4 This is the accurate depiction of the motions of the star Mars, which it traversed from the year 1580 until 1596, on the assumption that the earth stands still, as Ptolemy and Brahe would have it. These motions, continued farther, would become unintelligibly intricate, for the continuation is boundless, never returning to its previous path. Take note, too, that since the circle of Mars requires such a vast space, the spheres of the sun, Venus, Mercury, the moon, fire, air, water, and earth, have to be included in the tiny little circle around the earth A, and in its little area B. In addition, the greatest part even of this little space is given to Venus alone, much greater in proportion than is given to Mars here out of the whole area of the diagram. Moreover, we are forced to ascribe similar spirals to the remaining four planets if the earth stands still, and Venus's spiral would in fact be much more complicated. Ptolemy and Brahe




[5] The Latin is *panis quadragesimalis*, that is, 'bread of the forty [days]', or lenten bread. Pretzels were invented by monks of southern Germany, who adopted the practice of giving them to children as treats during lent.

offer explanations of the causes, order, permanence, and regularity of these spirals, the former using individual epicycles carried around on the eccentrics of the individual planets, in imitation of the sun's motion, and the latter by having all the eccentrics carried around upon the single orb of the sun. Nevertheless, both leave the spirals themselves in the heavens. Copernicus, by attributing a single annual motion to the earth, entirely rids the planets of these extremely intricate spirals, leading the individual planets into their respective orbits, quite bare and very nearly circular. In the period of time shown in the diagram, Mars traverses one and the same orbit as many times as the 'garlands' you see looped towards the centre, with one extra, making nine times, while at the same time the earth repeats its circle sixteen times.

Again, however, it was noticed that these loops in each planet's spirals are unequal in different signs of the zodiac, so that in some places the planet would retrogress through a longer arc of the zodiac, at others through a shorter, and now for a longer, now for a shorter time. Nor is the increment of brightness of a retrograde planet always the same. Also, if one were to compute the times and distances between the midpoints of the retrogressions, neither times nor arcs would be equal, nor would any of the times answer to its arc in the same proportion. Nevertheless, for each planet there was a certain sign of the zodiac from which, through the semicircle to the opposite sign in either direction, all those things successively increased.

The sun has only a single inequality, with respect to the time within which it is completed. But as for the causes of this inequality, the same two factors combine as much for the sun as for the other planets, as will be explained below.

From these observations it came to be understood that for any planet there are two inequalities mixed together into one, the first of which completes its cycle with the planet's return to the same sign of the zodiac, the other with the sun's return to the planet.

Now the causes and measures of these inequalities could not be investigated without separating the mixed inequalities and looking into each one by itself. They therefore thought they should begin with the first inequality, it being more nearly constant and simple, since they saw an example of it in the sun's motion, without the interference of the other inequality. But in order to separate the second inequality from this first one, they could proceed no otherwise than by considering the planets on those nights at whose beginning they rise while the sun is setting, which thence were called ***akronychioi***, or night rising. For since the presence and conjunction of the sun makes them go faster than usual, and the opposition of the sun has the opposite effect, before and after these points they are surely much removed from the positions they were going to occupy through the action of the first inequality. Therefore, at the very moments of conjunction with and opposition to the sun they are traversing their own true and proper positions. But since they cannot be seen when in conjunction with the sun, only the opposition to the sun remains as suitable for this purpose.

Terms.
*The sun's apparent position is that which it is perceived to occupy through its inequality.
The mean position is that which it would have occupied if it had not had its inequality.

But since the sun's mean and apparent motions* are two different things, for the sun, too, is subject to the first inequality, the question is raised which of these removes the second inequality from the planet, and whether the planets should be considered when at opposition to the sun's apparent position or its mean position. Ptolemy chose the mean motion, thinking that the difference (if any) between taking the mean sun and the apparent sun could not be perceived in the observations, but that the form of computation and of the proofs would be easier if the sun's mean motion were taken. Copernicus and Tycho followed Ptolemy, carrying over his asumptions. I, as you see in ch. 15 of my *Mysterium cosmographicum*, take instead the apparent position, the true body of the sun, as my reference point, and will vindicate that position with proofs in parts 4 and 5 of this work.

But before that, I shall prove in this first part that one who substitutes the sun's apparent for its mean motion sets up a completely different orbit for the planet in the ether, whichever of the more celebrated opinions of the world he follows. Since this proof depends upon the equivalence of hypotheses, we shall begin with it.

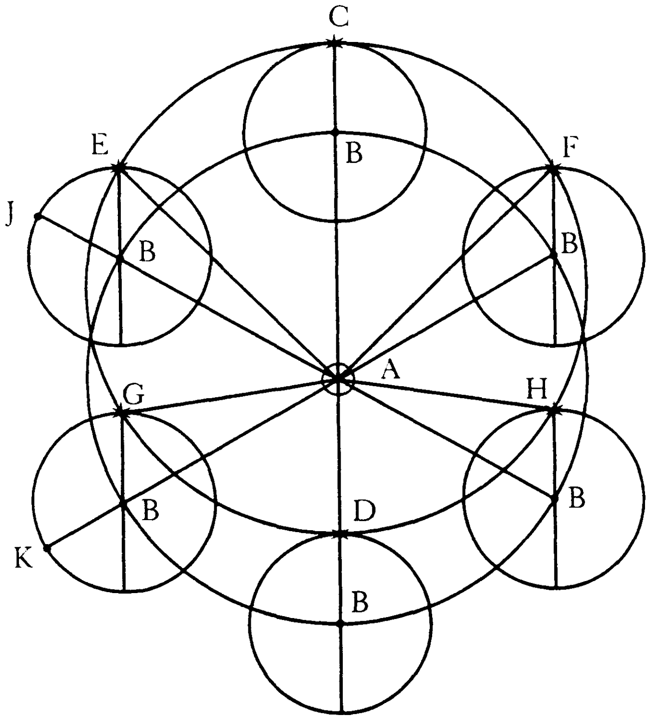
# 2

## On the first and simple equivalence, that of the eccentric and the concentric with an epicycle, and their physical causes

And now, to begin, I take up the equivalence of hypotheses adopted to save the appearances of the first inequality, which were demonstrated by Ptolemy in Book III and by Copernicus in Book III Ch. 15.

There, an eccentric is shown to be equivalent to an epicycle on a concentric, provided, that is, that the line of apsides in the eccentric and the line through the centre of the epicycle and the planet on the concentric always remain parallel, and that the semidiameter of the epicycle in the latter is equal to the eccentricity in the former, while the semidiameters of eccentric and concentric are equal. And also provided that, in the former, the planet is moved uniformly on its eccentric, so as to traverse equal arcs in equal times.

*First, let* A *be the position of the observer and the centre of the concentric* BB *on which is the epicycle* BC, BE. *Let the arcs between two* B*'s, or the angles* BAB, *be equal, and the planet be first at* C, *then at* E *and* G, *with the lines* BE, BG *parallel to* BC. *Next, let* $\beta$ *be the centre of the eccentric* $\gamma\zeta$, *with* $\beta\gamma$, $\beta\epsilon$ *equal to* AB, *and let* $\alpha$ *be the point at which the observer is, and* $\beta\alpha$ *(the eccentricity) be equal to the semidiameter of the epicycle* BC, BE, *and parallel to them. Also let the arcs* $\gamma\epsilon$, $\gamma\zeta$, *that is, the angles* $\gamma\beta\epsilon$, $\gamma\beta\zeta$, *be equal both among themselves and to the former angles* BAB. *I say that the distances* AC, $\alpha\gamma$, *are equal, and likewise* AE *to* $\alpha\epsilon$, AG *to* $\alpha\eta$, AD *to* $\alpha\delta$, AH *to* $\alpha\theta$, *and* AF *to* $\alpha\zeta$; *that the angles* EAC, $\epsilon\alpha\gamma$ *are equal; and that in each instance the planet, although its motion is uniform, appears from* A, $\alpha$ *to be slow at* C, $\gamma$, *and swift at* D, $\delta$. *As I said, Ptolemy demonstrated this in Book III, so there is no need for further discussion. To*


C
E
F
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B
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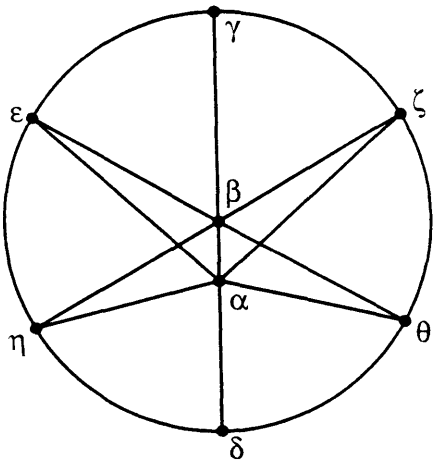
B


γ
ε
ζ
β
α
η
θ
δ

*geometers, the diagram speaks for itself, and others may go to Ptolemy.*

As for the physical accounts of these models, there is a greater difference between the two. That this may be very clear, it must be researched at greater depth, once from Peurbach[1] using Aristotle's principles, and again from Tycho.

Ptolemy has described these circles to us in their bare form, such as geometry applied to the observations shows. Peurbach set up a way for them to move around which follows Aristotle, who in turn had in mind the geometrical suppositions which Eudoxus and Calippus had used in their astronomy.

And while these authors had used 25 orbs to demonstrate all the inequalities of the planets, Aristotle (since he believed the heavens to be filled with solid orbs) thought that 24 others had to be interposed in order to free each lower orb from the revolving effect which, on account of the contiguity of surfaces, it was destined to receive from the orb above it. So, having thus accumulated 49 orbs in all (or 53 or 55, according to Calippus), he attributed to each its own mover. Each of these would be responsible for the perfectly uniform motion of its own orb and all inferior ones which it encompassed. This motion would take place inside the closest surrounding superior orb, as if in a sort of place[2], and from it would proceed a constant determination [*ratio*] of both the direction in which the motion was to occur, and the swiftness with which the orb was to return to its starting position. Moreover, since that philosopher held the motion to be eternal, he also stated that the movers were eternal. Since they created motion for an infinite time, and since Aristotle knew that nothing material could receive the form of infinity, he intended that they also be immaterial and separate principles of motion, and consequently immobile. Also, since he had evolved the world's eternity from the eternity of motion, and this duration had been essential, the goodness and perfection of the whole world, as opposed to destruction, which would have been bad, he therefore attributed to those principles the

[1] Georg Peurbach (1423–1461), an astronomer remembered for his work in trigonometry, as well as for his textbook on planetary theory. He had as his disciple Johannes Mueller of Konigsberg, better known by the Latinized form of the name of his home city, 'Regiomontanus'. In 1472, Regiomontanus published his mentor's chief astronomical work, entitled *Theoricae novae planetarum*, to which Kepler is referring.

[2] '. . . *Orbem suam proxime ambeunte, tanquam in loco quodam*, . . .'. This is a paraphrase of the Aristotelian theory of place, to which Kepler adhered, and would be recognized as such by all contemporary readers. See Aristotle, *Physics*, book IV ch. 4.

highest perfection and the understanding thereof, and from good understanding the will to see it accomplished, lest the good not be done well. In this way, he introduced to us separate minds which, it turned out, were gods, as the perpetual administrators of the heavens' motions. They also bestowed a moving soul [*anima motrix*], more closely attached to the orbs and giving them form, so that the mind would only have to give assistance. This was because it seemed necessary that the mover and the moving thing have some common ground, or so that the potentiality, considered in relation to the distance to be traversed, not be infinite, just as there exists no infinite motion, but only motion through a certain space in a certain time. They therefore made over this potentiality for creating motion to a soul, for the meantime becoming subject to matter for the purpose of inhering in the orbs of the heavens.

Now this coupling of mind and soul is indeed quite in agreement with the detailed considerations of the astronomers, even though the philosophers' arguments are chiefly metaphysical. For it is the same in humans: the moving faculty is different from the will, which makes use of the moving faculty, according to the indications of the senses. These differ from the moving faculty both in the means they use and in the excellence of their structure, which in the organs of sense is more admirable than in the seats of the motive faculty. Similarly, in proposing these Aristotelian orbs as objects of contemplation, two things will present themselves to us: 1) the motive force, sufficient for the round orb, from whose activity and constant strength the time of revolution arises; 2) the direction in which it acts. The former is more correctly ascribed to the animate faculty, and the latter to its intelligent or remembering nature. Now it is quite true that through this solidity of orbs all motions or celestial appearances are so provided for that nothing is left to the providence of the presiding movers. Indeed, the whole variety of motions is a consequence of the number and disposition of the orbs, nor is anything else required but that the moving souls receive and retain their activity, and be set going from the first moment of creation in whatever direction is theirs, sent forth from their prisons, as it were, into space. Nevertheless, it must be kept in mind that the function of the supreme mind is just this: to launch any of the planets in its direction, as if into its fixed and proper province. Aristotle, who knew nothing of the world's beginning and did not believe in it, of necessity ascribed this function instead to the governors of the motions. The followers of Aristotle,

including Scaliger[3], who professes to be a Christian, openly contend that this motion of the orbs is voluntary, and that the principle of that[4] volition is intellectual intuition and desire.

So, to return to Peurbach, certain others along with him (chiefly authors of books on the sphere), explain the first model by imagining for themselves a solid concentric orb of the thickness of the whole
8 epicycle, with an epicycle in it, and in the epicycle a planet. Then they attributed two moving souls to these two orbs (if they carried through with their physical considerations), both with the same amount of power, proportionally, so that they might complete their periods in the same time, although they are moving in opposite directions.

The other model requires two deferents (which remain motionless so long as we keep the motions simple, mentally removing the progression of the apogees), and one orb with the thickness of the planetary body. In this orb is a soul which drives it around with a uniform effort in that direction in which it was projected in the beginning. Thus, if the solidity of the orbs and the other assumptions be granted, in the first model BC and BE will remain parallel, and in the second, the orb $\gamma\epsilon$ will go around the centre $\beta$, even though the movers in the former pay no attention to AC, nor in the latter, to $\beta$. For they are governed by material necessity or by the arrangement and contiguity of the orbs.

But, with arguments of the greatest certainty, Tycho Brahe has demolished the solidity of the orbs, which hitherto was able to serve these moving souls, blind as they were, as walking sticks for finding their appointed road. This entails that the planets complete their courses in the pure aether, just like birds in the air. Therefore, we shall have to philosophize differently about these models.

Let it then be taken as a first principle, that each force by which motions of this sort are administered dwells in the body of the planet itself, and is not to be sought outside it.

Now the planet must execute a perfectly circular path in the pure aether by its inherent force, epicyclic in the first model and eccentric in the second. It is therefore clear that the mover is going to have two

3 Julius Caesar Scaliger (1484–1558), a mercenary turned scholar who is remarkable chiefly for his introduction of siege tactics into scholarly debates. He attacked the views of the mathematician, physician, philosopher, and gambler Girolamo Cardano (1501–1576) in a great, rambling work entitled *Exercitationum exotericarum adversus Cardanum liber quintus decimus* . . ., Paris 1557. The title, incidentally, suggests (falsely) that this is merely the fifteenth volume in a series of such onslaughts. On the present subject, see especially *exercitatio* 61.5, and 359.8.

4 Reading *illius* instead of *illis*.

jobs: first, it must have a faculty strong enough to move its body about, and second, it must have sufficient knowledge to find a circular boundary in the pure aether, which in itself is not divided into such regions. This is the function of mind. Please don't tell me that the motive faculty itself, as a member of the family of simple and brute souls, has a native aptitude for circular motion, exactly like a stone's nature to descend in a straight line. For I deny that God has created any perpetual non-rectilinear motion that is not ruled by a mind. Even in the human body, all the muscles move according to the principles of rectilinear motions. They either swell by contracting into themselves, so that the member approaches the muscle, or
9 stretch out, the ends moving apart, so that the member recedes. This takes place similarly, though in its own way, in the circular muscles which are set up in passages as guards. When they are extended by the circular filaments, they relax and open the passage, and constrict it when the filaments contract into the form of a smaller circle. There is no member whatever that rotates uniformly and without impediment. The bending of the head, feet, arms, and tongue is expressed in certain mechanical devices by the contraction of many straight muscles carried across from one place to another. In this way it is brought about that the motive faculty, which by its own nature tends in a straight line, swings its member in a gyre. Likewise, certain machines raise water to great heights, not because the nature of the body, which produces its motion, tends to an exalted position, but because, by an arrangement of channels, it is brought about that the water necessarily gives way upwards when a greater weight tends downwards. And even if the motion of certain members were perfectly circular, it nonetheless could not be perpetual. There should be no great wonder at this, since in the human body mind presides over the animate faculty. Surely, then, if there had been any way of so constructing some moving facility that some body might be able to rotate, it would not have been neglected in the human body.

Besides, it is quite impossible for any mind to manifest a circular path without the guidepost either of a centre or of some body which might appear under a greater or smaller angle according to its approach or recession. For a circle is both defined and brought to perfection by the same criterion, namely, equality of distance from the middle. No matter how many of these motive faculties you set up, a circle, even for God, is nothing but what was just said. Geometers do, of course, show how, given three points on a circumference, to

form a continuous circle, but this itself presupposes that some portion of the circumference (that which passes through the three points) is already constructed. Who, then, will show the planet this starting place, in conformity with which it will make the rest of its path? This is impossible unless the planet's mover (as in Avicenna's opinion) imagines for itself the centre of its orb and its distance from it, or is assisted by some other distinguishing property of a circle in order to lay out its own circle.

We will therefore now form the physical hypothesis of these two models in another way. In the latter, simpler one, if our supposition is valid that the mover driving the planet around the path $\gamma\epsilon\delta$ is in the planet itself, it would have been necessary that there accrue to the planet's mover some sort of notion of the apparent magnitude of the body at $\alpha$ seen (or as if seen) from $\gamma$, $\epsilon$, $\eta$, $\delta$. This is needed in order that the planet endeavour both to move forward uniformly (this is seen to by the undivided and unimpeded forces of the moving soul), and to exhibit all the distances $\alpha\gamma$, $\alpha\epsilon$, $\alpha\eta$, $\alpha\delta$, in such order as follows from the eccentric $\beta\gamma$ according to geometrical laws. To this end, the mover should also know how much longer $\alpha\gamma$ is than $\alpha\delta$; that is, by how much the path which is to be traversed is eccentric from the body at $\alpha$ around which it is to go. The planet's mover will thus be occupied with many things at once. To escape this conclusion, one must assert that the planet pays attention to the point $\beta$, entirely empty of any body or real quality, and maintains a constant distance from that point.

**10** The prior model is explained physically thus. Let a motive power be conceived which, seated on the concentric B and itself without body, moves around the body at A with a uniform exertion of forces, maintaining a constant distance from that point. Let there be another power in the body of the planet C, capable of holding its attention on the incorporeal power at B, estimating and maintaining its distance from that power, and moving uniformly around it. Thus, as before, this power again will have numerous tasks. But it is also incredible in itself that an immaterial power reside in a non-body, move in space and time, but have no subject, moving itself (as I said) from place to place. And I am making these absurd assumptions in order to establish in the end the impossibility that every cause of a planet's motions inhere in its body or somewhere else in its orb. In this way, I hope to clear the way for a persuasive presentation of other less difficult forms of motions.

I have presented these models hypothetically, the hypothesis being astronomy's testimony that the planet's path is a perfect eccentric circle such as was described. If astronomy should discover something different, the physical theories will also change.

The equivalence of these hypotheses lies not so much in the equality of the apparent angles at A and $\alpha$ as in the identity of the actual paths of the planets through the surrounding ether. For the size and shape of the planet's arc from C to E through the angle CAE are the same as the size and shape of the arc from $\gamma$ to $\epsilon$ through the equal angle $\gamma\alpha\epsilon$.

# 3

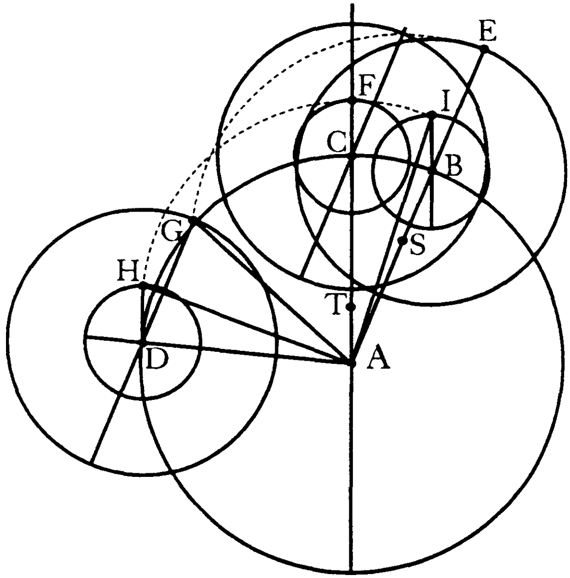
## On the equivalence and unanimity of different points of observation, and of quantitatively different hypotheses, for laying out one and the same planetary path

Next, I must show how one and the same planetary motion, while itself remaining the same, can display one or another appearance, and how the pair of forms are equivalent here.

*About centres* A *and* γ, *with equal radii* AC, γε, *let the circles* CD, εζ *be described with* CA, εγ *drawn through the centres parallel to one another, and other lines* AB, γδ, *and* AD, γζ, *through the centres inclined to the former, both pairs likewise parallel. Also, about* B *let an epicycle be described, with radius* BE, *and another about* D *with radius* DG *equal to* BE. *Let the planet be placed at* E *and* G, *with* DG
11 *and* AB *parallel. On the line* δγ *let a segment equal to* BE *be set out on the side opposite* δ, *and let it be* γβ. *Let* GA *and* ζβ *be joined. The hypotheses will be equivalent, by the preceding chapter, and to an observer placed at* A *and* β, EAG *and* δβζ *will be equal.* EA *and* δβ *will also be equal, as well as* GA *and* ζβ. *And finally, the arcs* EG *and* δζ *will be equal.*

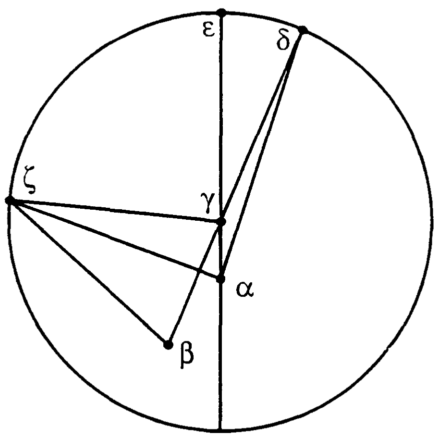
*Now let a smaller epicycle be described upon* BCD *with radii* BI, CF, DH, *and let* AC *be extended to* F, *and* BI *and* DH *be parallel to* CF. *And let the planet be on* IFH. *Again, by ch. 2, the circle* IFH *will be equal to the circle* δζ.

*Next, extend the arc* IF *from the point* δ, *so as to end at* ε, *and from* ε *through* γ *draw* εγ, *such that* εγ *is parallel to* CA. *And let a magnitude equal to* CF *be set out on the line* εγ, *and let it be* γα, *on the side opposite* ε. *Let* I *and* H *be connected to* A, *and also* δ *and* ζ *to* α. *Again, therefore, the hypotheses will be equivalent by the preceding chapter, and to an observer placed at* A *and* α, FAH *and* εαζ *will be equal, as*




*well as* FAI *and* $\epsilon\alpha\delta$. FA *and* $\epsilon\alpha$ *will also be equal, as well as* HA *and* $\zeta\alpha$, *and* IA *and* $\delta\alpha$. *And finally, the arcs* FH *and* $\epsilon\zeta$ *will be equal and similar, as are also* FI *and* $\epsilon\delta$, *by construction.*

Therefore, if the path of the planet remain the same while the observer is moved from $\beta$ to $\alpha$, different appearances will be



produced at the same moments of time. For the same places, δ and ζ, are beheld in different ways from β and α. On the other hand, if the observer remain at A, and the planet's path EG, IH remain quantitatively the same while changing its place, the planet will again appear in different places, even when at the same place on its path, because the entire path has been shifted. Accordingly, since the planet, whether viewed from α or β, is at δ, or at ζ, at the same moment in each observation, and the hypotheses are entirely equivalent, it must also be said that I and E, which are positions of different epicycles, are occupied by the planet at the same moment. The same is true of G and H. The only difference is that in the first diagram the planet's path is shifted by changing the position of its epicycle, the observer remaining in the same place, while in the second diagram, the position of the planet's path remains fixed while the observer's position is changed by the same amount in the opposite direction. It is, however, possible, if required, to keep the path in the former, and the observer in the latter, fixed, by shifting what is now fixed, in accordance with the demonstrations of the preceding chapter.

12 This demonstration will be put to use below. For if the first inequality of the superior planets could be accounted for by the simple hypothesis of the second chapter, no difficulty would arise as to whether one should examine the first inequality at mean opposition to the sun, or at apparent opposition. For the actual path would in fact remain the same, and in both models the planet would be at the same points of the path at any given moment. Only the position of this path, in the first model, would be moved through a distance equal to the sun's eccentricity, while in the second, although the path would stay fixed, the point whence the eccentricity is reckoned would also be shifted by the same amount.

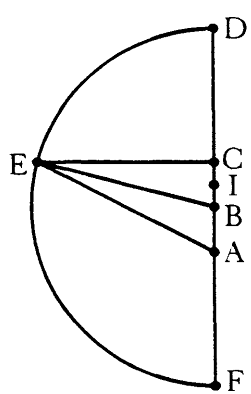
In the physical account, the above characteristics remain unchanged. Only the quantities change as the motive powers are intensified.

# 4

## On the imperfect equivalence between a double epicycle on a concentric, or eccentric-epicycle, and an eccentric with an equant

That is how it would be if the simple hypothesis of chapter three could be used to account for the first inequality of the superior planets. However, in demonstrating the first and simple inequality of the planets, Ptolemy makes use of a more complex hypothesis.

*About centre* B *let an eccentric* DE *be described, with eccentricity* BA, A *being the place of the observer. The line drawn through* AB *will indicate the apogee at* D *and the perigee at* F. *Upon this line, above* B, *let another segment* BC *be extended, equal to* BA. C *will be the point of the equant, that is, the point about which the planet completes equal angles in equal times, even though the circle is set up around* B *rather than* C.



In Book V ch. 4, as well as Book IV ch. 7[1], Copernicus notes this hypothesis among other things in this respect, that it offends against physical principles by instituting irregular celestial motions. *For let a point* E *be chosen on the circle which the planet is physically traversing, and let it be connected with* C, B, *and* A. *Now let* DCE *be a right angle, as well as* ECF. *Now since these angles are equal (for they are traversed in equal times), and the exterior angle* DCE *is equal to the interior angles* CBE *and* CEB, *therefore, when the part* CEB *is subtracted, the remainder* CBE *or* DBE *will be less than* DCE. *Consequently,* FBE *will be greater than* DCE *or* FCE. *But the arc* DE *measures the angle* DBE, *and the arc* EF *measures the angle* EBF. *Therefore,* DE *is smaller than* EF, *and the planet passes over them in equal times. Therefore, the same solid orb (Copernicus believed in them)*[2] *in which the planet inheres is slow when the planet borne by the orb proceeds from* D *to* E, *and fast when the planet goes from* E *to*
13 F. Therefore, the entire solid orb is now fast, now slow. This Copernicus rejects as absurd.

Now I, too, for good reasons, would reject as absurd the notion that the moving power should preside over a solid orb, everywhere uniform, rather than over the unadorned planet. But because there are no solid orbs, consider now the physical coherence of this hypothesis when very slight changes are made, as described below. This hypothesis, it should be added, requires two motive powers to move the planet (Ptolemy was unaware of this). It places one of these in the body A (which, in the reformed astronomy will be the very sun itself), and says that this power endeavours to drive the planet around itself, but possesses an infinite number of degrees corresponding to the infinite number of points of the distance from A. Thus, as AD is the longest, and AF the shortest, the planet is slowest at D and fastest at F, and in general, as AD is to AE, so is the slowness at D to the slowness at E[3], as will be demonstrated at great length in Part III

[1] The passages in which Copernicus presents the argument to which Kepler refers are in Book IV ch. 2 and Book V ch. 2.

[2] Again, it should be remarked that it is probably anachronistic of Kepler to ascribe this opinion to Copernicus. Although it is true that Copernicus believed in the physical truth of his hypothesis, and consequently in the reality of the orbs, it does not follow that the orbs had to have the terrestrial properties of hardness and impenetrability. On the contrary, natural philosophers contemporary with Copernicus generally held (with Aristotle) that the heavens are devoid of such qualities. For a thorough discussion of Copernicus's orbs, see N. Jardine, 'The Significance of the Copernican Orbs', *Journal for the History of Astronomy* 13 (1982), especially pp. 174–180.

[3] Kepler measures the degree of slowness by finding the elapsed time over a certain arc or path element. See ch. 32, footnote 6 to the Introduction, and the beginning of ch. 40.

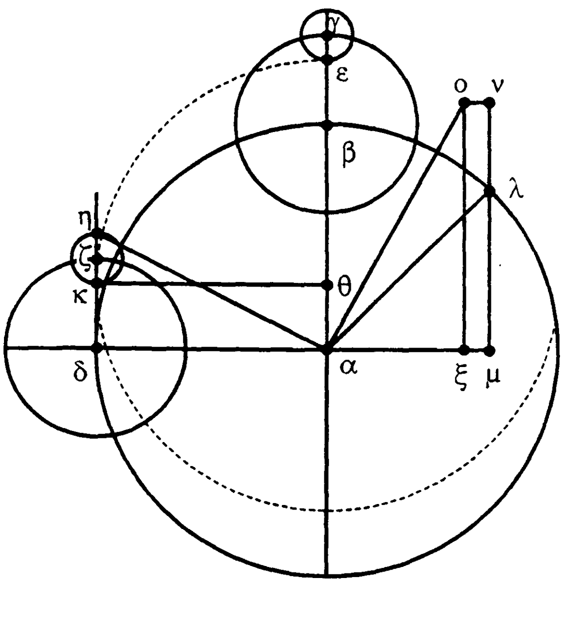
below. The hypothesis attributes another motive power to the planet itself, for which it functions to adjust its approach to and recession from the sun, either by the strength of the angles or by intuition of the increase or decrease of the solar diameter, and to make the difference between the mean distance and the longest and shortest equal to AB. Therefore, the point of the equant C is nothing but a geometrical short cut for computing the equations from an hypothesis that is clearly physical. But if, in addition, the planet's path is a perfect circle, as Ptolemy certainly thought, the planet also has to have some perception of the swiftness and slowness by which it is carried along by the other external power, in order to adjust its own approach and recession in such accord with the power's prescriptions that the path DE itself is made to be a circle. It therefore needs both an intellectual comprehension of the circle and a desire to effect it. Also, the ratios of its own slowness and swiftness must differ from the degrees of intensity of the external power. However, if the demonstrations of astronomy, founded upon observations, should testify that the path of the planet is not quite circular, contrary to what this hypothesis asserts, then this physical account too will be constructed differently, and the planet's power will be freed from these rather troublesome requirements.

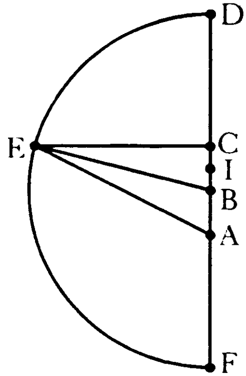
But let me return to Copernicus. He avoided the above-mentioned absurdity, which arose as a consequence of his beliefs, by substituting another epicycle for the equant, in the following way. *About centre* $\alpha$ *with radius* $\alpha\beta$ *equal to* BD *let the concentric* $\beta\delta$ *be described, with the observer at* $\alpha$*; let* $\alpha\beta$*, parallel to* BD*, be extended in both directions; and let the angle* $\beta\alpha\delta$ *be set up equal to* DCE. *Now let* BC *be bisected at* I, *and about the centres* $\beta$ *and* $\delta$ *with radii* $\beta\gamma$ *and* $\delta\zeta$ *equal to* AI *let the first or greater epicycle be described, and let* $\delta\zeta$ *be parallel to* $\alpha\beta$. *Next, about centres* $\gamma$ *and* $\zeta$*, but with radii* $\gamma\epsilon$, $\zeta\eta$ *equal to* IC, *let the second epicycle be described, and let its motion be eastward, with twice the speed of the motion of the first. And let the westward motion of the first epicycle be equal to the motion of the eccentric. And since* $\gamma$ *is on* $\alpha\beta$,
14 *let the planet be at* $\epsilon$*, the point nearest* $\beta$. *And since* $\beta\alpha\delta$ *is right, let the planet be at* $\eta$*, the point farthest from the centre of the greater epicycle* $\delta$. This particular hypothesis of Copernicus is also followed by Tycho Brahe religiously, in all particulars.

Physically considered, this hypothesis is in any event valid if you grant solid orbs. If, however, you remove these, as Brahe does with good reason, it says something practically impossible. For in addition

to its attaching three movement-producing minds to a single planet, the others will be thrown into confusion by the motion and impulse of any one towards the body at α. For that any of them should pay heed to its own centre, which is not distinguished by any body and is mobile besides, cannot be represented even in thought. Further, while Copernicus strives to outdo Ptolemy in the uniformity of motions, he is in turn outdone by him in the perfection of the planetary path. For, in Ptolemy, the planet bodily traces out a perfect circle in the aethereal air. Copernicus, on the other hand, says in Book V ch. 4 that for him the path of a planet is not circular, but goes outside the circular path at the sides. This is easily demonstrated in the present diagram.

*If from ε, the planet's position at apogee, you extend the distance αβ, the semidiameter of the orb, to θ, and from θ construct θκ parallel to αδ, the circle εκ described about θ will indeed go through ε and the perigeal point opposite, but since it touches the straight line δη only at κ, while the planet goes through η, it does not stay on the circle εκ, but strays outside this track.* To this excursion of the planetary path from the perfection of the circle Ptolemy might well have objected against Copernicus, but I do not. For below, in Part IV, it will be demonstrated that by the agency of two physical powers with simple capabilities, acting in concert to move the planet, it necessarily






Terms: The mean anomaly is the time elapsed since the planet was at apogee, expressed according to an arbitrary rule: the whole time in which the planet makes one return from apogee to apogee is divided into 360 degrees, like a circle.

The true anomaly is the arc of the zodiac between the position of the apogee and the apparent position of the star, viewed from the centre of the zodiac.

The equation is the difference between the two anomalies.

happens that the planet shortly turns aside from the circle, though not by running outside of it, as in this Copernican hypothesis, but contrariwise, making an incursion towards the centre.

Besides, should Copernicus retain the liberty of setting up the ratios of the epicycles, it can happen that the planet's path would come out twisted, higher before and after apogee than at apogee itself, and lower before and after perigee than at perigee itself. This happened to Tycho in his lunar theory, inasmuch as he followed Copernicus.

That these two forms of hypothesis are not simply equivalent, I shall demonstrate numerically.

In the Ptolemaic form it can be computed more simply than Ptolemy did it in the following manner. *First, in triangle* CBE, *given the mean anomaly* ECB *or* DCE, *the side* CB, *the eccentricity of the equant, is also given, as well as the radius of the orb,* BE. *Therefore, as the radius of the orb is to the sine of* ECB, *so is* CB *to the sine of* CEB. *And since* ECD *is equal to the two opposite interior angles* CEB *and* CBE *taken together,* CEB *subtracted from* DCE *leaves* CBE. *Therefore, in triangle* EBA, *the angle at* B *is given, together with the sides about it. For* BA *is the eccentricity of the eccentric, while* EB *is the radius of the orb. Therefore, following the rule for this form of triangle, the angle* BEA *is given. But* CEB *was given before. Therefore, the whole equation* CEA *will be given.*

We shall now make use of numbers belonging to the motion of Mars. Although Ptolemy made CB and BA equal, Copernicus, freed from this rule, adopted other ratios, which Tycho Brahe undertook to

imitate. *Let* CB *be 7560,* BA *12,600, where* BE *is 100,000; and, first, let* DCE *be 45°, whose sine is 70,711. Therefore, as 100,000 is to 70,711, so is 7560 to 5346, the sine of 3° 4′ 52″, which is* CEB. *Subtracting this from 45° leaves* CBE, *41° 55′ 8″ whose half is 20° 57′ 34″, whose tangent is 38,304. And since* EB *is 100,000, while* BA *is 12,600, the difference, 87,400, multiplied by the radius and divided by the sum, 112,600, gives 77,620. Multiply this by the tangent found above (38,304). The product, 29,732, is the tangent of the arc 16° 33′ 30″, which, subtracted from the half of* CBE, *found above, leaves 4° 24′ 4″ which is the angle* BEA. *Therefore, the whole,* CEA, *is 7° 28′ 56″,* in the Ptolemaic form. In the Copernican form, although the ordinary means of finding the equation is clearly presented in Tycho's lunar tables in Vol. I of the *Progymnasmata*, and in Copernicus himself, let me nonetheless follow a different, less usual procedure, which is peculiarly adapted to an anomaly of 45°. *Let* $\beta\alpha\lambda$ *be 45°, and* $\lambda\nu$ *or* $\beta\gamma$ *be 16,380,* $\gamma\epsilon$ *or* $\nu o$ *be 3780, and* $o\nu\lambda$ *be right, that is, twice* $\beta\alpha\lambda$. *Now let* $\nu\lambda$ *be parallel to* $\beta\alpha$, *and let* $\nu\lambda$ *and* $\delta\alpha$ *be extended, so as to meet at* $\mu$. *From* $o$ *let* $o\xi$ *be dropped parallel to* $\nu\mu$. *Therefore,* $\lambda\alpha\mu$ *is 45°, and consequently* $\alpha\mu$, *and also* $\mu\lambda$, *are 70,711. Add* $\lambda\nu$, *16,380, and* $\mu\nu$ *or* $o\xi$ *will be 87,091. And because* $\gamma\epsilon$, $\nu o$, *and* $\xi\mu$ *are equal, subtract* $\xi\mu$ *from* $\alpha\mu$. *The remainder,* $\alpha\xi$, *is 66,931. Therefore, as* $o\xi$ *is to* $\xi\alpha$, *so is the whole sine*[4] *to the tangent of* $\alpha o\xi$ *or* $o\alpha\beta$, *76,852, giving an angle of 37° 32′ 37″, which differs from the arc 45° by 7° 27′ 23″. Therefore, the difference between the Copernican and the Ptolemaic equations at this position is 1′ 33″, a very small difference indeed.*

*Again in the Ptolemaic hypothesis, let* DCE *be 90°. Therefore, since* ECB *is right, and* EB *is 100,000,* BC *will be the sine of the angle* CEB, *or 4° 20′ 8″. Therefore,* EBC *is 85° 39′ 52″, and* EC *is 99,713. Now, as* EC *is to* CA, *so is the radius to the tangent of* CEA, *20,218. Hence, the equation,* CEA, *is 11° 25′ 48″. But in the Copernican form, the whole magnitude* $\eta\delta$, *equal to* CA, *becomes the tangent, because* $\eta\delta\alpha$ *is right and* $\delta\alpha$ *is the radius. Therefore,* $\eta\alpha\delta$ *is 11° 23′ 53″. The difference is 1′ 55″.*

Terms: The equation of the eccentric is in the first inequality. The equation of the orb is in the second inequality. Likewise the annual equation of the centre.

Thus you see that, as far as the eccentric equation is concerned, there is a very slight difference preventing the two forms of hypothesis from being equivalent.

They are different, however, in the distances of the planet from the observer at $\alpha$, and as a consequence, in the annual equations of the

[4] That is, the radius.

centre as well. For, in the Ptolemaic form, as the sine of the angle AEC is to AC, the whole sine is to AE, which becomes 101,766 when
16 DCE is 90°. But in the Copernican, $\eta\alpha$ is the secant of angle $\eta\alpha\delta$, that is, 102,012. The difference is 246 parts, and this can have a somewhat greater effect upon the equation of the centre for the annual orb, as will be clear below in Part IV. We can eliminate even this extremely slight difference in the equations by introducing into the Ptolemaic form 20,103 as Mars's eccentricity where Brahe, in the Copernican form, found it to be 20,160. However, the distances in the Copernican form cannot be made equal to those in the Ptolemaic unless the equation be altered by 43′. In the hypothesis for Tycho's lunar tables, I attempted a reconciliation by transposing those two Copernican epicycles into such a Ptolemaic eccentric with an equant. Nevertheless, I added yet another epicycle on account of another inequality, peculiar to the moon.

Finally, in accord with chapter 2, the greater epicycle with its concentric in the Copernican form can, by virtue of its complete equivalence, be transformed into an eccentric whose eccentricity is equal to the semidiameter of the greater epicycle. Therefore, when a smaller epicycle is added to this Copernican eccentric, an eccentric with an epicycle is created which is in every respect equivalent to the double epicycle on a concentric, and which differs from the Ptolemaic eccentric with an equant by no more than does this double epicycle.

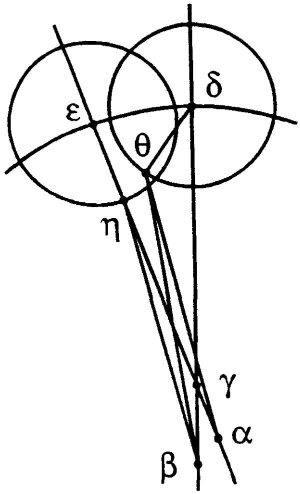
# 5

## The extent to which this arrangement of orbs, using either an equant or second epicycle, while remaining entirely one and the same (or very nearly one and the same), can present different phenomena at one and the same instant, according to whether the planets are observed at mean opposition to the sun, or at apparent opposition

17 This is done in two ways: one, in which the Ptolemaic and Copernican forms are equivalent, and another which is peculiar to the Copernican form. This latter, as it is further from what we are doing, we shall explain first, for it remains comparatively aloof from other considerations.

Term. The word 'eccentric' has a peculiar denotation here.

*About centre $\gamma$, with radius $\gamma\delta$, let an eccentric be described, upon which, in the first instance, let $\alpha\gamma$ be the line of apsides, with $\alpha$ the observer, and let this line be extended to $\epsilon$. And let $\gamma\alpha$ be the magnitude of the eccentricity, or the radius of the greater Copernican epicycle (the equivalence of the two was discussed at the end of chapter 4, preceding). Next, about centre $\epsilon$, with radius $\epsilon\eta$, let the smaller epicycle be described, and since its centre is at $\epsilon$, let the planet be at $\eta$, on the line $\epsilon\gamma$, so that the eccentric $\epsilon\delta$ passes through the centre of the epicycle bearing the star rather than the star itself. Thus, by chapter 4, it is the Copernican form that is expressed here. Upon it, we shall set up another, which, by what was said in chapter 3, is actually or virtually equivalent to the true planetary path, but differs from it in the appearances produced. This we shall do by moving the observer from $\alpha$. By what was said at the end of chapter 3, we could have done the same thing even if the observer were to remain at $\alpha$, by moving the eccentric, keeping all lines parallel, in such a way that the size of the*




*eccentric remains constant and only its position changes. But we shall carry it to completion as we have begun it. Choosing a position for the observer not on the prior line of apsides (let it be* β*), such that* βγ *has a magnitude different from that of* αγ *(that is, a new eccentricity or radius of the greater epicycle), let us draw a new line of apsides* βδ *through* βγ*, and about* δ *let us describe an epicycle equal to the former. Here, although the centre of the epicycle is at the apsis* δ*, we shall nonetheless not place the planet at the point nearest* γ*, as before, but, taking the measure of the angle* εγδ*, we shall set out the angle* θδγ *twice that size, in the direction of* ε*, and shall place the planet at* θ*, when the epicycle is at the apsis* δ*. For this is where the planet would be, were the observer at* α *and the epicycle at* δ*.* In this way, the true compound planetary path remains the same to a hair's breadth, while the appearances are altered. For when the lines of sight are inclined to one another, as βθ and αθ here, or βη and αη, they have different sidereal positions.

You may object that even when the lines of sight are parallel, they have different sidereal positions, and it is therefore irrelevant whether or not they are inclined to one another. I answer, this is indeed true, but in that case the distance on the fixed stars between the two lines is not perceptible to the power of sight unless the distance between the parallels is appreciable in relation to the radius of the fixed stars.

In the physical account, this must be added to what was said in chapter 3, in order to establish the identity of the path while the appearances are altered: the mind to which the smaller epicycle is committed must pay attention to a point different from the one regarded by the mind of the greater epicycle. For in the second instance, the greater epicycle or the eccentricity returns to its starting point on the line $\beta\delta$, while the smaller epicycle does so on the line $\alpha\epsilon$ which does not pass through the place of the observer. This is because, in the second instance, the observer is located at $\beta$, while in the first instance, where the observer is located at $\alpha$, both epicycles
18 return to their starting points on the same line $\epsilon\alpha$. Therefore, the form of the hypothesis does not remain simply the same physically, so that the planet has the same path. But suppose you were going to create the same path artificially in the second instance by having both epicycles return to their starting point at the same line of apsides $\beta\delta$. If so, while the eccentric as well as the epicycle stay the same in both instances, the position of the planet on the epicycle will not be the same at one and the same moment. Thus, although in the second instance the form of Ptolemaic hypothesis presented is the same to a hair's breadth, the actual path of the planet is altered. Hence, it is inferred below, even though the first inequality of the planets may be entirely accounted for by the compound hypothesis of chapter 4, it cannot happen that the first inequality have the same measure at the planet's mean opposition to the sun as at apparent opposition, unless the planet's orbit change its position at the same time (unlike the circles in the theory of the sun), or the Ptolemaic form of chapter 4 be changed.

Maestlin made use of this form of transposition in constructing the table in chapter 15 of my *Mysterium cosmographicum*. For when Copernicus transformed the Ptolemaic hypothesis into his own general form, he supposed the observer to be stationed at some nearly motionless point near the sun, distant from the sun's own body by the entire eccentricity of the solar orb. I, however, in adapting Copernicus to the subject matter of that book, made use of a different fiction. The observer was to be imagined as transported from that point to the very centre of the solar body, and from there (that is, from the body of the sun) the departures of the planetary bodies were to be computed, moving on the same path which the suppositions of Copernicus formed out. But, as has just now been shown, in respect to the particular times my translation of the line of apsides did not

effect exactly the same path. The difference, however, was very slight, and was clearly of no importance in that little book. For there the question concerned only the position of the path, which this procedure did not affect.

Term. What the word 'eccentric' will signify henceforth.

As for the rest, in order to avoid confusion in what follows, I shall no longer make use of this Copernican eccentric, described by the centre of the epicycle rather than the star. It differs from the planet's true path, which is higher at perigee and lower at apogee. The term 'eccentric' from now on we will use only to designate the actual path of the planet, or of the point to whose motion the first inequality belongs. In adopting this procedure, we are restricting ourselves to imagining only the Ptolemaic eccentric, or something like it. For it was shown in the fourth chapter that our computation of the equation, based upon the Ptolemaic form, is going to differ from the Copernican by only two minutes at most. Then, too, the procedure for computing the first inequality is easier in the Ptolemaic form than in the Copernican. Finally, as was said, the Ptolemaic form of the first inequality is better accommodated to nature herself and to our speculations in the third and fourth parts. However, because of the
19 equivalence, anyone who so chooses can always read in the Copernican eccentric-cum-epicycle considered up to this fifth chapter.

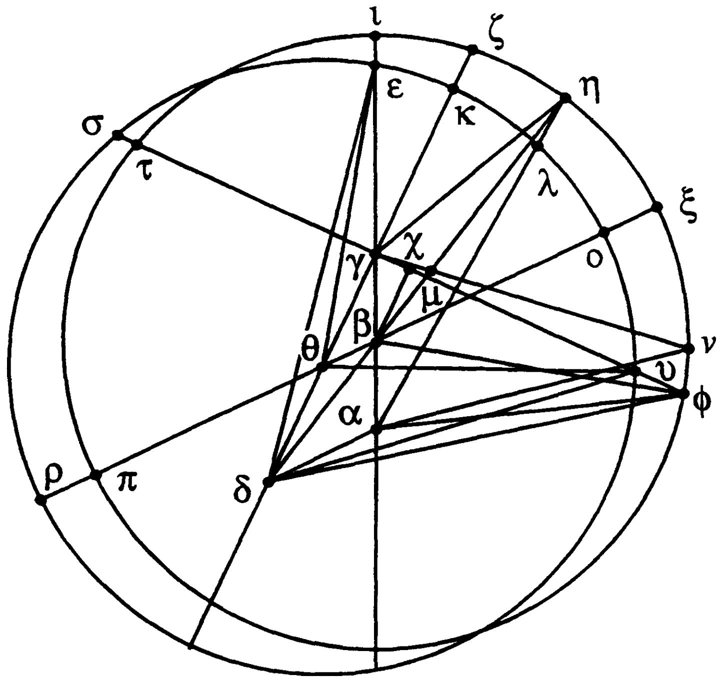
I now proceed to the prior of the two sorts of equivalence that I have proposed to establish, the one common to the particular hypotheses of the authorities. I shall demonstrate this first in the Ptolemaic form.

*About centre* $\beta$ *let the Ptolemaic eccentric* $\iota\zeta\eta$ *be described, with* $\iota\beta$ *the line of apsides,* $\alpha$ *the observer, and* $\gamma$ *the equalizing point*[1].

Now when I say that the observer is at $\alpha$, I mean it either as a fiction or as truth. Physically speaking, it is not so much the observer which is to be placed at $\alpha$ as the power which brings about a planetary circuit around itself that is swifter or slower according to the ratios of its proximity to $\alpha$, as was said above. *Let some point on the circumference not on the line of apsides (say,* $\eta$ *) be connected with* $\gamma$, $\beta$, *and* $\alpha$. *It shall so be that about as many angles* $\iota\alpha\eta$ *may be computed by this hypothesis throughout the entire circuit as are observed from* $\alpha$, *after certain periods of time, which the angle* $\eta\gamma\iota$ *measures uniformly. Later, in the second part, it will be shown how one can find, through astronomical observations, how great the angle* $\eta\alpha\iota$ *should be for any*

[1] *Punctum aequatorium.*

*given* ηγι. *Again, let the observer or moving power be at some point not on the line* ια , *and let this be* δ. *Also, let us be given the apparent angles about* δ *that would be apprehended by astronomical observations at certain times; that is, how much the planet would appear to move forward in sidereal position in a given time when seen from* δ. *Let this be given as well: that these appearances at* δ *square with a hypothesis in conformity with the previous one, with only the magnitude of the eccentricity altered. But since it is certain that at one and the same time the planet traverses one and the same path in the heavens, not one seen from* δ *and another from* α, *it is also certain, as a consequence, that the planet cannot appear to both observers (both the one at* α *and the one at* δ) *to be equally moved in the same time. For let* ιη *be a portion of the planet's true path, and let the planet traverse this in a given time, say twenty days. Now since* α *is nearer* ιη *than* δ *is,* ιη *will appear greater at* α *than at* δ, *by what is demonstrated in optics. Therefore, during the same twenty days the planet will appear to make greater progress to one who is at* α *than to one who is at* δ. *And since for each planet there is a fixed and constant number of days which it takes*



*to return to the same sidereal position, the slowness has to be matched by a compensating speed. Therefore, since in the portion $\iota\eta$ the planet appears slower to one at $\delta$, it will in some other portion appear swifter to the one at $\delta$ than to one at $\alpha$. Hence it happens that it appears slowest to the one at $\delta$ in one place, and to one at $\alpha$ in another. Nevertheless, the planet itself can be truly slowest in but one place on its orbit.*

20 With these as preliminaries, the question is raised whether one and the same true path of the planet in the heavens (this is presupposed) can present two sets of appearances, one to the observer at $\delta$ and another to the one at $\alpha$, both proper to those places and both such as comply with and admit the Ptolemaic form of computation.

If the planet were of equal speed at all parts of its orbit, the answer, according to what was said in chapter 3, is yes. But since, in terms of real and true elapsed time[2], the planet is slowest at one point on the eccentric, and fastest at the opposite point, the answer must therefore be, clearly not.

The reason for this is that the two retardations are intermingled, one real and physical, occurring at one place on the eccentric, and the other optical and apparent, and not occurring at a single place, but in the place most distant from whatever position is chosen for the observer. *Now, when the observer $\alpha$ lies upon a line through the centre of the eccentric $\beta$ and the centre of the equant $\gamma$ on the side of $\beta$ opposite the centre of the equant $\gamma$, then both retardations occur about the same sidereal position $\iota$. But when the observer departs from this line, as at $\delta$, then a straight line drawn from $\delta$ through the centre of the circle $\beta$ marks the place of the optical retardation, $\eta$, while the true and physical retardation is at $\iota$. Furthermore, each of these inequalities or retardations dilutes the other, and their sum is greatest at an intermediate point between $\iota$ and $\eta$, as would be if a line were to be drawn from $\delta$ through $\gamma$ to the point $\zeta$. Consequently, were one to adopt a form of computation in which $\delta\beta$ were the line of apsides of the eccentric and $\beta\gamma$ the line of the equant's eccentricity, then even though the planet's true path $\iota\eta$ remained the same, it would be represented differently at $\delta$ than at $\alpha$. For to the observer at $\delta$ the planet would be slowest at $\zeta$, and to the one at $\alpha$ it would be slowest at $\iota$. In addition, the appearances, as represented at $\delta$, would not be such as, according to our presuppositions above, ought to have been represented by a hypothesis of the same form as the previous one. For the forms of the hypotheses differ*

[2] *Mora*. See the Glossary.

*in that in the former,* $\beta$ *is the midpoint between* $\alpha$ *and* $\gamma$ *(as physical considerations require, if the moving power is in* $\alpha$*), while in the latter the centre of the eccentric* $\beta$ *would not be the midpoint between* $\delta$ *and* $\gamma$*, nor would the line of the equant's eccentricity pass through the observer* $\delta$*, as before. Even if it did pass through* $\delta$*, as* $\delta\gamma$*, it would not cut the eccentric into two equal parts, because it would not pass through the centre* $\beta$*, and the planet would not appear fastest and slowest, respectively, at opposite places.*

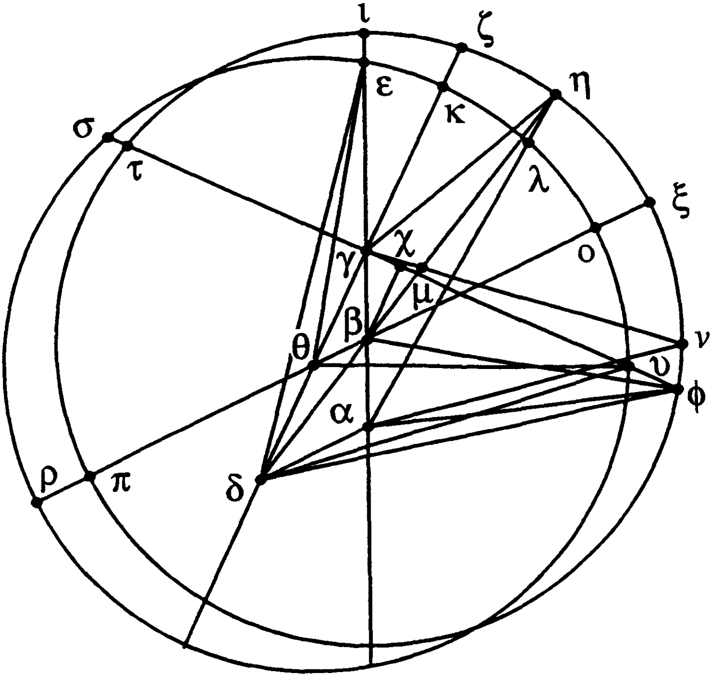
It is thus established that when a planet's path in the heavens remains in all respects unchanged, the form of hypothesis cannot persist entirely unchanged. The question is now raised, how much the path of a planet would have to be changed from its prior position if the same form of hypothesis is set up about $\delta$, and how much this newly established hypothesis at $\delta$ will be at variance with the appearances as seen at $\alpha$. *First, if the centre of the equant be transferred from* $\gamma$ *to the line* $\delta\beta$*, and* $\beta\mu$ *be made equal to* $\beta\gamma$*, the position of the planetary path is quite unchanged, but the planet is slowest, in physical terms, at* $\eta$ *rather than* $\iota$*. This changes what cannot be changed in the planet's*
21 *path, because, unlike the optical retardation, the physical retardation is independent of the observer's viewpoint. Even though the planet would traverse the same path* $\iota\eta$ *in twenty days (which path appears greater at* $\alpha$ *and smaller at* $\delta$*), nevertheless, if you consider the parts of this time, their ratio in comparison with the parts of this path will be violently perturbed, and much more so at other places, not between* $\iota$ *and* $\eta$*. In particular, the quantities of the equations will be changed noticeably for the observer at* $\alpha$ *if, for the observer at* $\delta$*, the planet is no longer slowest at* $\iota$*; that is, if you transfer the equant from* $\gamma$ *to* $\mu$*. For if you draw a straight line through* $\gamma\mu$ *to the point* $\nu$ *on the circumference and connect* $\alpha\nu$*, this equation* $\alpha\nu\mu$ *alone will be equal to the prior,* $\alpha\nu\gamma$*. Above* $\nu$ *the equations about* $\mu$ *will be smaller, and below* $\nu$*, greater. For example, at* $\eta$ *the angle* $\mu\nu\alpha$ *is much less than* $\gamma\eta\alpha$. But then what we proposed to do has not been done, for the prior form of hypothesis has not quite been established. *For* $\alpha\beta$ *is not to* $\beta\gamma$ *as* $\delta\beta$ *is to* $\beta\mu$*, since* $\beta\mu$ *is equal to* $\beta\gamma$*, while* $\delta\beta$ *is greater than* $\alpha\beta$*. But if, on the other hand, you make* $\delta\beta$ *be to* $\beta\mu$ *as* $\alpha\beta$ *is to* $\beta\gamma$*,* $\beta\mu$ *becomes greater than* $\beta\gamma$*.* Whence it follows that the observer at $\alpha$ is going to be much more in error as to the equation, even when it is at a maximum, on account of the increased eccentricity. Not only will the planet be slowest in a different place than before, but in addition the measure of its true slowness will be different, and also greater. It appears, therefore,

that the equivalence we have been seeking cannot be established by drawing the line of apsides from $\delta$ through the centre of the eccentric $\beta$. And since it has at the same time become clear how important it would be to keep the same equant point $\gamma$, a breakthrough must in all events be made here, or nowhere.

But what will happen if a new line of apsides be drawn from $\delta$ through the old equant point $\gamma$, and a new hypothesis of the same form as the old be set up? That is, if the centre of the eccentric be transferred from $\beta$ to the line $\delta\gamma$, and $\delta\theta$ be made to $\theta\gamma$ as $\alpha\beta$ is to $\beta\gamma$, $\theta$ thus being the centre of the eccentric? Obviously, the result will be this: the path of the planet in the heavens will not be entirely the same. *About* $\theta$ *let the eccentric* $\epsilon\kappa\lambda$ *be described, equal to the previous one, and through* $\theta\beta$ *let a straight line be extended to the circumference, on one side to* $\xi o$ *and on the other to* $\rho\pi$. *Therefore,* $\xi o$ *and* $\rho\pi$ *are both of the same magnitude as* $\theta\beta$, *and the planet is this much closer to* $\beta$ *at* $o$, *and more remote at* $\rho$, *than it would have been had it traversed the previous eccentric. However, the region in which the planet is slowest is different, for previously the apsis was at* $\iota$, *and now it is at* $\kappa$. Through this felicitous combination it is brought about that the observer previously stationed at $\alpha$ has his observations pretty much unchanged, which is the only thing sought for here. But now we shall prove it with numbers belonging to Mars's motion, although they are somewhat different from Brahe's. This should prove no impediment to us, since we are only performing a preliminary exercise here.

*Let the magnitudes on* $\delta\gamma\alpha$ *be taken as follows: let* $\delta\alpha$ *be the quantity of the sun's eccentricity, 3584;* $\delta\gamma$ *the eccentricity of Mars, 30,138 of the same parts, and the angle* $\alpha\delta\gamma$ *47° 59¼′, which is the angular difference between the sun's and Mars's apogees. Now, from these three given quantities,* $\gamma\alpha$, *Mars's new eccentricity, is also given, and is 27,971, while the angle* $\delta\gamma\alpha$ *is 5° 27′ 47″. On the supposition that the old apogee of Mars* $\delta\gamma$ *is positioned at 23° 32′ 16″ Leo, Mars's new apogee* $\alpha\gamma$ *falls at 29° 0′ 16″ Leo.*

22 *Now let* $\beta\xi$ *be 100,000 and* $\alpha\gamma$ *be 18,034 of the same parts. Before, it was 27,971, in units of which* $\delta\gamma$ *was 30,138. Therefore, in these units,* $\delta\gamma$ *will be 19,763. Next, let both be divided at* $\theta$ *and* $\beta$ *in such a ratio that* $\delta\theta$ *is to* $\theta\gamma$, *and* $\alpha\beta$ *to* $\beta\gamma$, *as 1260 is to 756:* $\delta\theta$ *will be 12,352,* $\theta\gamma$ *7411; and* $\alpha\beta$ *11,271,* $\beta\gamma$ *6763. In this way, a Ptolemaic hypothesis for the first inequality may be set up about both* $\delta$ *and* $\alpha$. *Then, in the former units, of which* $\delta\alpha$ *is 3584,* $\theta\beta$ *or* $o\xi$ *will be 1344, but in units of which* $\beta\xi$ *is 100,000,* $\theta\beta$ *or* $o\xi$ *will be 880. These should be kept in mind.*




To find the basis of a computation whereby we may investigate the amount the appearances will be changed for the observer at δ by transposing the eccentric from ρθο to πβξ, we proceed as follows. *Since γ is the common centre about which the times are indicated, the line γει will indicate the same moment in both hypotheses. Therefore, if the planet is traversing the eccentric εο, it will at that moment be at ε with equation δεγ, but if it is traversing ιξ, it will be at ι with no equation, since the line of apparent motion αι coincides with the line of mean motion γι. Again, after a certain amount of time, whose measure shall be ιγζ or εγκ, whose vertical angle is δγα, just found to be 5° 27′ 47″, let a common moment be taken, represented by γκζ. At that time, the planet traversing the eccentric εο will be at κ with no equation, while the planet traversing ιξ will be at ζ with equation γζα.* Thus, in both instances, the planet is always on a line drawn from γ, at the points where that line cuts the respective eccentrics. If the observer were at γ, there would be no difference in the appearances, whether the planet were at κ or at ζ. But since in the present model the observer is

placed at $\delta$ by the theorists and at $\alpha$ by myself, the question arises at what point on the circumference the distance between the eccentrics is perceptibly a maximum for the observer at $\delta$. *In order that this difference become perceptible, three factors must concur. First, the distance itself must be large (that is, in the region of* $o\xi$ *and* $\rho\pi$, *where it is a maximum). Second, as nearly as possible it should be presented directly to an observer at* $\delta$ *(it vanishes, for example, at* $\zeta\kappa$ *and the point opposite, according to optical principles). Thus, it will reach an apparent maximum at the intermediate regions, below* $\xi$ *and above* $\rho$. *Third, it must be close to* $\delta$ *(for example, it is closer above* $\rho$ *than below* $\xi$, *because the centre of the other eccentric* $\beta$ *lies off to the right of* $\delta$). *If we construct a line from* $\gamma$ *at right angles to* $\gamma\delta$, *this perpendicular from* $\gamma$ *to the circumference will bring us close to the place where the apparent magnitude is greatest. Let* $\sigma\phi$ *be drawn through* $\gamma$ *perpendicular to* $\delta\gamma$, *intersecting the eccentric about* $\theta$ *at* $\sigma$ *and* $\upsilon$, *and the other at* $\tau$ *and* $\phi$, *and to this let the perpendicular* $\beta\chi$ *be drawn. Therefore, at the moment* $\gamma\sigma$ *the planet will be at* $\sigma$ *and* $\tau$, *and at the moment* $\gamma\phi$, *at* $\upsilon$ *and* $\phi$. First of all, the quantity $\upsilon\phi$ must be found. *Let* $\theta\upsilon$ *and* $\beta\phi$ *be joined. In* $\theta\upsilon\gamma$, $\theta\upsilon$ *is given as 100,000, because* $\theta$ *is the centre of the eccentric* $\upsilon$. *The magnitude* $\theta\gamma$ *is 7411, and* $\theta\gamma\upsilon$ *is right. Therefore,* $\gamma\upsilon$ *is*
23 *99,725. The same is to be done in* $\beta\gamma\phi$. But first, $\beta\chi$[3] has to be found. *This will be revealed by the triangle* $\beta\gamma\chi$, *in which* $\beta\chi$ *is parallel to* $\theta\gamma$, *the angle at* $\chi$ *is right, and* $\gamma\beta\chi$ *is equal to* $\theta\gamma\beta$ *(5° 27′ 47″) and* $\beta\gamma$ *is 6763. Hence, the sides are found to be:* $\gamma\chi$ *644,* $\beta\chi$ *6732. Therefore, in the right triangle* $\beta\chi\phi$, *since* $\beta\phi$ *is 100,000 (*$\beta$ *being the centre of the eccentric* $\phi$*) and* $\chi\beta$ *is 6732,* $\chi\phi$ *will be 99,773. Add* $\chi\gamma$, *644, and* $\gamma\phi$ *is given, 100,417. But* $\gamma\upsilon$ *was 99,725.* Therefore, the quantity sought, $\upsilon\phi$, is 692.

With $\upsilon$ and $\phi$ joined to the position of the observer $\delta$, the magnitude of angle $\upsilon\phi\delta$ is found as follows. *Above,* $\delta\gamma$ *was 19,763 in the new units, and the angle at* $\gamma$ *right. Therefore, as* $\delta\gamma$ *is to* $\gamma\phi$ *and* $\gamma\upsilon$, *so is the whole sine to the tangents of the angles* $\gamma\delta\phi$ *and* $\gamma\delta\upsilon$. *These come out to be 78° 51′ 54″ and 78° 47′ 30″.* The difference of these angles, the angle $\upsilon\delta\phi$, is 4′ 24″. The angle $\sigma\delta\tau$ will be much less, because $\sigma\tau$, as it is closer to the intersection of the eccentrics, is smaller than $\upsilon\phi$.

You see, then, how nearly the appearances remain unchanged for the observer at $\delta$, despite the substitution of a new planetary path

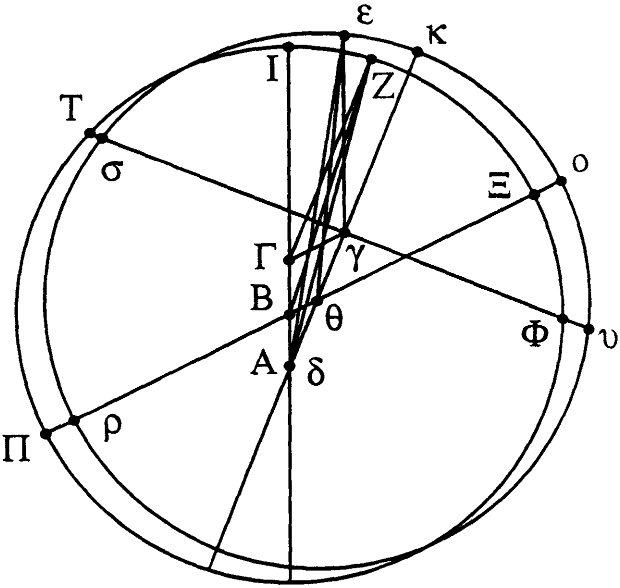
[3] Incorrectly denoted as $\beta\gamma$ in all editions.

in the heavens by transposition of the observer and change of hypothesis. Moreover, it is still within the theorist's power to vary the mean motion and the ratio of the eccentricities both among themselves and in relation to the radius of the orb, should this become desirable, for the purpose of obliterating this discrepancy of some five minutes.

This equivalence pertains chiefly to the first inequality, that is, to the appearances at $\delta$, close to the centre of the eccentric. However, in the second inequality (the equation for the annual orb), it makes a big difference whether the planet traverses $\xi\pi$ or $o\rho$, as was also noted above in the other equivalence. And there we could not ignore the 246 units (the difference between the Ptolemaic and the Copernican hypotheses). Much less can we overlook these 880 units, which are 1344 of the old units. In the next chapter we shall see how much of a difference in Mars's apparent position this would occasion.

Hitherto, we have been transposing the observer from $\delta$ to $\alpha$. Now it will be demonstrated that very nearly the same things happen if the observer remains fixed while the point of the equant is transposed. It will thus be apparent that what could be done in a simple eccentric (at the end of chapter three above) can also be done in this chapter with an eccentric which has an equant. In the former, the result was the same whether the observer or the centre of the eccentric were transposed, while here, likewise, the result is almost the same whether the observer or the centre of the equant be
24 transposed. It is, however, necessary to present the demonstration in a variety of ways on account of the great dissimilarity of the opinions followed by the theorists in demonstrating the second inequality of the planets, which opinions will be keeping us in court in the next chapter.

*Let the points* $\alpha$ *and* $\delta$ *merge into one, so that the observer remain in the same place. Let* $\delta$, $\theta$, *and* $\gamma$ *remain the same, but let the line* $\gamma\beta\alpha$ *of the previous diagram be replaced by* $\mathrm{AB\Gamma}$ *parallel to it and passing through the point* $\delta$ *or* A. *Let the segments* AB, $\mathrm{A\Gamma}$ *be equal to the previous segments* $\alpha\beta$, $\alpha\gamma$. *Therefore,* $\Gamma\gamma$ *will be the transposition of the equalizing point* $\gamma$, *equal to the previous transposition of the observer* $\alpha\delta$. Once more, two eccentrics or planetary paths through the aethereal air will be described, about B and $\theta$. All the letters on each circle will be carried over, and the magnitudes of the lines will remain precisely the same. The only difference is that the two points on the two eccentrics at which the planet is to be placed at a given




moment are no longer determined by a single line, but by parallel lines drawn from the two equant points $\Gamma$, $\gamma$, each set out in its own eccentric. *For example, when the eccentric* $\theta\kappa$ *has its planet at* $\kappa$, *the eccentric* BI *will have it at* Z, *where* $\gamma\kappa$ *and* $\Gamma$Z *are parallel. And when the former has the planet at* $\epsilon$, *the latter will have it at* I, *where* $\gamma\epsilon$ *and* $\Gamma$I *are again parallel. The rest will be clear from the diagram without demonstration.*

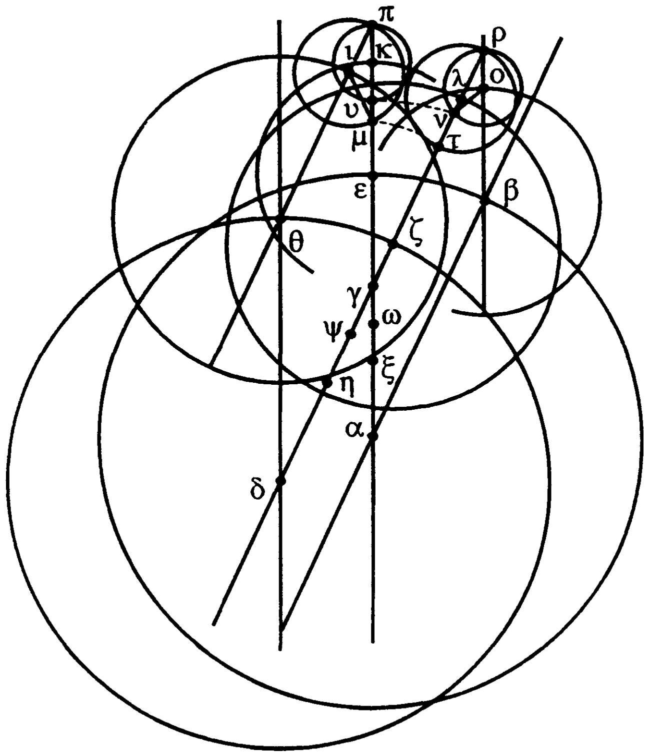
Now, suppose it is not permissible to shift the observer (and it is not permitted by those who make the earth the centre of the world, as will be remarked in the next chapter), and that the planet has been observed in several positions on the zodiac, always at opposition to the sun's mean position, and that the theorist uses the positions and the intervals of time between them to construct this sort of hypothesis, with the observer at $\delta$, $\delta\theta$ the eccentricity of the eccentric $\theta\kappa$, $\theta\gamma$ the eccentricity of the equant, and $\kappa$ the apogee. Comes now Kepler, wishing to change the observed positions and times; that is, he would observe the moments and points at which the planet is at opposition to the apparent position of the sun rather than its mean position. From these positions and times he will have come up with another hypothesis, in which the observer would be left unmoved at $\delta$ or A, but where the eccentricity would come out to be AB in a new eccentric BI, and the eccentricity A$\Gamma$ of a new equant $\Gamma$,

and there would be a new apogee I. The question now is whether, if the prior theorist combines the new eccentric BI with his original equalizing point γ, the computed equation and sidereal position of the planet will turn out much different from what he formerly had found using his eccentric γκ. (It is the first inequality that is in question; this discussion is not concerned with the second inequality, and the nature and magnitude of the changes which this procedure would effect therein.) The answer, arising from the equivalence of transpositions, is that the discrepancy is going to be extremely small. Its maximum, reached in the neighborhood of the points υ and Φ, will not exceed five minutes, exactly as before when the observer was transposed, except that now the line υΦ is closer to the observer δ than is its endpoint υ. Consequently, the angle υδΦ, which previously was 4′ 24″ is now 4′ 43″. The opposite happens at σT.

It has thus been demonstrated in a Ptolemaic eccentric what sort of disturbances would arise if one were to transpose either the observer or the orb and construct a new eccentric, making use of the planet's oppositions to the sun's apparent position.

I do not think there is any need to repeat the arguments and demonstrate the same equivalence in the Copernican or Tychonic form, which makes use of two epicycles. I shall only show, by what was established at the end of chapter III, how to express the eccentric-cum-equant that suits the planet, and its transformation using different magnitudes and a different position for the observer, in terms of the Copernican double epicycle. This is done in such a manner that, while the observer is transposed, the path of the planet through the aethereal air is invariant, as nearly as possible, in accord with what has been said in this fifth chapter. (This is the possibility adumbrated in chapter III).

*Let the triangle δγα be constructed equal to the previous one, with corresponding lines parallel, and through α let αβ be drawn parallel to δγ, and through δ, δθ parallel to αγ . And about centres α, δ let two concentrics be described, equal to the previous eccentrics δθ, αβ. Let δγ be extended to ζ and λ, and αγ to ε and κ, and let δζ and αε be semidiameters, as before, and lines of apsides (since both go through the same point γ). Now let δγ and αγ be cut at η and ξ in the same ratio as before, and let ηγ and ξγ be bisected at ψ and ω. Then, with radius δψ and upon centres θ and ζ let the epicycles ι and λ be described, and let ζλ be parallel to θι. Then, about centres ι and λ, with radius ψγ, let epicyclets be described through πμ and ρτ.*




*Again, let the epicycles* $\kappa$ *and* $o$ *be described about centres* $\epsilon$ *and* $\beta$ *with radius* $\alpha\omega$, *and with* $\beta o$ *parallel to* $\epsilon\kappa$. *Next, about centres* $\kappa$ *and* $o$ *with radius* $\omega\gamma$, *let epicyclets be described through* $\pi\upsilon$ *and* $\rho\nu$, *and let* $\theta\iota\mu$ *and* $\beta o\nu$ *be double* $\delta\gamma\alpha$. *Let the planet on epicyclet* $\kappa\pi$ *be at* $\upsilon$ *on the*
26 *side nearest* $\epsilon$, *and the planet on epicyclet* $\lambda\rho$ *be at* $\nu$ *nearest* $\zeta$. *Therefore, according to the hypothesis about* $\delta$ *the planet moves along* $\tau\mu$, *while according to the hypothesis about* $\alpha$ *the planet moves on* $\nu\upsilon$. *Here you see that the points* $\mu$ *and* $\upsilon$, *as well as* $\tau$ *and* $\nu$, *hardly differ when seen from* $\delta$ *and* $\alpha$, *respectively, when the planet is near the apsides. In the middle longitudes these points will be separated only by the amount that separated* $\upsilon$ *and* $\Phi$ *in the previous diagram. The magnitudes will all be nearly equal and the demonstrations entirely the same. For if* $\theta\iota$ *and* $\epsilon\kappa$ *be extended to meet at* $\pi$, *and* $\zeta\lambda$, $\beta o$ *to meet at* $\rho$, *the triangles* $\theta\pi\epsilon$ *and* $\zeta\rho\beta$ *will be congruent to triangle* $\delta\gamma\alpha$, *and corresponding sides will be parallel.*

These demonstrations will be perplexing enough in themselves, so it is not really advisable to make them more involved by a heaping up of Copernican or Brahean epicycles. Therefore, in what follows, we shall count the Copernican or Brahean form as belonging to the first inequality. For the procedure for treating the hypotheses of the second inequality, since it is always going to concern each of the three, will furnish us with a great abundance of things to do.

I now immediately state it as a postulate that whatever we shall have demonstrated using the Ptolemaic equant-cum-eccentric also be taken as demonstrated in the Copernican or Brahean concentric-cum-double-epicycle, or eccentrepicycle. For in chapter 4 above, the difference was found to be very small.

# 6

## On the equivalence of the hypotheses of Ptolemy, Copernicus, and Brahe, by which they demonstrated the second inequality of the planets, and how each changes when accommodated to the sun's apparent motion instead of its mean motion

27 The discussion so far has concerned the hypotheses of the planets' first inequality, which completes its cycle each time the planet returns to the same sign of the zodiac. Now we pass on to the other inequality, which completes its cycle not at a single constant sign of the zodiac but with the sun's opposition to, or conjunction with, the planet. People have wondered exceedingly at this, different ones proposing different reasons why a planet in conjunction with the sun becomes swift, direct, high, and small, and opposite the sun, retrograde, low, and large, while in between it becomes stationary and of a medium size.

The Latin authors considered that in the sun's aspects and rays there is a force by which the other planets are in fact attracted. Their opinion cannot be shown numerically, because it is not astronomical. But it is also improbable, now that the true causes have been found, and manifestly false, since Saturn begins to retrogress at quadrature with the sun, or beyond; Jupiter, at trine; Mars, at biquintile or before sequiquadrate, and all at variable distances[1].

Ptolemy said that at a determined point on the planetary circle that serves for the first inequality, there is fixed, not the planet itself, but the centre of an epicycle bearing the planet fixed upon its circumference, which is in turn borne by the planet's chief circle. The motion has the following form: if the centre of the epicycle be in conjunction with the sun, the planet is also at the highest point of the epicycle and is moved along with the sun in the same direction, [and] when the sun,

[1] Quadrature is 90° of angular difference: trine, 120°; biquintile, 144°; and sequiquadrate, 135°.

which is faster, departs from the centre of this epicycle, the planet simultaneously descends on the epicycle. But since the epicycle's motion about its centre is faster than the motion of its centre about the earth, when the planet traverses the lower parts of the epicycle while the epicycle's centre is at opposition to the sun, the compounding of motions makes it actually retrograde. Thus Ptolemy made his opinions correspond to the data and to geometry, and has failed to sustain our admiration. For the question still remains what cause it is that connects all the epicycles of the planets to the sun, so that they always complete their periods when their centres are in conjunction with the sun.

Copernicus, with the most ancient Pythagoreans and Aristarchus, and I along with them, say that this second inequality does not belong to the planet's own motion, but only appears to do so, and is really a by-product of the earth's annual motion around the motionless sun. In this way, just as in chapter 1 the diurnal motion was separated from the motion proper to the planets, the second inequality of the planets is now separated from the first by Copernicus, and in quite the same way. For some theorists admit that the first motion is not really proper to the planets, but still think it is in fact in the planets, implanted in them, so that the planets, too, are moved with the same motion. Copernicus holds that it is neither intrinsic to the planets nor extrinsically implanted, but only attached to them through an optical illusion. For while the earth rotates upon its axis from west to east, it appears to our eyes that the rest of the world rotates from east to west. It is, I claim, in just the same way that Copernicus asserts that the planets do not really become stationary and retrograde, but only
**28** appear so. For he says that since the earth has in addition the annual motion in a very large circle (which he calls the *orbis magnus*), those who believe that the earth is at rest think that the planets and the sun are carried in the opposite direction; and he says that when the sun is between the planet and the earth, the motions of the earth and the planet are added in the appearances, whence the planet appears to be swift; and when, on the other hand, the earth is between the sun and the planet, the planet is apparently left behind and thus retrogresses, owing to the earth's being swifter than the planet.

Tycho Brahe holds something in common with the Latins: although the sun does indeed not attract the planets through the aspect, the planets do fawn upon the sun. For he says that they strive to keep the sun (although it is moving) nearly in the middle of their

circuits, and indeed, that they arrange their real paths around the sun as if it were motionless. Thus any given planet, besides its own path, describes the sun's path in the aethereal air, and out of these compounded with one another there is formed exactly what Ptolemy had (that is, a spiral), as described in chapter 1. In astronomical terms, Ptolemy put epicycles on eccentrics, while Brahe put eccentrics on a single epicycle, which is the sun's orb.

I, in the demonstrations that follow, shall link together all three authors' forms. For Tycho, too, whenever I suggested this, answered that he was about to do this on his own initiative even if I had kept silent (and he would have done it had he survived), and on his death bed asked me, whom he knew to be of the Copernican persuasion, that I demonstrate everything in his hypothesis.

Furthermore, we shall demonstrate, both right here and through the entire book (in the course of doing other things), that these three forms are absolutely, perfectly, geometrically equivalent. For the present we must carry out what we set out for ourselves and what is to be demonstrated, namely, that there is a very great error indeed in the second inequality if the apparent motion of the sun is replaced by its mean motion, to which the planet is at opposition at the beginning of this second inequality.

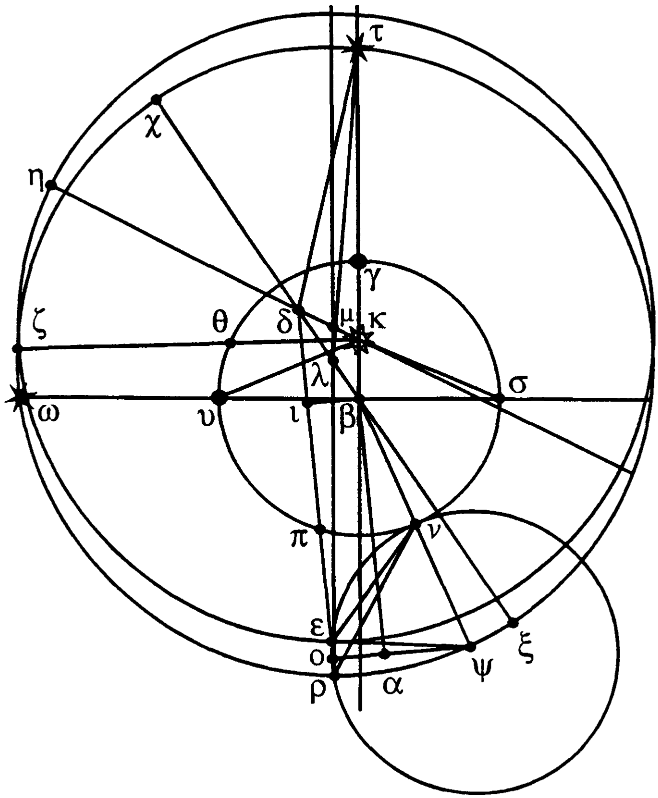
I shall begin with the Copernican opinion.

*About centre β let the earth's eccentric γυ be described, such as Copernicus, putting his trust in Ptolemy, imagined. Let γβ be its line of apsides, κ the position of the motionless sun, and β the point about which the earth's motion is uniform.*

*Through β and perpendicular to βγ let υβσ be drawn, intersecting the circumference at points υ and σ, and let υ and σ be joined to κ.*

Copernicus, intending to carry over the Ptolemaic numbers into his own form of hypothesis, reckoned the eccentricities of the planets from the conjectural centre β of the earth's uniform motion rather than from the sun κ. *For if lines be drawn from β (as βγ, βυ, βσ), whenever a planet and the earth lie upon one of these, the planet has been supposed to be divested of the second inequality, to which it was subject on account of the earth's motion, as, for example, if the earth were at υ and the planet were found on βυ extended.*

29 In effect, by adopting this procedure, Copernicus established a fictitious observer at the point β. *For provided that the planet is on the line βυ, it makes no difference for the purpose of designating its sidereal position whether it be viewed from σ or from β. The same may*




*be truly said of the lines* βγ, βσ, *and all the other infinite lines intersecting at* β. Therefore, β is the point of intersection of all the lines of vision, and is thus the fictitious common point for all observers. In fact, however, the point of vision is the earth, our home, which is found at one or another point on the circle σγυ at various times.

So Copernicus believed the planet was freed from the second inequality whenever the planet and the earth were found on any one line passing through β. Consequently, he used mathematical instruments to endeavour to find the apparent sidereal position of a planet at those moments of opposition to the mean position of the sun. For if the planet's position was found on one of the nights near the planet's opposition to the sun, and if at that time the computed mean position

of the sun was found to be at the point precisely opposite, then that was the moment of time desired. However, if there was still a little distance between them on that night, then this moment of time, and the point or position held by the planet at this moment, were tracked down by a comparison of two or more nights and of the diurnal motions of Mars and the earth over the interval. When he had done this as many times and in as many places on the zodiac as he thought necessary (suppose, for example, $\beta\gamma$, $\beta\upsilon$, $\beta\sigma$ ), the theorist now used
30 these sidereal or zodiacal positions of the planet, $\beta\gamma$, $\beta\upsilon$, $\beta\sigma$, as the starting point of an investigation of the hypothesis of the first inequality. This involved finding the magnitude of the planetary circle's eccentricity from the selected point $\beta$, and the zodiacal position of the apogee, by comparing the angles which the observed positions set up about the point of observation $\beta$, with the time intervals between them. The method of this undertaking, however, I shall present clearly in its proper place, below.

Suppose these things already done, giving a line of apsides $\beta\delta$, eccentricity of the equant $\beta\delta$, and the centre of the eccentric on this line at the point $\lambda$, and let this hypothesis correspond to all positions observed at the moment of opposition to the sun's mean position.

Now, Kepler, what more could you ask of Copernicus? Are you denying that this hypothesis corresponds entirely to observations or to astronomers' experience[2]? That is indeed not at present in question. Nor was I, when I first entered upon this undertaking, tempted by the observations to take up a different opinion. But it is this that I have been wanting: *Let* $\beta\delta$ *be extended so as to intersect the eccentric at* $\chi$ *and* $\xi$, *and near* $\chi$ *let some point* $\tau$ *on the eccentric be chosen, and lines be drawn from* $\tau$ *to* $\delta$ *and* $\lambda$. *Now* $\chi\tau$ *is the measure of the angle* $\chi\lambda\tau$, *while the angle* $\chi\delta\tau$ *is greater than the angle* $\chi\lambda\tau$ *by the amount* $\delta\tau\lambda$, *and* $\delta$ *is the point of temporary uniformity*[3]. *Therefore, the time designated by* $\chi\delta\tau$ *is greater, with respect to the whole periodic time designated by four right angles, than is the arc* $\chi\tau$ *with respect to*
31 *the whole circumference. The planet, then, is actually slower over the arc* $\tau\chi$ *(this is not just an optical illusion), and fast over the opposite arc; and at* $\chi$ *it is slowest, and at* $\xi$ *it is fastest. Nevertheless, it is not*

2 *Experimentis*. The distinction between *experimentum* and *experientia* is not the same as our distinction between 'experiment' and 'experience'. The Latin words mean very nearly the same thing, rather like our 'observation', although *experimentum* is more nearly related to proving something, while *experientia* may denote the skill gained through practice or experience.

3 The malapropism ('temporary' instead of 'temporal') is present in Kepler's Latin.

*farthest from the sun* κ *when it is at* χ, *nor is it nearest* κ *at* ξ. But all the arguments, even the testimony of the very hypothesis set up upon β which I am refuting, make it probable that this real slowing down of the planet arises from its moving away from the body of the sun, and the speeding up from its approach to the sun itself, seated at κ. On the other hand, it is impossible even to conceive of how a force could inhere in point β, which has no body, rather than in κ, quite nearby, in which there is the sun, the heart of the world, which force would move the planet more swiftly or slowly according to its approach and recess. Furthermore, one who is not prepared to admit that the retardations and accelerations arise physically from the close interconnection of the eccentrics, must consequently assert that these affects of motion are naturally under the control of the motive faculties residing in the body of the planet, and we will again reach the same probable conclusion. For what would be the reason why those minds would bypass the point κ (which has a geometrical affinity for motion, being invested with a body of no small magnitude) and pay attention to the point β, only four semidiameters (diameters, according to the authorities)[4] of the solar body distant from the sun, empty, and propped up by nothing but imagination alone? Even Copernicus himself admits, in Book 5 chapter 16, that the sun is really fixed at κ, wherefore the eccentricity δκ is constant, while he asserts that the point β, which he takes as the centre of the annual orb, is displaced over the ages, thus making βδ shorter. Thus either β is no longer in the centre of the world today, or it was once not there. But it is probable either that the motion originates from the centre of the world, or that the moving minds pay attention to the centre of the world, and thus not to β but to κ, which Copernicus says is fixed, as the centre of the world should be.

Led by these probable arguments, I concluded that the line of apsides taken in order to account for the planet's first inequality, ought to go right through κ itself rather than β. But we shall obtain this result when we make use of the sidereal positions which the planet has at the moment when the sun's apparent position and its own are opposite.

*And when the points* κ *and* β *are collinear with the earth* γ, *and when the planet also falls on the same line, as at* τ, *it is at the same moment at*

[4] Kepler is referring to his reduction of the solar eccentricity, introduced in Part III of the present work. See especially chapters 27–31.

*opposition to both the sun's mean position and its apparent position. Its position is the same whether it be designated by $\beta\tau$ or by $\kappa\tau$ extended out to the fixed stars, and the planet is truly stripped of its second inequality, whether it depends upon the earth's apparent motion or its mean motion. But when the earth comes to the side of its eccentric, to the middle longitudes, a fairly large difference arises between them. For let the earth have travelled from $\gamma$ to $\upsilon$ (that is, let the sun opposite have moved from perigee in Capricorn to Aries), and let the line of the sun's mean motion $\upsilon\beta$ be found in Aries, while the line upon which the planet is observed in Libra be exactly opposite, namely, $\upsilon\omega$. Now since $\upsilon\kappa$ is farther eastward beyond $\upsilon\beta$, the sun's apparent position is beyond opposition to the planet. And since $\upsilon$ is the observer's home, the earth, and $\omega$ is the planet, and both are going down towards $\xi$, with the earth $\upsilon$*
32 *the swifter, the line $\upsilon\omega$ will at a later time be still more inclined to the line of the sun's visible position $\upsilon\kappa$. Therefore the apparent opposition precedes the mean. So, at a time preceding the moment designated by $\beta\upsilon$ (call it $\beta\theta$), the planet falls upon a line drawn from $\kappa$ through $\theta$, that is, at $\zeta$.* And then the planet's line of vision $\theta\zeta$ (as the less experienced should diligently note) lies farther eastward beneath the fixed stars than the line $\upsilon\omega$ of the later time. This is because, although $\theta\zeta$ precedes the line $\upsilon\omega$ in being farther west, it is nonetheless exactly as if $\theta$, $\upsilon$, and absolutely all points on the earth's circle were a single point and were the centre of the sphere of the fixed stars. Therefore, it is not the distance of the endpoints $\theta$ and $\upsilon$ but the inclination of the lines $\theta\zeta$ and $\upsilon\omega$ that causes the lines to strike upon different zodiacal positions, since they would be perceived as coinciding if they were parallel. *But that $\zeta$ is inclined towards $\omega$ is clear from the supposition that the planet is moved from $\zeta$ to $\omega$ in the same time that the earth is moved from $\theta$ to $\upsilon$. For the earth is swifter than the planet. Therefore, the earth traverses a greater space $\theta\upsilon$ than does the planet along $\zeta\omega$.*

But it can be shown even more easily that at an earlier time the planet is farther east, since at opposition it is retrograde, as is clear to everyone. It therefore appears that in this transition from the mean to the apparent motions of the sun, something is altered in the positions stripped of the second inequality.

*For, at $\tau$ and the point opposite, the original positions remain; at $\zeta$ or $\omega$ an addition is made to the observed position, since $\theta\zeta$ (as was said) is farther eastward than $\upsilon\omega$. A subtraction is made in the intervening time, because $\theta\zeta$ is the line of vision at an earlier time than is $\upsilon\omega$. At the opposite position the outcome is the contrary, that is, an addition is*

*made to the time and a subtraction from the position. Accordingly, these positions of the planet differ considerably from the original ones. And therefore, the operation set up in this new way produces quite different results.* That is, since we have transferred the fictitious point of observation to the sun κ *(by virtue of our having viewed the planet when it was at* τ *and* ζ *and the earth was on the lines* κτ *and* κζ, *at the points* γ *and* θ), the eccentricity will now originate at the point κ. *But in chapter 5 above, it was shown that when the observer is shifted from* β *to* κ *and a new line is drawn from* κ *through the original point of uniformity* δ, *although this new hypothesis does result in a new eccentric, all the appearances at* β *are left almost completely undisturbed. So, by joining* δκ, *dividing it at* μ *so that* δλ *is to* δμ *as* δβ *is to* δκ, *setting up a new eccentric* ηε *about* μ *equal to the previous one* ξχ, *and drawing a new line of apsides through* κδ, *a new hypothesis is formed, whose apsis is at* η. Previously, however, we had improperly called χ the apogee, because the Copernican centre β on the line χβ was the successor to the Ptolemaic position of the earth. Now, following my own notions, we shall call η the aphelion (since we are in the Copernican hypothesis), and the point opposite it perihelion, because the sun's distance from η is a maximum.

Terms: What are aphelion and perihelion?

It has been said how my opinions and those of the authorities differ as regards physics. It has also been shown how each is to be constructed geometrically in the Copernican form. Third, it has been forcefully established that in astronomical terms the two do not differ in any important way at the moments of conjunction and opposition[5]. The next thing for me to do is to demonstrate what remained unexplained in ch. 5 above, that there is a considerable difference between the two hypotheses if you have to use them to compute the planet's position when it is not acronychal.

33 *If a line be drawn through the centres of the eccentrics* λ, μ, *parallel to* βκ, *and extended to intersect each eccentric in two points, above and below, there will be set up, below, the maximum space* ερ *between the two, equal to* λμ. *But because it is lines from* δ, *not those from* λ, *that designate one particular moment of time, which is what we need here, let* δρ *be drawn intersecting the eccentrics at* ε *and* ρ, *so that at one and*

[5] Kepler clearly shows here for the first time the three distinct areas in which he requires a comprehensive theory to operate: the physical, the geometrical, and the observational. The entire work can fruitfully be viewed as a thoroughgoing effort to bring these three into harmony with one another. For more on this 'Keplerian synthesis', see the translator's introduction.

*the same moment the planet on one might be at* $\epsilon$ *and on the other at* $\rho$. *When the earth is on the line* $\delta\rho$, *at* $\pi$, *the planet, whether it is at* $\epsilon$ *or* $\rho$, *will be seen at the same place on the zodiac. For, optically considered, the line* $\epsilon\rho$ *appears as a point. But when the earth departs towards either side of this line, the magnitude of the line* $\epsilon\rho$ *appears greater and greater, since it is viewed obliquely.*

It is required to find the point on the earth's orb from which the lines of vision passing through $\epsilon$ and through $\rho$ are at their greatest distance from one another and form the greatest angle of vision, and at which the error would be greatest if the planet were placed at $\rho$ when it should have been placed at $\epsilon$.

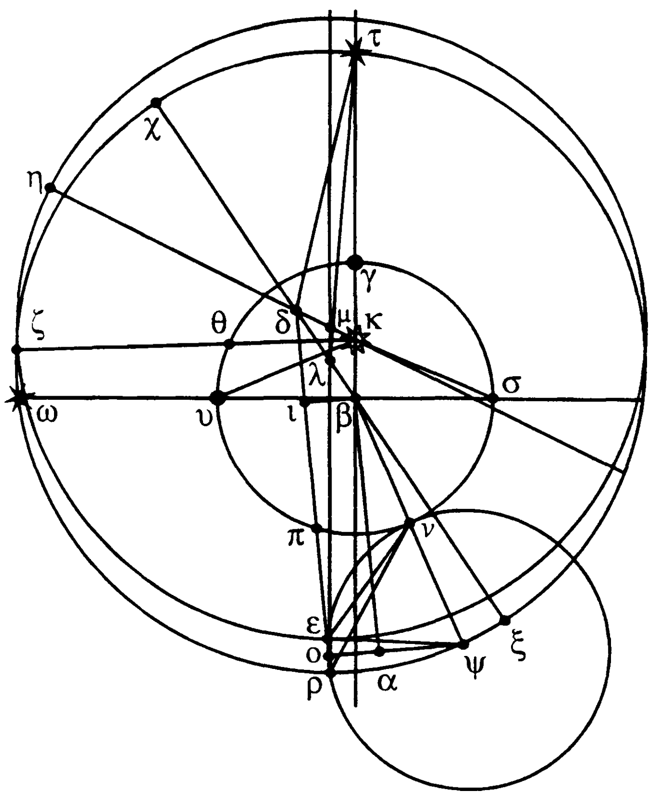
*First, the angle will be greater down at* $\epsilon$ *than up near* $\tau$ *because the earth's orbit, described about* $\beta$, *moves the observer nearer to* $\epsilon\rho$ *than to* $\tau$. *Next, since* $\delta\rho$ *is beyond* $\tau\beta$, $\epsilon\rho$ *is seen more obliquely from the left side than from the right. It will consequently appear less from the former than from the latter even when the distances of the earth from the line* $\delta\rho$ *are equal. Therefore, the point we are to find is on the right*
34 *side.* I say that $\epsilon\rho$ subtends a maximum angle of vision when the observer is stationed at the point where a circle drawn through $\epsilon$ and $\rho$ is tangent to the earth's circle. *For let such a circle be described through* $\epsilon\rho$ *tangent to the circle* $\upsilon\sigma$ *on the side towards* $\sigma$, *and let the point of tangency be* $\upsilon$. *From* $\epsilon$ *and* $\rho$ *let lines be drawn to the tangent point* $\upsilon$ *and also to several other points of circle* $\upsilon\sigma$ *before and after the point of contact. Now since circles touch one another in one and only one point, the sides of all angles drawn from* $\epsilon$ *and* $\rho$ *to points on circle* $\upsilon\sigma$ *will therefore be cut by the circle through* $\epsilon\rho$, *with the exception of those that terminate at the tangent point of the circles,* $\upsilon$. *But those whose sides from* $\epsilon$ *and* $\rho$ *are cut by the circle* $\epsilon\rho$ *before they intersect would have formed a greater angle had they intersected at either of the points of section*[6], *by Euclid's* Elements *I. 21. And by Euclid III. 21, all angles at the circumference set up on segment* $\epsilon\rho$ *are equal. Therefore, the one at* $\upsilon$ *(the point of contact) is greater than all the others. Q. E. D.*

Next, to investigate its magnitude in numbers belonging to the theory of Mars, we need to find $\epsilon\rho$ and also the perpendicular from $\beta$ to $\delta\rho$.

*We shall find both by solution of the triangles* $\delta\lambda\rho$ *and* $\delta\mu\epsilon$. *Now we*

[6] That is, points of intersection with the circle $\epsilon\rho$.

*said above*[7] *that in* δλρ, δλ *is 7411*[8] *where* λρ *is 100,000, and* ρλβ *is 47° 59′ 16″. This gives* ρδλ *a value of 44° 59′ 10″, and* δρ *105,123. Therefore, in* εδμ, *since* εδλ *is 44° 59′ 10″ and* λδμ *earlier came out to be 5° 27′ 47″ the whole* εδμ *is 50° 26′ 57″ and* δμ *above was 6763 where* με *is 100,000. Therefore, in* εδμ, *with three magnitudes given the rest are given:* εμκ *is 53° 26′ 17″ and through this,* δε *is 104,170. But earlier,* δρ *was 105,123. Therefore, the remainder,* ερ, *is 953.* Above, λμ was




[7] In ch. 5, p. 147.

[8] As a result of a slight error in the passage cited in the previous footnote, this magnitude is too large. In particular, the angle δγα (in the diagram in ch. 5) should have been 5° 29′, not 5° 27′ 47″, and so the magnitude of θγ (or δλ in the present diagram) should be 7314, not 7411. However, as the intention of the present calculation is to show only that the two hypotheses give significantly different results, this error is of no importance. Kepler says that ρνε is 1° 3′ 32″, although it really is 0° 55′ 12″: in any event, it is about one degree, which is a difference that cannot be overlooked.

880, to which $\epsilon\rho$ would be equal were $\epsilon$ and $\rho$ on the line $\mu\rho$. But because here $\epsilon$ is on the line $\delta\rho$ which is inclined to $\mu\rho$, you should not be surprised that $\epsilon\rho$ is longer than $\mu\lambda$[9]. *Now let* $\beta\iota$ *be drawn from* $\beta$ *perpendicular to* $\delta\rho$. *In triangle* $\delta\beta\iota$, *the angle at* $\iota$ *is right, and* $\beta\delta\iota$ *is 44° 59′ 10″, and* $\beta\delta$ *was found to be 19,763, above. Therefore, the required perpendicular,* $\beta\iota$, *is 13,971, and* $\delta\iota$ *is 13,978. Consequently,* $\iota\rho$ *is 91,145. It is also necessary to have a quantitative idea of the radius* $\beta\upsilon$ *in the same units. Above, when the line corresponding to the present* $\beta\kappa$ *was taken to be 3584 parts,* $\beta\upsilon$ *was, by presupposition, 100,000. Now, however,* $\lambda\rho$ *is 100,000 by presupposition, and* $\lambda\rho$ *is to* $\beta\upsilon$ (*taken above*) *as approximately 61 to 40, whence the other ratios are extrapolated. Thus 60 is to 41 as 100,000 is to 65,656½, the appropriate magnitude for* $\beta\upsilon$.

*Next, let a circle passing through* $\epsilon$ *and* $\rho$ *touch the circle* $\beta\upsilon$ *at point* $\nu$, *and,* $\epsilon\rho$ *being bisected at* o, *let* $\psi$o *be set up perpendicular to* $\iota\rho$, *and* $\beta\nu$ *be extended so as to intersect* o$\psi$ *at* $\psi$. *The centre of the circle will be* $\psi$. *For the centre of the circle is on the line passing through the centre of one of the tangent circles and the point of tangency, by Euclid III. 11: hence, it is on the line* $\beta\psi$. *Again, by Euclid III. 3, the centre of the circle is on the perpendicular bisector of the chord* $\epsilon\rho$, *which connects the points of intersection* $\epsilon$ *and* $\rho$. *Therefore, the centre is on the line* o$\psi$, *and hence is at the point* $\psi$ *common to the two lines. Let* $\epsilon\psi$ *be joined, and from* $\beta$ *parallel to* $\iota\rho$ *let* $\beta\alpha$ *be drawn intersecting* o$\psi$ *at* $\alpha$. *Therefore,* $\beta\alpha$ *is equal to* $\iota$o, *and* $\alpha$o *is equal to* $\beta\iota$. *But* $\beta\iota$ *was just found to be 13,971, while* $\iota$o *is known through* $\iota\rho$ *and* $\epsilon\rho$. *And* $\iota\rho$, *above, was 91,145, and* $\epsilon\rho$ *was 953. But* o$\rho$ *is half of* $\epsilon\rho$, *and therefore,* o$\rho$ *is 476½. So, when* o$\rho$ *is subtracted from* $\iota\rho$, *the remainder* $\iota$o *or* $\beta\alpha$ *is 90,668. Now since* $\alpha$ *is a right angle,* $\beta\psi$ *is the power*[10] *of the two,* $\beta\alpha$, $\alpha\psi$. *However,* $\beta\psi$ *is composed of* $\beta\nu$, *which is known (65,656), and* $\nu\psi$. *But because* o *is a right angle,* $\nu\psi$, *that is,* $\epsilon\psi$, *is the power of the known* $\epsilon$o (*476½*) *and* o$\psi$ *which is composed of the known* o$\alpha$ *and* $\alpha\psi$, *which is unknown but was also noted previously. Therefore,* o$\psi$ *must be made long enough that when you add the powers of* $\psi$o *and* o$\epsilon$, *the side* $\epsilon\psi$ *or* $\psi\nu$ *will be just so long that when the power of the sum of* $\beta\nu$ *and* $\nu\psi$ *is diminished by the power of* $\beta\alpha$, *it leaves the power of* $\psi\alpha$ *of*

[9] Since the angle at $\rho$ is only 3°, the inclination of the lines is not enough to account for $\epsilon\rho$'s being longer than $\mu\lambda$. However, if Kepler's errors are corrected, the computed magnitudes of both are the same, 870.

[10] *potentia*. For an explanation of this term see the glossary.

*such a magnitude that when it is compounded with* $\alpha o$ *the sum is equal to* $o\psi$ *which was taken at the beginning*[11].

*I take* $\psi o$ *as a figured unit [x]. Its square will also be a figured unit [$x^2$]. Add the square on* $\epsilon o$, *227,052, and the sum of the two will be the square of* $\psi\epsilon$ *or* $\psi\nu$. *But the square of* $\beta\nu$ *is 4,310,747,475. If you add this to the square of* $\psi\nu$ *and complete the rectangle, the result will be the square of the whole* $\psi\beta$. *Then each rectangle is the root of 4,310,747,475* $x^2$ *+ 978,763,835,536,363. And thus the square on* $\beta\psi$ *is obtained for the first time*[12].

*Now since* $\alpha o$ *is 13,971,* $\psi\alpha$ *will be represented by the figured unit diminished by 13,971. Its square will be* $x^2 - 27{,}942\,x + 195{,}188{,}841$. *Add to this the square of* $\beta\alpha$, *8,220,686,224, so that the square on* $\beta\psi$ *may be established for a second time:* $x^2 - 27{,}942\,x + 8{,}415{,}875{,}065$. *Previously it was* $x^2 + 4{,}310{,}974{,}527$ *augmented by double the root of 4,310,747,475* $x^2$ *+ 978,763,835,536,363. Subtract* $x^2$ *from both, and also 4,310,974,529. In the former, the remainder is* $-27{,}942\,x +$
36 *4,104,900,538, and in the latter, twice the root of 4,310,747,475* $x^2$ *+ 978,763,835,536,363, and these are equal. Therefore, the simple root in the former is equal to* $-13{,}971\,x + 2{,}052{,}450{,}269$. *And since this is equal to the latter's root, its square will be equal to the quantity itself. This square is* $+195{,}188{,}841\,x^2 - 57{,}349{,}565{,}416{,}398\,x + 4{,}212{,}552{,}106{,}718{,}172{,}361$. *Subtract from each 195,188,841* $x^2$ *and*

[11] In algebraic terms,

$$(\beta\nu + \sqrt{(\epsilon o^2 + o\psi^2)})^2 - \beta\alpha^2 = \alpha\psi^2,$$

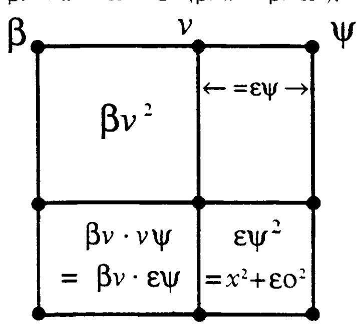
where $\alpha\psi = o\psi - o\alpha$.

[12] The argument makes use of the geometrical 'algebra' of Euclid II. 4. The square on $\beta\psi$ is made up of the squares on its parts plus twice the rectangle formed by its parts. Each rectangle is:

$$\beta\nu \cdot \epsilon\psi = \beta\nu\sqrt{(x^2 + \epsilon o^2)} = \sqrt{(\beta\nu^2 x^2 + \beta\nu^2\epsilon o^2)},$$

which is what Kepler has here. The whole square on $\beta\psi$ is:

$$\beta\nu^2 + x^2 + \epsilon o^2 + 2\sqrt{(\beta\nu^2 x^2 + \beta\nu^2\epsilon o^2)},$$




or:

$$4{,}310{,}974{,}527 + x^2 + 2\sqrt{(4{,}310{,}747{,}475\,x^2 + 978{,}763{,}835{,}536{,}363)}.$$

*978,763,835,536,363, and add to each 57,349,565,416,398 x. The two will remain equal, the former being 4,115,558,634* $x^2$ + *57,349,565,416,398 x, while the latter, 4,211,573,342,882,635,998. In least terms,* $x^2$ + *13,934 x is equal to 1,023,329,690. Solving the equation gives the figured unit* οψ *the value of 25,772.*

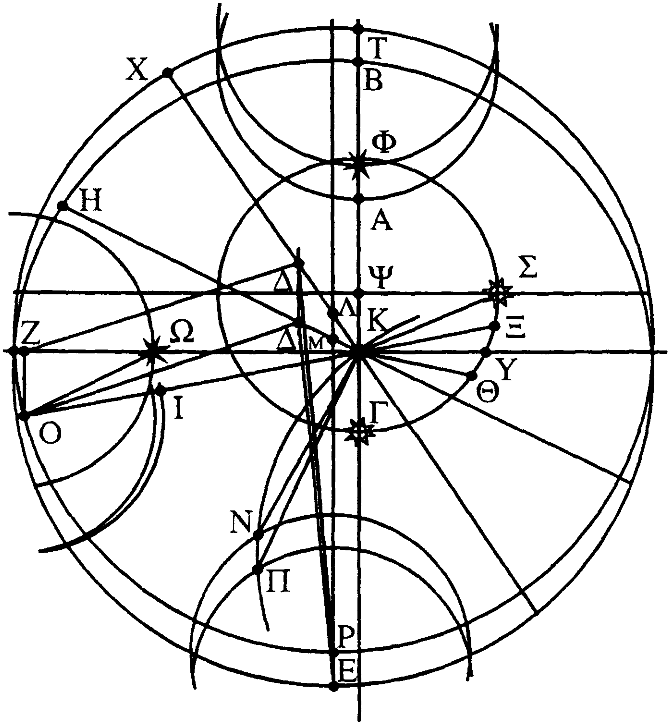
Now that the semidiameter of the circle is known, the angles are easily obtained. *From* ψο *subtract* οα, *13,971. The remainder,* ψα, *will be 11,801. And* βα *is 90,668½, and* βαψ *is right. Therefore,* αβψ *is 7° 30′ 10″. But above,* αβ *or* ρδ *were inclined to* ρλ *or* βκ *by 3° 0′ 6″, which latter lies at 5½° Cancer. Therefore,* ρι *or* αβ *will be at 8½° Cancer, and* ψβ *is consequently at 16° Cancer.* Therefore (assuming these numbers), when the sun is passing through 16° Cancer while the planet, according to its mean and uniform motion, is at 8½° Capricorn, and appears in the neighborhood of 27° Scorpio, ερ will appear maximum. If the planet is beyond 8½° Capricorn, that is, beyond ρε, even though ρε is then diminished, its apparent size can increase when viewed from a point beyond ν owing to the greater propinquity of the orbs.

37 This apparent size is now obtained explicitly. *For since* οψ *was found to be 25,772, and* ορ *476½,* οψε *will be 1° 3′ 32″*[13]. But ρνε *(which is what we have been seeking) is equal to* οψε, *by Euclid III. 20. This is because the whole angle at the centre* ρψε *is twice the angle at the circumference* ρνε, *and at the same time* οψε *is half of* ρψε. *But if* βδ *and* κδ *be bisected [at* λ *and* μ*], and* λμ *were to be taken as half* βκ *(on which more will be said below), then* ρε, *and consequently its angle at* ν *as well, could possibly become greater by one fourth.* So, at last, you see the magnitude of the disturbance occasioned in the annual parallax by my transposition of the hypothesis from the mean to the apparent motion of the sun.

A door is therefore open to us for deciding by means of the observations what I had deduced *a priori* by consideration of moving causes: that the planet's line of apsides, which is the only line bisecting the planet's path into two semicircles equal both in size and in vigour, – this line, I say, passes right through the centre of the solar body and not (as the theorists have it) through some other point. In the course of the work I shall demonstrate this by means of observations, in parts IV and V.

Now, to the extent that this is possible, I shall deduce the same things in the Ptolemaic hypothesis.

[13] The translator's recomputation shows the correct value for this angle to be 0° 55′ 12″, a change which does not affect the force of Kepler's argument here. See footnote 8, above.




*About centre* Ψ *let the sun's eccentric* Γ *be described, upon which let* ΨΓ *be the line of apsides, with the motionless earth at point* K *on the side nearest* Γ, *and* Ψ *be the point about which the sun's motion is reckoned as being uniform. From* Ψ *and* K *let the perpendiculars* ΨΣ
38 *and* KΥ *be set up, and let* ΣK *be joined. Let* KΣ *be the line of the sun's apparent motion, and* KΥ *the line of the sun's uniform motion.*

Now Ptolemy measured out the courses of the planets, not on the lines KΣ, but on the lines KΥ drawn from K parallel to the lines ΨΣ passing through the sun's body. For whenever the planet fell upon these lines KΥ on the side opposite the sun, it was supposed to have been divested of its second inequality, which fell to it (in Ptolemy's opinion) because of the epicycle. Then the planet's sidereal position was sought out by means of instruments, and the centre of the epicycle was supposed to be found at that time on the same line. This was done a number of times at various places on the zodiac: say, KΓ, KΥ, and the points opposite. From three of this sort of position of the

planet (or of the centre of the epicycle, which, according to Ptolemy, accounts for the second inequality), the theorist set out to investigate the hypothesis for the first inequality by comparing these angles which the observed positions form about K, the centre of the earth and of observation, with the intervening times. The method for this enterprise is found in Ptolemy Book IX.

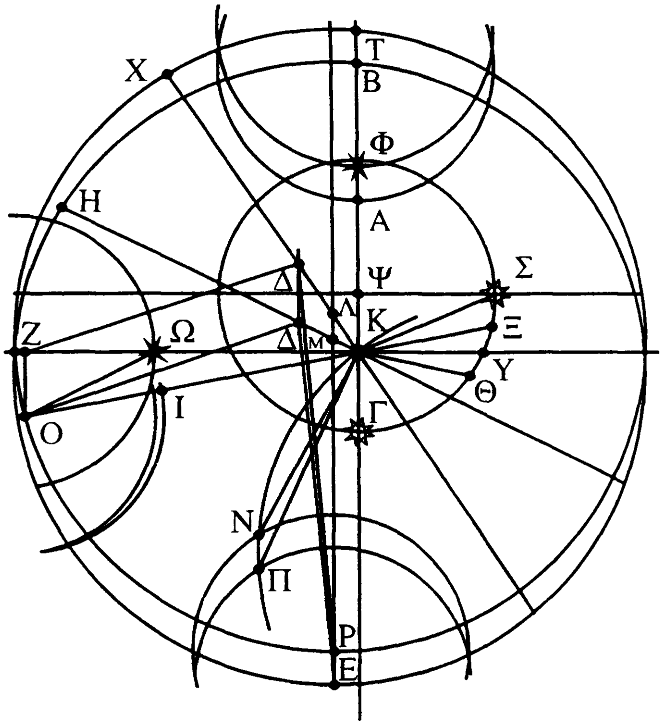
Suppose all these things done, and let the eccentric's line of apsides come out to be $K\Lambda\Delta X$, with $\Delta$ the equalizing point, the centre of the eccentric upon this line and at point $\Lambda$, and the eccentric XZ. And let this hypothesis correspond to all positions observed at the time of the planet's opposition to the sun's mean position.

Here, the objections I raised against Copernicus concerning the physical likelihood of the motions are also not clearly in agreement with Ptolemy. For here, the centre of the epicycle which accounts for
39 the second inequality is borne slowly or swiftly according to its approach to or recession from the earth K, performed on the circle XZ, just as the planet was previously. Furthermore, to assert that there is in the earth K (just as there was before in Copernicus's sun, the heart of the world) a motive force which drives the centres of this kind of epicycle around, is absurd and monstrous. The hypothesis can also be discredited by another physical argument. For this form of hypothesis involves, in one way or another, the property of solidity in the orbs, and since this has been destroyed (by Tycho Brahe's observations of comets), the hypothesis appears, so to speak, to collapse under its own weight. For it would be asserting that a moving force resides in the centre of the epicycle (not in a body, but in a mathematical point), and to rouse itself to move from place to place, by an unequal amount in equal times, and at the same time to draw the planet along with itself at the distance of the epicycle's diameter, all the while causing it to gyrate around the mover, by equal amounts in equal times. That this enormous variety cannot fall to a single moving mind (unless it be God), Aristotle gives supporting arguments in the *Metaphysics* Book XII chapter 8, where he states that the individual minds preside over individual, perfectly uniform and simple circular motions. Besides, how will some power sit in a non-body, and flow out of the non-body into the planet? Even if you divide up the tasks, placing one motive intelligence at the centre of the epicycle and another in the body of the planet, the one at the centre will use the earth (a body, of course) as its reference, moving around it nonuniformly in a circle, while the one at a point on the

circumference (that is, in the planet's body) will move around an incorporeal centre and do so uniformly. The question, then, is (as before), by what means would it move around the incorporeal point? Not by geometrical imagination, for it does not admit of geometrical imagination of its own. Nor can a mobile point subsist in a non-body even in imagination. We humans, in imagining points of this sort, have recourse to the assistance of tablets or paper, which we draw upon with our hands or at least remember ourselves to have drawn once. But it likewise cannot happen through a physical flowing of power (which is in the centre of the epicycle) out to the circumference and the planet's body. For we are putting up with this outflow of power on the supposition that the tasks of the compound motions are divided between two minds. Why not also question whether in the first, eccentric motion, some natural force capable of producing motion might subsist in some point entirely devoid of any body? And even more, whether this kind of incorporeal power can move itself around the earth, going from place to place? And most of all, whether it can communicate or transfer another's motion through an efflux from itself, although this is not supported by anything that could serve as its nest. Those sublime considerations of the essence, motion, place, and operations of the blessed angels and of separate minds which some people will want to raise in opposition to me, are irrelevant. For we are arguing about natural things which are far inferior in dignity, about powers not endowed with a will to choose how to vary their action, and about minds which are not in the least separate, since they are yoked and bound to the celestial bodies which they are to bear. These, then, are the general objections that can be raised against Ptolemy.

But something further may be said to Ptolemy, on account of which in particular he would wish to abandon his mean motion of the sun and embrace the apparent with us. For if the power moving the planet (whether single or double) acts with respect to the sun, so that the planet is at the lowest point of the epicycle whenever the centre of the epicycle is opposite the sun, I ask (as above) why it moves with respect to the imaginary point Y (which sometimes precedes and sometimes follows the sun, here marked Σ, and is sometimes above and sometimes below it), rather than the sun's body itself? Or how it is that power can have any perception whatever of the motion of Y around the earth K, since there is no body at Ψ? Is it not more probable that the epicycle adjusts its return to the lines KΣ of the

The epicycle referred to here is the Ptolemaic epicycle that accounts for the second inequality.




sun's apparent position, when these pass through the centre of the epicycle?

Let us then see what is changed on the eccentric by substitution of the sun's apparent motion. *Again (as before) when the sun* Γ, *the centre of the sun's eccentric* Ψ, *and the earth* K *are on the same line, so that the line of the sun's apparent motion* ΨΓ *and the line of its mean motion* KΓ *coincide, the centre of the epicycle* T *is in the sidereal position designated by either* KT *or* ΨT, *and the planet is really on the line* KT *or* ΨT *and at the lowest point on its epicycle* Φ, *since here it is nearest to both* Ψ *and* K. *The planet is thereby truly divested of its second inequality. But when the sun comes to the side of its eccentric (that is, to the middle longitudes), a great enough difference arises. For let the sun have gone from* Γ *to* Σ, *and the line of the sun's mean motion* KY *be found in Aries, and the planet's line of vision* KΩ *precisely*
41 *opposite in Libra, so that* YKΩ *is a single line. Now since Ptolemy stated that the planet* Ω *on this line of vision* KΩ *is divested of its second inequality, he places the centre of the epicycle* Z *on the line* KΩ. *But*

*since* KΣ *has gone beyond* KY, *the sun's apparent position is beyond opposition to the planet. Also, in the last part of the time,* KΩ *is not descending, so as to be opposite* KΣ, *but ascends towards* KΦ, *because the lowest parts of the epicycle* Ω *are retrograde and swifter than the centre* Z, *and it is there, of course, that the planet is at opposition to the sun. Therefore, this apparent opposition precedes the mean one.*

*Therefore, at some time preceding the moment designated by* KY *(let it be* KΘ*) when the sun is seen on the line* KX, *the planet will be seen opposite it (suppose it to be at* I*) on* KI *which forms a straight line with* KΞ. *Also, because it is now laid down that the second inequality is stripped away at this true opposition, the centre of the epicycle will also be seen on this line* ΞK, *say at* O. *And because the planet is retrograde, at the time* KΘ *prior to* KY *the planet is on the line* KI *later than at* KΩ. *But* KI *and* KΩ *are parts of the lines* KO *and* KZ. *Therefore,* KO *too is east of* KZ.

Thus it appears what would be changed on the line of the epicycle's centre by this reduction from the sun's mean motion to its apparent motion. *For at* T *and the opposite point on the original line the motions of the epicycle's centre remain unchanged. At* Z *this line, and the centre of the epicycle upon it, is moved forward and the intervening time is subtracted. At the opposite place the contrary happens: an addition is made to the time, and the line of motion of the centre of the epicycle is moved back westward. And thus these lines of the centre of the epicycle differ much from the original lines.* For this reason also, when we investigate the causes and measure of the first inequality by a new and repeated operation using these several observed positions of the centre of the epicycle (that is, using the observed positions of the planet, after which we suppose the centre of the epicycle to lie upon the same line), the outcome of this operation differs much from that of the preceding one. *That is, since the time in the semicircle containing the apogee has been diminished, so that the planet is correspondingly faster, the eccentricity of the equant will come out smaller. And since in the greater quadrant* BZ *of the semicircle containing the apogee the time has been diminished by an amount equal to the diminution in the other, smaller part, the planet has been made to go proportionally much faster in the remaining part of the semicircle. Therefore, the perigee has come closer to the latter, and the apogee has descended from* X *towards* Z.

Quantitatively, the new hypothesis will be made clear thus. *It is presupposed that the planet arrives at* Ω, *on the line drawn from the*

*centre of the epicycle* Z *through the earth* K, *only when* KZ *is collinear with* KΣ, *the sun's apparent position. Therefore,* KΣ *and the line drawn from* Z *through the body of the planet move in such a way as to remain always parallel. Moreover, we have just taken it from Ptolemy that at the time when the line of the sun's mean motion was* KY, *drawn through* Ω, *the planet was observed on the line* KΩ, *and nonetheless we disagree with him in that we deny that the centre of the epicycle* Z *is on* KΩ *at the same time. Therefore, in accord with our position, let a line be drawn from the planet's position* Ω *parallel to* KΣ, *and let this be* ΩO. *We are supposing that the centre of the epicycle is at this moment on the line* ΩO, *or some other line nearby and parallel, according as* Ω *(representing the planet) is closer to or farther from* K *on the line* KZ. *Let* ΩO *be drawn from whatever point on the line* KZ *(now, it is* Ω*) equal to* ΩZ. *From* O *let a line be drawn to* ZK *parallel to* KΨ, *and let this be* OZ. *Now since* ZΩO *is equal to* KΣΨ, *and* KΣ *is imperceptibly*
42 *greater than* ΨΣ *or* ΩO *(because* KΨΣ *is right, and the angle at* Σ *is not greater than 2° 3′, so that if* ΨΣ *is 100,000,* KΣ *is 100,064),* OZ *is also imperceptibly less than* KΨ. *Let* ZΔ *be joined, and let a line be drawn at* O *parallel to* ZΔ. *Now the moment of time at which the centre of the epicycle is placed at* Z *by Ptolemy and at* O *by me, is the same, and is designated by both of us by the line* KY *in the theory of the sun. In the theory of Mars, that moment should be designated in the former theory by* ZΔ, *because* Δ *is the equant point, while in the new theory it will be designated by a line parallel to* ZΔ. *Therefore, there will be a new point of equality about which the time is counted, lying on this parallel through* O.

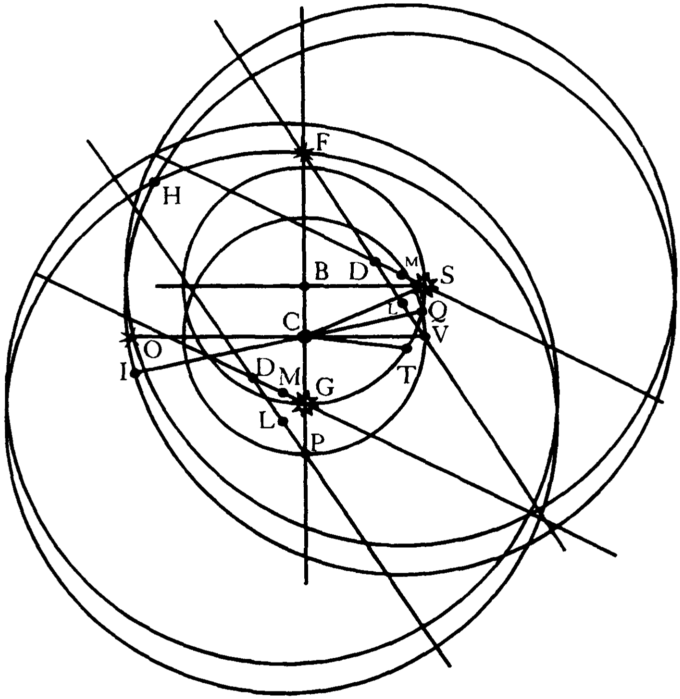
*The same things will occur when the centre of the epicycle (in Ptolemy's account) is at the other end of the line of the sun's mean motion in the neighborhood of* KY *(which, for the sake of brevity, I shall not prove). Therefore, if again a line is drawn parallel to the Ptolemaic line of the epicycle's mean motion, a line* ΔΔ *drawn from* Δ *to where the two new parallels intersect will be parallel to* ZO *or* ΨK, *equal to* ZO, *and very nearly equal to* ΨK, *and the new* Δ *will be the common point of equality in the new hypothesis.*

43 But at the end of ch. 5 above, it was shown that if through Δ a line ΔΔ be drawn parallel to KΨ, KΨ being equal to ΔΔ, and if the new Δ be joined to K, and the new line KΔ be cut at M in the same ratio in which the previous KΔ was cut at Δ, then by this new hypothesis a new eccentric is constructed, that is, one having a different position from the former, which would still leave all the appearances furnished by

the former hypothesis to an observer at K very nearly undisturbed. *Let such a new eccentric be described about* M, *equal to the former, and let* KM *be extended in both directions. The new apogee will be* H, *and the centre of the epicycle will be on the points* B, O, *of the new eccentric, with the planet nearer at* Λ *and farther at* I *than before.* However, in positions in which the second inequality is involved, the former observations are thoroughly and vehemently perturbed by this new eccentric introduced into their hypothesis. This is clearly so if an epicycle equal to the sun's eccentric is attributed to the planet, as it must be if we wish to carry over the force of Copernicus's and Tycho's discoveries into the Ptolemaic form. *The reason is not that the point of equality* Δ *does not remain the same, but that near the positions of the sun's apsides the centres of the two eccentrics, the Ptolemaic and ours, are distant by an interval* ΛM. *This distance of the centres also naturally gives rise to an equal distance of the positions of the planetary body.* Furthermore, this discrepancy is not greatest when the centre of the epicycle is about the sun's middle longitudes. *For it has been said that at those places the position of the centre of the epicycle on either eccentric is nearly the same, even though their distances are measured by parallels from* ΔΔ. It is therefore greatest near the sun's apsides, *and greater near perigee on* MΛ *extended so as to intersect the eccentrics at* P *and* E. *For* PE *is of the same magnitude as* MΛ. *But this line* MΛ *does not designate a unique moment, since it is not* M *or* Λ *that is the point of equality, but* Δ. *Therefore, let parallels be drawn from* ΔΔ *towards* P *and* E, *which shall designate the same moment, and let them be* ΔP *and* ΔE. *Also, let the epicycles* N *and* Π *be described about* P *and* E.

The question now is, where on the circumference of the epicycle would this discrepancy appear greatest? It is certain that this does not occur at the parts of the epicycle nearest K, the earth, because these parts would be in the same direction as K, nor at the highest parts of the epicycle, because they would be too far away. Therefore, this will occur at the parts of the epicycle near perigee, but also when the sun, and the planet with it, are not exactly at perigee but nearby: namely (to put it briefly) at the points N and Π corresponding to the same moment of time, such that the small circle containing them and K is a minimum. The centre of this small circle is, moreover, on the line through K which, extended upward so as to meet PΔ likewise extended, makes an angle of $7\frac{1}{2}$ degrees.

*Let anyone who disagrees with this adapt the previous demon-*




*stration to the present conditions. The numbers are just the same, except for Ptolemy* ΜΛ *is greater than the value taken above for* μλ. *For this reason, the difference in apparent position,* ΝΚΠ, *is also greater.*

45 *For previously,* βκ *was to* λμ *as* δβ *was to* δλ, *which is less than half* δβ. *To Ptolemy,* ΔΔ *(equal to* βκ*) would be to* ΜΛ *as the whole* ΚΔ *is to the half,* ΚΛ.

Although for Brahe the orb of Mars intersects the sun's orb, I have nonetheless found it preferable to exclude this intersection, because I am presenting general considerations in

Finally, I shall deduce the same thing in the Tychonic hypothesis. *About centre* B *let the solar eccentric* GS *be described, with line of apsides* BG, C *the position of the immobile earth, and* B *the point of equality, following the opinion of the authorities.* For it will be shown in due course that the point of equality and the centre of the eccentric are not the same in the theory of the sun. *Upon* BC *let the perpendiculars* BS *and* CV *be erected, and let* SC *be joined, so that* CV *may be the line of the sun's mean motion, and* CS, *of its apparent motion.*

Now, although Tycho Brahe had never finally decided whether to refer the planets to the lines CV, or to CS instead, in his initial conception he used the lines CV, according to the explanation he left

this first part, which belong to all planets. For this would bring to birth much obscurity in the diagram.

in the *Progymnasmata*, vol. I p. 477 and vol. II p. 188. This is the way shown him by the footprints of Ptolemy and Copernicus. Of this path followed by Tycho (if we carry on with Ptolemy's views), it should be said that whenever a planet is on one of the lines of the sun's mean motion CV, on the side opposite the sun, the planet is divested of its second inequality, which, in Brahe's opinion, falls to it on account of the motion of the centre of the eccentric around the earth in the same time as the sun does so.

This common point, with respect to which all the planets are said to perform their eccentric motion, and at which the whole planetary
46 system is conceived as being attached to the sun's orbit, – this point, I say, is always on the line of the sun's mean motion, distant from the earth C by a constant distance BS, and describing a concentric V equal to the eccentric GS. This was Tycho Brahe's opinion – except that he denied solid orbs. So what we said about the attachment of the whole system to the sun's orbit, we said for the comprehension of those who believe in solid orbs. *Let* VC *be extended, and let the planet be on that line beyond* C. *In this configuration, Brahe will place the point at which the planetary system is attached at* V. *The planet will be observed on the line* VC. *So, even though the observer is on the earth,* C, *it is exactly as if he were at* V, *the point upon which the first inequality depends. Next, let the planet's sidereal position be taken with instruments whenever it has been seen at some point on the line* CV *opposite* V *beyond* C *(let this be on the lines* CV, CG, *and points opposite). Thus the centre of the planetary system will be on the circle* VP, *the sun at* S *and* G, *and the body of the planet opposite it at* O *and* F. *(In the theory of Mars, the planet's eccentric is so small in proportion to the sun's eccentric that Mars's eccentric and the points* O *and* F *are nearer to the earth* C *than is the sun* S, which was one of the reasons why Brahe would deny the solidity of the orbs.) From many positions of this sort, as many, in fact, as he could obtain, Tycho Brahe used to investigate the hypothesis of the first inequality, by removing the magnitude of the orb VP and treating it as a single
47 point, as though in the meanwhile the centre VP of the planetary system, the point at which the system is attached, had remained at rest. So he set up a comparison of the elapsed times with the angles which VO and PF, drawn from a single point (V and P coinciding), would form. These are in fact identical to the angles OCF and VCP.

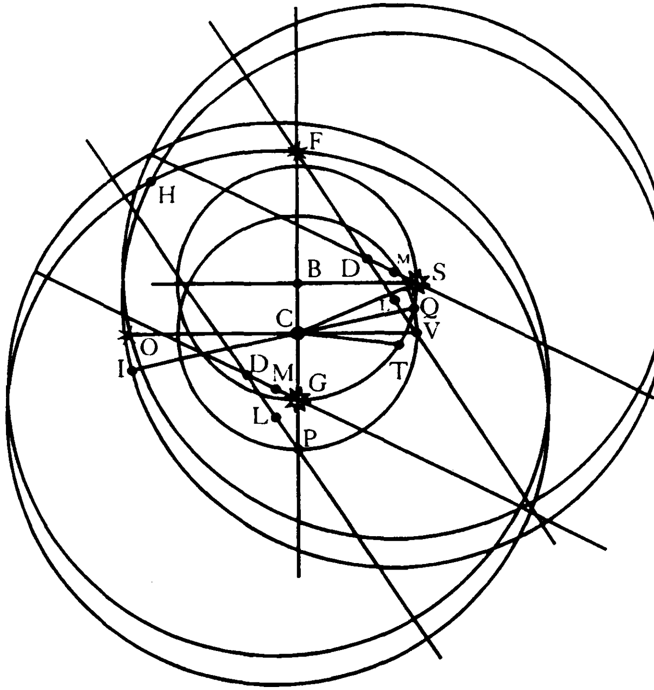
Suppose these things done, and that the line of apsides of the eccentric come out to be VLD or PLD, D the equalizing point, L the

centre of the eccentric on this line, and the eccentric HO and FH. Let this hypothesis correspond to all positions of the planet observed at the moment of the planet's opposition to the sun's mean position.

For the present, I am postponing a more careful consideration of whether this sort of hypothesis is in accord with physical principles – a hypothesis, that is, in which the sun, through its motive mind paying attention to the earth, moves around the earth, and of itself (since it lacks an orb) drives itself forward nonuniformly according to its approach to or recession from the earth (unless you would make the earth more important than the sun and ascribe to the earth the sun's motive force); while this same sun (as in Copernicus) sends out a motive force to all the planets, sweeping them around itself with a degree of speed corresponding to their degree of nearness to the sun; the planets meanwhile striving to accomplish their approach to and recession from the sun on a small epicycle, and at the same time to follow the sun, wherever it is supposed to be, in those same tracks not proper to them; and thus any planet, and most of all the sun, must attend to many things at once, and the actual trajectories of the planets through the aethereal air (as with Ptolemy) make spirals, such as were depicted in chapter 1. Whether, I say, these are fitting, we shall consider elsewhere when the occasion arises. Here, this form of hypothesis is supposed true in its general features. The question is, whether it is also fitting, in particular, that the planets follow the sun's body S, G, or whether they instead follow the point V, P, void of any body, four solar semidiameters (no more) from the centre of the sun, which is now above the sun and now below, now before and now behind. And further, whether it is more fitting that the force that drives the planets in an orb around the sun be in the body of the sun S, G, or make its nest in some other point such as VP void of body. In brief: if the axis (to use the coarse analogy of a wagon) of the planetary system, to which the orbs of the planets are fastened as with a nail, – if this, I say, is near the sun, why not in the sun itself? If this axis or point of attachment travels around the earth both close to the sun and in the same period, why does it describe its own peculiar path? Why does it not hold to the very same path as the sun?

Term. Axis or centre of the planetary system. Elsewhere, the point or centre of attachment.

I therefore wholeheartedly conclude that if, perchance, Tycho Brahe's opinion on the world system be universally true, it must be accepted in such form that the centre of the planetary system lies upon SG, exactly on the sun's path, not upon VP; and finally, that it is




in the sun itself, and that the first or eccentric inequality is to be liberated from the second by making use of the planet's oppositions to the apparent position of the sun, not the mean. Brahe himself in his final days embraced this procedure unreservedly. Let us, then, see what would be changed on the eccentric. *Once again, as before, when the sun is on the line* BC, *as at* G, *and the planet is at* F *opposite point* P,
48 *the planet* F *will be opposite the sun itself,* G. *Consequently, the sidereal position of the planet will appear to be the same, along the line GF, whether the continuation of the line of vision be* CP *or* CG, *because both have been made into one line. Therefore, according to either procedure, the planet is divested of the second inequality. But when the sun comes to the side of its eccentric (that is, to the middle longitudes), an appreciable difference arises. For let the sun have gone from* G *to* S, *let the line of the sun's mean motion* CV *be in Aries, and the planet's line of vision* CO *be exactly opposite in Libra, so that* VCO *is one line. Since* CS *is beyond* CV, *the sun's apparent position is beyond opposition to the planet. But, because of my alterations, the centre of the planetary system is not at* V *but at* S *when the planet is seen along* CO. *Therefore,* SO *being connected, the earth* C *will not lie on*

*the line* SO, *and hence, the planet's apparent position on the line* CO *still has an intermingling of the second inequality. Nor will* CO *be farther east at a later time, so as to be at opposition to* CS. *Instead, it ascends toward* CF, *because the sun's motion, and with it the motion of the centre of the planetary system and of all its parts (including the planet* O *and the centre of the eccentric* L*), is from the line* CO *upward towards F, and is much faster than the motion of the eccentric or planet at* O *about* L *from the point* H *towards the bottom. Consequently,* O *is drawn backward by a motion extraneous, not proper, to the eccentric, moving appreciably westward; and indeed, it is well known that the planets are retrograde at opposition to the sun. Therefore, at a time preceding the moment designated by* CV *(let it be* CT*), when the sun appears on line* CQ, *the planet will be seen at the point opposite its*
49 *apparent position, at* I. *And now, since in the present case the second inequality is supposed to have been removed,* QCI *will be collinear; that is, the point whence the eccentricity arises will be on the line* CQ. *But* CI, *the retrograde planet's line of vision, is beyond the later (and hence more westerly) line of vision* CO *at the earlier time. Therefore,* CQ *will also be beyond* CV, *and* Q *will be the system's new centre, beyond the old,* V. *Moreover, the line* IQ *was made to be east of the line* OV *by the angle* OCI, *while the line of apsides* VD *or* PD *(from which the motion begins) remains parallel to itself in the entire circuit. As a result, it appears to be established that the planet goes farther in a shorter period of time around the centre* Q *of the system than it previously did in a greater period of time about the centre V of the system.*

It thereby becomes apparent what is changed in the apparent eccentric motion by this reduction from the sun's mean motion to its apparent motion. For when the centre of the system is at G and the point opposite, the line of the apparent eccentric motion remains unchanged, at Q it is moved forward, and opposite Q it is moved back, for at Q the time is diminished, and opposite Q it is increased. Also, these lines differ greatly from the original ones. Accordingly, when we repeatedly use a new operation, based upon these several apparent positions of the planet (opposite which we are supposing the centre of the planetary system to be found; that is, in the sun itself), to investigate the causes and measure of the first inequality, the result of the operation differs greatly from the previous result.

*For we have just transposed the point of attachment from the circle* VP, *upon which Brahe had it move around, to the circle* GS, *that is, to*

*the body of the sun. This new centre always stands on a line parallel to* CB, *at a distance* CB *from the original Brahean point; for example, above* V *and* P *at* S *and* G. *Therefore (with the equant point* D *fixed, so that* CV *represents the same moment), in order for it to be possible both for the planet to be at* O *and for the point of attachment to be at* S, *a new line of apsides needs to be drawn through* D *and* S *or* G. *Consequently, according to the demonstrations of chapter 5 (which we have brought forward above, in explaining the Copernican form), with* DS *or* DG *drawn and divided at* M *in the ratio in which* DP *or* DV *are divided at* L, *let there be described about centre* M, *with the same radius as before,* a new eccentric. This will not only account for the later observations, upon which it was constructed, but, introduced into the prior hypothesis, it will also save the observations previously brought in, with a precision of within five minutes.

However, computations which are carried out upon this new eccentric and the previous one at positions other than acronychal can in some places (particularly near the sun's perigee) differ by more than one degree, if we use numbers fitting and proper for Mars, which have been furnished by Brahe.

50 *There is no need to repeat the demonstration. The drawing is easiest in the Copernican diagram, if from the earth* $\nu$ *you set up a line parallel to* $\beta\kappa$, *measure off upon it, at an interval* $\beta\kappa$ *above* $\nu$, *the centre of the sun's eccentric, and upon this centre set up the Brahean eccentric of the sun through* $\kappa$, *deleting the Copernican centre of the earth's eccentric.*

The differences of the hypotheses, and their equivalence in the first inequalities but discrepancy in the second, have now been expounded. Let us then conclude the first part, which, as I see it, is the most difficult of the entire work, because of the almost inescapable labyrinths of opinion and the perpetual ambiguities of words and extremely tiresome circumlocutions. What necessity it was, however, that forced me to prefix this body of instruction, will now directly appear in chapter 7. Anyone who is not so bright can defer the whole part until he understands the easier parts.

THE COMMENTARIES
ON THE MOTION OF THE STAR MARS

51 PART II

# On the star Mars's first inequality, in imitation of the ancients

52

# 7

# The circumstances under which I happened upon the theory of Mars

It is true that a divine voice, which enjoins humans to study astronomy, is expressed in the world itself, not in words or syllables, but in things themselves and in the conformity of the human intellect and senses with the sequence of celestial bodies and of their dispositions. Nevertheless, there is also a kind of fate, by whose invisible agency various individuals are driven to take up various arts, which makes them certain that, just as they are a part of the work of creation, they likewise also partake to some extent in divine providence.

When, in my early years, I was able to taste the sweetness of philosophy, I embraced the whole of it with an overwhelming desire, and with no special interest whatever in astronomy. I certainly had enough intelligence, nor did I have any difficulty understanding the geometrical and astronomical topics included in the normal curriculum, aided as I was by figures, numbers, and proportions. These were, however, required courses, and did not suggest a particular inclination for astronomy. And since I was supported at the expense of the Duke of Wurttemberg, and saw my comrades, whom the Prince, upon request, was sending to foreign countries, stalling in various ways out of love for their country, I, being hardier, quite maturely agreed with myself that whithersoever I was destined I would promptly go.

The first to offer itself was an astronomical position; however, to tell the truth, I was driven to take on this task by the authority of my teachers. I was not frightened by the distance of the place, for (as I have just said) I had condemned this fear in others, but by the low

opinion and contempt in which this kind of function is held, and the sparsity of erudition in this part of philosophy. I therefore entered upon this better furnished with wits than with knowledge, and protesting loudly that I would never willingly concede my intention to follow another kind of life which seemed more splendid[1]. What came of those first two years of study may be seen in my *Mysterium cosmographicum*. The additional goads which my teacher Maestlin gave me towards embracing the rest of astronomy, you will read of in the same book, and in that man's prefatory letter to Rheticus's *Narratio*[2]. I had the very highest opinion of the discovery, and all the more so when I saw that Maestlin, too, held it in similar esteem. It was not so much his untimely promise to the readers of what he called a 'uranic opus' of my universe that spurred me on, as it was my own ardour to seek, through a reworking of astronomy, whether my discovery would stand comparison with observations made with perfect accuracy. For it had then been demonstrated in that book that the discovery was consistent with the observations within the limits of accuracy of ordinary astronomy.

53 So from that time I began to think seriously of comparing observations. In 1597, I wrote Tycho Brahe asking his opinion of my little book[3], and when he, in reply, mentioned among other things his own observations, he ignited in me an overwhelming desire to see them. Moreover, Tycho, who was indeed himself a large part of my destiny, did not cease from then on to invite me to come to him. And since I was frightened off by the distance of the place, I again ascribe it to divine arrangement that he came to Bohemia. I thereupon came to visit him at the beginning of 1600 in hopes of learning the correct eccentricities of the planets. But when I found out during the first week that, like Ptolemy and Copernicus, he made use of the sun's mean motion, while the apparent motion would be more in accord with my little book (as is clear from the book itself), I begged the master to allow me to make use of the observations in my own manner. At that time, the work which his aide Christian Severinus[4] had in hand was the theory of Mars. The occasion had placed this in

[1] Kepler had hoped eventually to obtain an ecclesiastical post.

[2] Maestlin, who saw to the printing of the *Mysterium cosmographicum*, took the liberty of appending Georg Joachim Rheticus's *Narratio prima* (1540) to aid the reader in understanding the Copernican hypothesis.

[3] Letter to Tycho Brahe, 13 December 1597, letter number 82 in KGW 13 p. 154.

[4] Better known under the name 'Longomontanus', the Latinization of his birthplace, Longberg.

his hands, in that they were busy with the observation of the acronychal position or opposition of Mars to the sun in 9° Leo. Had Christian been treating a different planet, I would have started on it as well.

I therefore once again think it to have happened by divine arrangement, that I arrived at the same time in which he was intent upon Mars, whose motions provide the only possible access to the hidden secrets of astronomy, without which we would remain forever ignorant of those secrets.

A table of mean oppositions was worked out, starting with the year 1580. A hypothesis was invented which, it was proclaimed, represented all these oppositions within a distance of two minutes in longitude. (The numbers I used in chapter 5 were taken from among these, or differed from them only slightly.) The apogee at the beginning of the year 1585 was placed at 23° 45′ Leo. The maximum eccentricity, made up of the semidiameters of the two small circles, was 20,160, in units of which the semidiameter of the greater epicycle would have been 16,380. Therefore, in the Ptolemaic form of the first inequality, the eccentricity of the equalizing point was 20,160 or a little less. From this hypothesis, a table of eccentric equations for individual degrees was constructed, as well as a table of the corrected mean motion, made by adding $1\frac{3}{4}'$ to the mean motion of the Prutenic Tables. These mean motions, apogees, and nodes were extended over a period of forty years, exactly as was done for the solar and lunar motions in Book I of the *Progymnasmata*. It was only in the latitude at acronychal positions and also the parallax of the annual orb that Christian got stuck. There was, actually, a hypothesis and table for the latitudes, but they failed to elicit the observed latitude. This same result was destined to be a problem for the lunar theory as well.

Now since I suspected what proved to be true, that the hypothesis was inadequate, I entered upon the work girded with the preconceived opinions expressed in my *Mysterium cosmographicum*. At the beginning there was great controversy between us as to whether it were possible to set up another sort of hypothesis which would
54 express to a hair's breadth so many positions of the planet, and whether it were possible for the former hypothesis to be false despite its having accomplished this so far over the entire circuit of the zodiac.

I consequently showed, using the arguments presented already in

Part I, that an eccentric can be false, yet answer for the appearances within five minutes or better, provided that the centre of the equant be correctly known. As for the parallax of the annual orb, and the latitudes, that prize is not yet won, and besides, was not attained by 55 their hypothesis. What remains, then, is to find out whether they,

# 8

## Tycho Brahe's table of observed and computed oppositions of Mars to the line of the sun's mean motion, and an examination thereof

The table mentioned above is the following:

P denotes the Paduan observation carried out by Magini with Brahe's student Gellius Sascerides. N denotes our observation (that is, Brahe's), carried out at Uraniborg.

An accurate rendering of the motion of the planet Mars on its eccentric from trustworthy acronychal observations in a variety of positions, carried out sedulously over a period of twenty years (1580 to 1600) using our instruments.

| Uniform Time of Mars | | | | | Obs. Long. with respect to Mars's Circle | | | | True obs. Latitude | | | | Long. with respect to the ecliptic | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Year | Month | D | H | M | ° | ′ | ″ | | ° | ′ | ″ | | ° | ′ | ″ |
| 1580 | November | 17 | 9 | 40 | 6 | 50 | 10 | Gemini | 1 | 40 | 0 | N | 6 | 46 | 10 |
| 1582 | December | 28 | 12 | 16 | 16 | 51 | 30 | Cancer | 4 | 6 | 0 | N | 16 | 46 | 10 |
| 1585 | January | 31 | 19 | 35 | 21 | 9 | 50 | Leo | 4 | 32 | 10 | N | 21 | 10 | 26 |
| 1587 | March | 7 | 17 | 22 | 25 | 5 | 10 | Virgo | 3 | 38 | 12 | N | 25 | 10 | 20 |
| 1589 | April | 15 | 13 | 34 | 3 | 54 | 35 | Scorpio | 1 | 6 | 45 | N | 3 | 58 | 10 |
| | | | | | 26 | 40 | 30P | | | | | | | | |
| 1591 | June | 8 | 16 | 25 | | | | Sagittarius | 3 | 59 | 0 | S | 26 | 32 | 0 |
| | | | | | 26 | 42 | 0N | | | | | | | | |
| 1593 | August | 24 | 2 | 13 | 12 | 35 | 0 | Pisces | 6 | 3 | 0 | S | 12 | 43 | 45 |
| 1595 | October | 29 | 21 | 22 | 17 | 56 | 5 | Taurus | 0 | 5 | 15 | N | 17 | 56 | 15 |
| 1597 | December | 13 | 13 | 35 | 2 | 34 | 0 | Cancer | 3 | 33 | 0 | N | 2 | 28 | 0 |
| 1600 | January | 19 | 9 | 40 | 8 | 18 | 45 | Leo | 4 | 30 | 50 | N | 8 | 18 | 0 |

with their means of computation, might not somewhere differ from the observations by five minutes.

I therefore began to investigate the certitude of their operation. What success came of that labour, it would be boring and pointless to recount. I shall describe only so much of that labour of four years as will pertain to our methodical enquiry.

| Difference | | | Simple Long. of Mars[1] | | | | Apogee of Mars[1] | | | | Our precess. of the equin. | | | Computed | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| ′ | ″ | | S | ° | ′ | ″ | S | ° | ′ | ″ | ° | ′ | ″ | ° | ′ | ″ |
| 4 | 10 | + | 0 | 27 | 29 | 46 | 3 | 25 | 21 | 40 | 27 | 58 | 50 | [illegible] | 50 | 40 |
| 5 | 20 | + | 2 | 11 | 34 | 56 | 3 | 25 | 22 | 17 | 28 | 0 | 38 | [illegible] | 51 | 26 |
| 0 | 36 | − | 3 | 22 | 37 | 46 | 3 | 25 | 22 | 55 | 28 | 2 | 25 | [illegible] | 9 | 41 |
| 5 | 10 | − | 5 | 3 | 27 | 46 | 3 | 25 | 23 | 32 | 28 | 4 | 10 | [illegible] | 4 | 50 |
| 3 | 35 | − | 6 | 16 | 53 | 7 | 3 | 25 | 24 | 10 | 28 | 5 | 55 | [illegible] | 54 | 33 |
| 10 | 20 | + | 8 | 7 | 47 | 30 | 3 | 25 | 24 | 48 | 28 | 7 | 47 | [illegible] | 40 | 23 |
| 8 | 45 | − | 10 | 10 | 53 | 50 | 3 | 25 | 25 | 26 | 28 | 9 | 40 | [illegible] | 34 | 36 |
| 0 | 12 | + | 0 | 8 | 26 | 47 | 3 | 25 | 27 | 35 | 28 | 11 | 27 | [illegible] | 57 | 14 |
| 6 | 0 | + | 1 | 24 | 55 | 47 | 3 | 25 | 29 | 5 | 28 | 13 | 20 | [illegible] | 32 | 20 |
| 0 | 45 | − | 3 | 6 | 46 | 16 | 3 | 25 | 30 | 6 | 28 | 15 | 5 | [illegible] | 19 | 57 |

[1]. These are apparently measured from 'the first star in Aries, in the Copernican manner', as Kepler puts it below.

The emended mean longitudinal motion of Mars at the beginning of 1585 was found to exceed the numbers provided by the Prutenic calculation by at least a minute and a half, or at most $1\frac{3}{4}'$, which by all indications appears more nearly correct. However, the position of its apogee then fell short of the Prutenic calculation at the same time, by $5^\circ\ 2'$, both being compared with the first star of Aries in the Copernican manner. Hence, owing to our removal of the vernal equinox westward from that star, which was then $28^\circ\ 2\frac{1}{2}'$[1] it is concluded that Mars's apogee was at $23^\circ\ 25'$ Leo. For the first date it was set at $23^\circ\ 20'$ Leo, and for the last, at $23^\circ\ 45'$ Leo.

Also, the maximum eccentricity, composed of the semidiameters of the two small circles, was found to be 20,160, of which the semidiameter of the greater epicycle, or distance between centres as given by Copernicus, is 16,380. In both of these, however, he differs both from himself and from Ptolemy. Care was taken, where appropriate, concerning refraction, using the solar parallax.

THIS, THEN, IS BRAHE'S TABLE.

56 We shall begin the examination of the sun's mean motion with the listed instants of equal time, as many as the table presents. For indeed, it is the sun's mean position in opposition to which the table says the star Mars was found, referred to the ecliptic.

You see here that the sun's mean position differs from opposition to Mars's apparent position on the ecliptic by as much as $13\frac{1}{4}'$, nearly thrice the error which could arise through a change of hypothesis. Therefore, the exactness of their hypothesis did not prevent my seeking another.

But they permitted this discrepancy advisedly. For, since the nodes are about $17^\circ$ Taurus and Scorpio, and the limits about $17^\circ$ Leo and

[1] In Tycho Brahe's *Stellarum inerrantium restitutio* (TBOO III p. 344), the first star in Aries (Gamma Arietis) had a longitude of $27^\circ\ 37'$ Aries at the end of 1600 (that is, at noon on 1601 January 1). Tycho's correction for the interval of 15 years 11 months would be $13'\ 32''$, putting the star at $27^\circ\ 23\frac{1}{2}'$ Aries at the end of January 1585. Hence, Kepler's longitude for the star appears incorrect. Nor is there any other star in Tycho's catalogue which would have the stated longitude at that time. This error appears to affect the figures in the column headed 'Our precess. of the equin.', as well as the positions of the apogee given in this paragraph. Note that the third entry in the 'precession' column is the stated longitude in question, and the other figures in this column reflect only the change required by the Tychonic rate of precession. The column headed 'Simple long. of Mars' may also reflect this curiously aberrant elongation for Gamma Arietis, since, according to the translator's computation, the third entry in this column differs from the Keplerian mean longitude for this date and time by $28^\circ\ 6'\ 22''$.

| Year | Day | Month | H | M | S | Sun's mean position ° | ′ | ″ | Star's obs. pos. on the ecliptic ′ | ″ | Difference ′ | ″ | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 1580 | 17 | November | 9 | 40 | 8 | 6 | 48 | 32 | 46 | 10 | 2 | 22 | – |
| 1582 | 28 | December | 12 | 16 | 9 | 16 | 50 | 58 | 46 | 10 | 4 | 48 | – |
| 1585 | 31 | January | 19 | 35 | 10 | 21 | 10 | 13 | 10 | 26 | 0 | 13 | + |
| 1587 | 7 | March | 17 | 22 | 11 | 25 | 5 | 57 | 10 | 20 | 4 | 23 | + |
| 1589 | 15 | April | 13 | 34 | 1 | 3 | 53 | 32 | 58 | 10 | 4 | 38 | + |
| 1591 | 8 | June | 16 | 25 | 2 | 26 | 45 | 24 | 32 | 0 | 13 | 24 | – |
| 1593 | 24 | August | 2 | 13 | 5 | 12 | 34 | 36 | 43 | 45 | 9 | 9 | + |
| 1595 | 29 | October | 21 | 22 | 7 | 56 | 17 | 56 | 15 | 0 | 2 | 22 | – |
| 1597 | 13 | December | 13 | 35 | 9 | 2 | 28 | 51 | 28 | 0 | 0 | 51 | – |
| 1600 | 19 | January | 9 | 40 | 10 | 8 | 18 | 43 | 18 | 0 | 0 | 43 | – |

Aquarius, as will be said below, the additions and subtractions are made chiefly at 17° Cancer, 25° Virgo, 4° Scorpio, 27° Sagittarius, and 13° Pisces, all intermediate points, and none at 21° Leo and 18° Scorpio, the nodes and the limit. They therefore had reason to believe that a planet is not divested of its second inequality unless the sun's departure from the node is as great as the planet's on its own orbit. Their intentions, moreover, were not consistent. For, in their way of thinking, the difference ought to be greatest at 3° Cancer, because Cancer is closest to the 45th degree[2], where this difference is generally greatest. But at 17° Cancer they subtracted 5 minutes, while at 3° Cancer, one minute only. Because of this, the following table, comparing the positions (referred to the orbit of Mars) with the mean positions of the sun at these moments, is presented.

| Minutiae of the sun's mean position ′ | ″ | Minutiae of Mars's apparent position on its orbit ′ | ″ | Difference ′ | ″ | |
|---|---|---|---|---|---|---|
| 48 | 32 | 50 | 10 | 1 | 38 | + |
| 50 | 58 | 51 | 30 | 0 | 32 | + |
| 10 | 13 | 9 | 50 | 0 | 23 | – |
| 5 | 57 | 5 | 10 | 0 | 47 | – |
| 53 | 32 | 54 | 35 | 1 | 3 | + |
| 45 | 24 | 42 | 0 | 3 | 24 | – |
| 34 | 36 | 35 | 0 | 0 | 24 | + |
| 56 | 17 | 56 | 5 | 0 | 12 | – |
| 28 | 51 | 34 | 0 | 5 | 9 | + |
| 18 | 43 | 18 | 45 | 0 | 2 | + |

[2] From aphelion.

Clearly, they did not compensate the whole difference in this way. We shall discuss this plan of theirs once more a little later.

Now, we shall pass on to examine Mars's mean motion, for the sake of which, see the following table.

Minutes and seconds of mean motion.

57

| By my computation from Brahe's tables | | Value presented | | Difference | | |
|---|---|---|---|---|---|---|
| ′ | ″ | ′ | ″ | ′ | ″ | |
| 29 | 9 | 29 | 46 | 0 | 37 | + |
| 35 | 26[4] | 34 | 56 | 0 | 30 | −[5] |
| 37 | 4 | 37 | 46 | 0 | 42 | + |
| 27 | 16 | 27 | 46 | 0 | 30 | + |
| 52 | 33 | 53 | 7 | 0 | 34 | + |
| 46 | 45 | 47 | 30 | 0 | 45 | + |
| 53 | 18 | 53 | 50 | 0 | 32 | + |
| 26 | 5 | 26 | 47 | 0 | 42 | + |
| 54 | 48 | 55 | 47 | 0 | 59 | + |
| 45 | 39 | 46 | 16 | 0 | 37 | + |

[4]. This should be 34′ 26″, according to the translator's recomputation.
[5]. Should be positive.

I am therefore missing something small in the mean longitude. For, that nearly everywhere there is half a minute too much, may be so because of my having computed the mean motions from the most recent table, in which something might possibly have been deliberately altered.

There follows a table of Mars's eccentric positions.

| By my computation from the Brahean tables | | Value presented | | Difference | | |
|---|---|---|---|---|---|---|
| ′ | ″ | ′ | ″ | ′ | ″ | |
| 49 | 37 | 50 | 40 | 1 | 3 | + |
| 52 | 59 | 51 | 26 | 1 | 33 | − |
| 9 | 47 | 9 | 41 | 0 | 6 | − |
| 4 | 49 | 4 | 50 | 0 | 1 | + |
| 54 | 46 | 54 | 33 | 0 | 13 | − |
| 34 | 45 | 40 | 23 | 5 | 38 | + |
| 33 | 59 | 34 | 36 | 0 | 37 | + |
| 57 | 37 | 57 | 14 | 0 | 23 | − |
| 31 | 48 | 32 | 20 | 0 | 32 | + |
| 45 | 39 | 46 | 16 | 0 | 37 | + |

All positions are tolerably accurate except 27 Sagittarius. Here, for various reasons, a small but appreciable quantity has been accumulated. First, the sun's position is 26° 45′ 24″ Gemini. Now the

computed position on Mars's orbit is 26° 24′ 43″ Sagittarius. In the opinion of the table, 10′ 20″ are to be subtracted from the latter to reduce it to the ecliptic. Therefore, the computed position on the ecliptic would be 26° 24′ 13″ Sagittarius, a difference from opposition to the sun of 21′ 11″.

# 9

## On referring the ecliptic position to the circle of Mars

But it is now time for us to discuss in detail this reference to the ecliptic or to the planet's orbit, upon which everything else is founded.

First, the table provides us the following information from the observations: the northern latitude takes its rise from 18° Taurus, at which it was five minutes, reached its observed maximum at 21° Leo,
58 decreased thereafter to become only $1\frac{2}{3}°$ at 3° Scorpio, but right away at 27° Sagittarius it was a larger-than-average 4°, and still greater at 13° Pisces. From this one can make a rough estimate that the ascending node is a little before 18° Taurus, and the descending node far beyond 3° Scorpio. The nodes will therefore be around 17° Taurus and 17° Scorpio, and the limits around 17° Leo and Aquarius. Since the plane of Mars's eccentric is inclined to the plane of the ecliptic, nearly the same thing that happens with the right ascensions of parts of the ecliptic will happen here: the observed arcs of one circle do not correspond to the same observed arcs on the other, except the ones beginning at a node and ending at a limit. I use the term, 'observed arcs', because here one must mentally separate out the planet's eccentricity, and proceed as though Mars's path were projected upon the orb of the fixed stars, exactly as is the ecliptic, and as though it really intersected the latter. And indeed, when asked what is the ecliptic position of a planet, astronomers define it thus: it is that point on the ecliptic at which the circle of latitude (at right angles to the ecliptic) passing through the sidereal position of the planet's body intersects the ecliptic.

Term: What is the ecliptic position of a planet, as opposed to the position on the orbit, or considered with respect to the orbit?

It is therefore clear from the demonstrations in Theodosius's *Sphere* that unless this circle passes through the poles of both circles (the ecliptic and the planet's path), its points of intersection will always cut off unequal arcs as measured from the point at which the two circles intersect. And since the circle of latitude is at right angles to the ecliptic, it will always be oblique to the planetary orbit if it does not pass through the poles of the orbit. Consequently, the arc between the planet's position on its orbit and the nearest node is always greater than that between its ecliptic position and the same node.

What does it mean to refer a planet to the ecliptic?

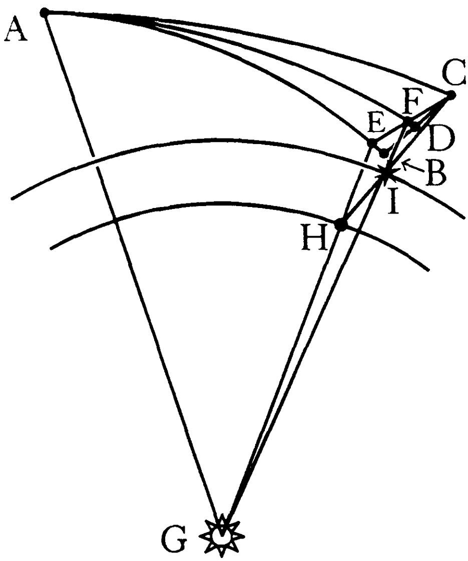
Now when we observe the planets, we do not feel convinced that we have defined their exact positions until we have referred them to the ecliptic. This is done by indicating the point on the ecliptic at which the circle of latitude passing through the planet is found. The ecliptic position is used, therefore, to aid our memory and comprehension. But when, on the other hand, we compute the planet in its own hypothesis, we are concerned with the exact path of the planet, and not with the ecliptic to which it is inclined. Therefore, to be able to compare the observed position with the computed position, we must either extend the arc between the ecliptic position and the nearer node, or abridge the arc between the body of the planet and the same node, so that from the former operation the position on the orbit might be given, and from the latter, the ecliptic position. This is actually accomplished by adding or subtracting, according as the node precedes or follows the planet's position.

Such care concerning the planets Ptolemy considered unnecessary. Copernicus did not forego it in treating the moon, and Tycho Brahe diligently embraced the cause of precision.

To continue: in the referring process which we have been considering, there are two things I would like to know, both of which I can seek using the same procedure and diagram.

**59** *Let* A *be the sidereal position of the node,* AB *the arc of the ecliptic, and upon it let* AC *be set equal to* AB, *and let the planet be observed beneath* C. *Further, from* C *let an arc be drawn perpendicular to the ecliptic, and let it be* CE.

*Now in the first place the ancients thought that since* E *is the position on the ecliptic and* C *the position of the orbit of the planet* I, *the planet is at the point opposite the sun when the sun is at* E, *the planet being observed at* C. However, as was said above, those who constructed the tables thought that the planet is not exactly at opposition to the




sun unless AC (the observed distance of the planet from the node) is equal to arc AB, the elongation of the place opposite the sun from the same node.

Now the truth of the matter is quite different. The planet is, indeed, seen to be exactly opposite the sun at that time, but it really is not, and the advantage we seek to obtain from the planet's opposition to the sun is vitiated by making AC and AB equal, rather than corrected, as they were hoping it would be. For why are the planets observed at opposition to the sun? In order, of course, that they then lack the second inequality of longitude. And when the point opposite the sun is at B and the planet is at C, both being between the nodes and the limits, the planet is more implicated in the second inequality of longitude than if the point opposite the sun were at E, the planet remaining at C. *For let* G *be the sun, the centre of the planetary system, at which all the orbs intersect the ecliptic, whether in the Copernican or the Brahean form. Let* G *be joined to* A *and* E, *points on the ecliptic, and let the earth be on the line* EG, *at* H. *Let* HC *be joined, and from* H *let the sun* G *be observed opposite* E, *while from the same point* H *let the planet be seen at* C, *its sidereal position along the line* HC. *Therefore, in this sighting, the planet is really on the line* HC. *It is, however, far below the fixed stars. Let it be at the point I on the line*

HC, *and let a straight line be drawn from* G *through* I, *which will intersect the arc* CE. *For the whole plane* CEHG *is beneath the arc* EC. *Let its point of intersection be* F, *and let a third arc* AF *be drawn from* A *through* F *to* BC, *cutting* BC *at* D. *It is obvious that the plane of the planet's eccentric viewed from* H *towards* C *is set beneath* AF, *not* AC, *and that when the point opposite the sun is at* E, *the planet will really be beneath* F, *while when the point opposite the sun is at* B *the planet is really going to be beneath* D, *although both do appear beneath* C. *But* AD *is shorter than the legs of the isosceles triangle* BAC. *Therefore, the point* B *opposite the sun is farther from* A *than is* D, *the position beneath which the planet is at the moment they have chosen.* Therefore, the sun really stands beyond the point opposite the planet's true position. This is contrary to what they proposed to do.

But it is likewise false that if the orbit of the planet were beneath AC, AB should on that account be taken equal to AC. *For the orbit's really being beneath* AD *is likewise not sufficient reason for taking* AD *equal to* AB. *For since the planet is observed at the point opposite the sun in order to divest it of the second inequality of longitude, while the longitude is to be reckoned on the planet's true orbit, or on* AD *which stands above it, surely, unless the point opposite the sun falls upon the arc drawn through the planet at right angles to the orbit (that is, unless* ADB *is a right angle), the point* B *opposite the sun will not coincide in longitude with* D. *But if* ADB *is a right angle,* AB *is longer than* AD. *They are therefore not equal.* Clearly, therefore, the equality which the table supposes to exist between arcs AC and AB, is destroyed.

60 However, for practical purposes, these differences are smaller than can be perceived. I therefore do not hesitate to allow the point opposite the sun to be at E, with AEF right and AFE consequently acute, even though it has just been demonstrated that AFE should be right instead. There is, however, another affectation of accuracy which must be countered with accurate arguments. What follows here is the new accusation arising from this accuracy.

In the second place, then, I wish to establish this: that in the table of reference they followed a procedure that is unsound. *For, given Mars's ecliptic position* E, *and apparent latitude* EC, *they computed the length of* AC, *and stated that the planet on its orbit was then distant from the node by the amount* AC. *Now the orbit of the planet (whose first inequality we are investigating) is not beneath* AC, *but beneath* AD, *as was just shown. Therefore, the arc* AC *has nothing to do with the first inequality, but adulterates the planet's true elongations from*

A. *And furthermore, the apparent latitude is* EHC, *while the true latitude of the point* F, *the inclination of the line* GF *to the ecliptic, is* EGF. *Thus, although the second inequality of longitude is swallowed up at opposition to the sun, the second inequality of latitude is nonetheless near its maximum there, and its measure is the angle* HIG. *Therefore, just as the whole latitude* EC *causes* AC *to be longer than* AE *by the arc* EB, *similarly, the apparent part of this latitude,* FC *or* HIG, *which is a result of the second inequality, makes* AC *longer than* AF. *So it is longer than it should be.* And this error cannot be ignored, as it can be as much as 9 minutes.

| ° | ′ |
|---|---|
| 4 | 58 |
| 4 | 58 |
| 4 | 0 |
| 4 | 33 |
| 5 | 29 |
| 6 | 20 |
| 6 | 26 |
| 4 | 30 |
| 4 | 22 |
| 3 | 10 |

This error could also have been perceived in the inconstancy of the angle BAC, which they attributed to the inclination of the planes of the ecliptic and of Mars's orbit. This is clear from the result obtained if you suppose the arc AC to be increased by the amount of the addition expressed in the table, and use this and AC to compute the angle EAC. For the angles come out as in the accompanying table, from which it appears that in the northern semicircle they suppose an angle of maximum northern latitude of 4° 33′, and in the southern, of 6° 26′ south. According to this, at the line connecting the nodes, which passes through the sun or earth, the plane of the eccentric would somehow be bent, since the upper part is less inclined than the lower. Or rather, the whole path or plane of the planet's eccentric would be full of twists and turns, just as is the path described beneath the fixed stars by the observed latitudes of Mars, which is no circle.

However, all this is in conflict with the simplicity of the celestial motions, as many examples from experience will attest.

Therefore, the true procedure for referring the inclinations to the orbit is this: from the planet's position E on the ecliptic, known from

the observations, to find the angle of inclination EGF for that position, using the method which follows below. Then, since the angle E is right, from AE and EF (the measure of the angle EGF) AF is found by trigonometry, or instead of EF the constant angle EAF
61 may be used. And since, from arguments which I shall present below, it appears that for the star Mars the angle EAF is not greater than about 1° 50′, the adjustment about 45° from the node (where it is maximum) accordingly does not exceed one minute, for which the table nonetheless has one add 8 or 10 minutes in certain places. So for this reason, too, the hypothesis can be in error by as much as 7 and 9 minutes, since the observations upon which it was founded suffered some vitiation through this adjustment. I am consequently subject to much less restraint than before in seeking out a new hypothesis.

# 10

## Consideration of the observations themselves, through which Tycho Brahe hunted for the moment of opposition to the mean sun

In an enquiry of such precision, I could not have foregone a deeper inspection of the foundations themselves. And Brahe had given me the opportunity to make use of his observations. This is what I found.

Term: What are horizontal variations?

I. On 1580 November 12 at $10^h\ 50^m$,[1] they placed Mars at 8° 36′ 50″ Gemini, without mentioning the horizontal variations, by which term I wish the diurnal parallax and the refraction to be understood in what follows. Now this observation is distant and isolated. It was reduced to the moment of opposition using the diurnal motion from the *Prutenics*. For in Maestlin,[2] on the twelfth at noon, Mars is put at 8° 20′ Gemini, and on the seventeenth, again at noon, it is at 6° 25′ Gemini. Therefore, the motion over five whole days would be 1° 55′. In Stadius,[3] it is 1° 52′. Therefore, on the seventeenth at the same hour of $10^h\ 50^m$, Mars ought to have been seen at either 6° 41′ 50″ Gemini, or 6° 44′ 50″. At $9^h\ 40^m$ (which Tycho gives as the moment of opposition), it is 1′ 4″ farther forward, at either 6° 42′ 54″ or 6° 45′ 54″. They put it at 6° 46′ 10″ Gemini.

As you see, this opposition, in its minutiae, is a little more uncertain because it makes use of a diurnal motion which is not observed but imported from elsewhere, and about which the different authors differ from one another by three minutes over these five days.

1 Times are reckoned from noon on the stated day.
2 Michael Maestlin, *Ephemerides ab anno 1577 ad annum 1590*, Tubingen, 1580.
3 Joachim Stadius, *Ephemerides ab anno 1554 ad annum 1606*, Cologne, 1556–1581. Both of these references are provided by Caspar in KGW 3.

II. On 1582 December 28 at $11^h$ $30^m$, they placed Mars at 16° 47′ Cancer by observation. The moment of opposition assigned by Tycho comes 46 minutes later, during which the planet receded less than one minute. Tycho therefore puts it at 16° 46′ 16″ Cancer. On an inserted sheet here, an attempt was made to correct for a refraction of two minutes. This was, I think, a premature product of the theory of refraction then being developed. Nevertheless, he followed the observed value unchanged, thus declining to consider the planet as something which could alter its position. Nor was there any need for
62 correction, since it was in Cancer, beyond the reach of refraction, and was in mid-sky where, in Cancer, there is no longitudinal parallax.

III. On 1585 January 31 at $12^h$ $0^m$, Mars was placed at 21° 18′ 11″ Leo. The diurnal motion, by comparison of observations, was 24′ 15″. The moment of opposition followed at $19^h$ $35^m$, 7 hours and 35 minutes later. To this period belongs 7′ 41″ of diurnal motion westward. Therefore, at the designated moment, it would have been at 21° 10′ 30″ Leo, which is what was accepted. There is no mention of parallax. Nothing had to be done about refraction, because Mars was high and at mid-sky. I therefore find the word of advice in the table about refraction (properly) ignored.

IV. On 1587 March 7 at $19^h$ $10^m$ they deduced the position of Mars from the observations, which will have been 25° 10′ 20″ Virgo. This they kept in the table, but changed the time to $17^h$ $22^m$. The difference of $1^h$ $48^m$ multiplied by a diurnal motion of 24′ gives the same number of minutes and seconds (that is, 1′ 48″), no more. It therefore should have been 25° 8′ 32″ Virgo, which also approaches nearer the point opposite the sun. The difference is of practically no importance.

V. On 1589 April 15 at $12^h$ $5^m$ they established the position of Mars very carefully at 3° 58′ 21″ Scorpio, and corrected for longitudinal parallax so as to make it 3° 57′ 11″. There remained $1^h$ $30^m$ until the designated moment of opposition, which, for a diurnal motion of 22′, bring the planet back 1′ 22″, so as to be at 3° 55′ 49″. They took the value of 3° 58′ 10″. The former is closer to the sun's mean position.

VI. On 1591 June 6 at $12^h$ $20^m$, Mars is placed at 27° 15′ Sagittarius. There remained 2 days 4 hours and 5 minutes until the designated moment. In four days it was found to be moved forward 1° 12′ 47″. Therefore, to $2^d$ $4^h$ $5^m$ correspond 39′ 29″. Consequently, at that moment Mars was at 26° 35′ 31″ Sagittarius. There is no need to consider any horizontal variations in longitude, since Mars is at mid-

sky and at the beginning of Capricorn. The table has 26° 32′ Sagittarius.

VII. On 1593 August 24 at $10^h\,30^m$ they report Mars as being at 12° 38′ Pisces with an observed diurnal motion of 16′ 45″, and this near the nonagesimal[4] where there is no longitudinal parallax. The moment designated for the opposition preceded this by $8^h\,17^m$ (for it was at $2^h\,13^m$), to which corresponds a motion of 5′ 48″ eastward. Therefore, the planet falls at 12° 43′ 48″ Pisces. And the table has 12° 43′ 45″.

VIII. On 1595 October 30 at $8^h\,20^m$, they found Mars at 17° 48′ Taurus, with a diurnal motion of 22′ 54″. The designated moment preceded by $11^h\,48^m$, to which corresponds a motion of Mars of 11′ 7″ eastward, so that it would be at 17° 59′ 7″ Taurus. But it was projected eastward on account of parallax. Therefore, possibly using another observation on the meridian, they put down 17° 56′ 15″ Taurus in the table.

IX. On 1597 December 10 at $8^h\,30^m$, they first placed Mars at 3° 30′ Cancer, and again at 4° 1′ Cancer, the mean being 3° 45½′ Cancer. The moment of opposition came 3 days $5^h\,5^m$ later, to which, from Magini, corresponds 1° 15′ westward. Therefore, Mars would have been at 2° 30½′ Cancer. In the table, it was put at 2° 28′. The reason for the rough measurement, carried out with a measuring staff, is clear from the date. Tycho had left the island, leaving all instruments but
63 the staff behind. Nevertheless, he did not wish to ignore this opposition completely. But I wish he had still been there, for this opposition was marvellous opportunity, not often recurring within a man's lifetime, for finding Mars's parallax.

X. On 1600 January 13/23 at $11^h\,50^m$, the right ascension of Mars was:

| | ° | ′ | ″ |
|---|---|---|---|
| using the bright foot of Gemini | 134 | 23 | 39 |
| using Cor Leonis | 134 | 27 | 37 |
| using Pollux | 134 | 23 | 18 |
| at $12^h\,17^m$, using the third in the wing of Virgo | 134 | 29 | 48[5] |
| The mean, treating the observations impartially: | 134 | 24 | 33[6] |

[4] The 90th degree from the horizon on the ecliptic.

Hence, Mars is at 10° 38′ 46″ Leo, at an adjusted time of $11^h\,40^m$ reduced to the meridian of Uraniborg. But on January 24/February 3 at the same time it was at 6° 18′ Leo. This gives a diurnal motion of 23′ 44″, and a position on January 19/29 at $9^h\,40^m$ of 8° 18′ 45″ Leo, just as they put it.

I have presented these discrepant values for the right ascension in order to show that even in the observations themselves there is an uncertainty of several minutes unless extreme care be exercised and all possible aids used. At that time the instruments (except the largest) had arrived in Bohemia, but they were still not well enough positioned, and were affected by the journey besides. However, even in observations at the island it often happened that right ascensions measured from two different stars differed by three minutes. When I asked Christian [Longomontanus], on this subject, whether I should consider this an effect of the limitations of observation or vision, he replied, 'This is not unusual.'

Finally, the reader should be advised that Tycho, in his table, claimed to have made use of the solar parallax in correcting the positions of Mars. But it will now shortly be made clear that the parallax of Mars is a slippery and imperceptible business. However, this does not much affect the certainty of the tables, as Mars can almost always be observed in mid-sky where it has no longitudinal parallax.

[5] Gamma Geminorum, Regulus, Pollux, and Beta Virginis, respectively.

[6] This should be 134° 26′ 6″. Kepler took the figures, including the average, straight from Tycho: they appear in TBOO 13 p. 221. Apparently, Tycho averaged in pairs, and then instead of averaging the averages, he averaged the first pair's average with the third figure alone (surely unintentionally).

# 11

## On the diurnal parallax of the star Mars

The beginning of my new elaboration and rendition of the motions was the point at which I have just stopped. For it is clear from Part I that the positions of Mars should be taken at the moments of true opposition to the sun. However, it is also clear that not every trace of the second inequality is thus removed, it being also necessary to refer the arcs measured on the ecliptic to the planet's orbit. But the planet's orbit must first be investigated by finding the nodes and the inclina-
64 tion of the planes. Again, the inclination and the nodes cannot be found without knowing the diurnal parallax, at least if this should turn out to be relatively large. One must therefore start with the parallax. I shall present two ways to find it.

The first way (and the one more familiar to others) will be examined using the Brahean observations.

In 1582, when Mars was opposite the sun in Cancer, I found incredible care in observing, with Tycho's manuscript title, 'For investigating the parallax of Mars', from which you will, however, deduce either no parallax at all, or one exceedingly small. It goes without saying that (as is customary) they compared the star Mars with nearby stars on the ecliptic, and frequently with ones at a great distance. Now it is usual to find the parallax of a mobile star (for Mars moves, with a retrograde motion when opposite the sun) by comparing morning and evening observations. It has thus happened that almost all the stars from which Mars's distance was observed in the morning are different from those by which it was observed in the evening. For a fixed star which is at hand in the morning (and higher than Mars), if it be near the ecliptic, has either set by evening (when

Mars is in the west) or is rendered useless by refraction for this delicate procedure. Another star therefore had to be substituted. But if the fixed stars are substituted for one another, there is always less trust in the procedure than if the same star had been retained.

Brahe, however, announced to the learned world in many places that from the observations of that year, the parallax of Mars was found to be considerably greater than that of the sun. I therefore very carefully scrutinized the whole book in order more deeply to examine his operation or computation. I did indeed find a chapter which professed to offer a procedure for investigating the parallax of Mars and of its year using observations. But here was something really surprising: they fitted the position of Mars found by observation into a Copernican diagram drawn very laboriously and carefully. In this diagram, they took up the immense labour of solving all triangles created by the double epicycle on the concentric, in numbers with a great many digits. Then, at last, they came to the end of the calculation, which was to issue a pronouncement that the parallax of Mars is indeed greater than the solar parallax. Brahe had thus asked one thing, but his assistants in calculation carried out something else. He wanted them to find out the parallax of Mars by comparing morning and evening observations with one another, but they had in fact found out how much parallax the Copernican diagram would bring about. Whether Brahe's pronouncement on parallax was founded solely upon his trust in his assistants, is unknown to me.[1]

As for us, we shall consult the observations themselves, insofar as they pertain to our undertaking.

In 1582 on the night between 23 and 24 November, the distances from the fixed stars remained the same at different times. This, then, was a station point.

On the following two days, the motion was 11′ and 15′.

On the night of 26 December, it passed between the second and seventh stars of Gemini[2], its distance (measured with the staff) from the head of the lower of the Twins (the second star) being 2° 25′ or 2° 26′, but from the seventh, 1° 6′ or 1° 7′, making the latitude, about 4°
65 9′. Then, at $8^h\ 28^m$, it was 44° 41′ from the eye of Taurus[3], whose

[1] Kepler's interpretation cannot be correct, as the computation in question exists in Tycho's own hand as well as in the copy made by assistants. Tycho apparently made no direct attempt to measure Mars's parallax, but concluded that Mars must be nearer than the sun at opposition by measuring its diurnal motion at opposition. See Owen Gingerich, 'Dreyer and Tycho's World System', *Sky and Telescope*, August 1982, pp. 138–140.

[2] Pollux and Kappa Geminorum, respectively.

[3] Aldebaran.

latitude is 5° 31′ south, longitude 4° 12½′ Gemini, in 1600. Hence, Mars's longitude as if the year were 1600 is 17° 53⅓′ Cancer[4], or, at the end of 1582, 17° 38′ Cancer, at an altitude of 40° 50′. It is thus beyond the effects of refraction.

Again, at $7^h\ 15^m$ on the morning of December 27, it was 36° 43′ from Cor Leonis, whose latitude is 0° 26½′; hence, its longitude at the end of 1582 is 17° 28⅓′ Cancer, altitude 14° 4′, and thus affected by refraction. Therefore, from $8^h\ 28\frac{1}{2}^m$ in the evening to $19^h\ 15^m$, an interval of $10^h\ 46\frac{1}{2}^m$, it was observed to retrogress 9⅔′.

For the diurnal motion, on the 29th at $7^h\ 47^m$, the distance of Mars from the southern foot of Erichthonius[5] was 29° 38½′. But on the 30th at $8^h\ 8^m$ the distance from the same star was 29° 13½′. Therefore, over $24^h\ 21^m$ it moved 25′. And this diurnal motion remained the same on the 27th. Therefore, to $10^h\ 46\frac{1}{2}^m$ there should have corresponded 11½′ of arc, but we found only 9⅔′. Let us consider this.

On the previous evening, when Mars was rising farther to the east (because it was retrograde), parallax moved it eastward, and in the morning, when it was setting and was farther to the west, parallax moved it westward. So, just as the moon's diurnal parallax apparently retards its motion, the same parallax in turn accelerates Mars's retrograde motion. Therefore, if parallax is perceived, it is perceived through an excessively large diurnal motion. But this motion is diminished. Therefore, there is no parallax. Again, refraction is perceived as contrary to parallax. Now the refraction at altitude 13° is 4′, from the table of fixed stars, and 8′, from the table of the sun[6], and only a very small part of this affects the longitude, as Cancer was descending quite obliquely. So the refraction in longitude comes to

[4] Since Mars's latitude is 4° 9′ north, and Aldebaran's is 5° 31′ south (both from Tycho's tables), their latitudinal difference is 9° 40′. If λ is the difference in longitude,

$$\cos \lambda = (\cos 44° 41')/(\cos 9° 40');\ \lambda = 43° 50\tfrac{1}{2}'.$$

Mars's longitude is the sum of Aldebaran's longitude and the difference in longitude just found:

| | | |
|---|---|---|
| 4° | 12½′ | Gemini |
| +43° | 50½′ | |
| 18° | 3′ | Cancer. |

This differs from Kepler's figure, 17° 53⅓′, by about 10′, suggesting that he erred in reading his tables. The error is important, since it increases the total retrograde motion to 19′, resulting in a total parallax of eight minutes, not including the 3′ correction for refraction.

[5] Beta Tauri, which also serves as the foot of Erichthonius (Auriga, also called Heniochus).

[6] Tycho believed the sun, moon and fixed stars each to have a different angle of refraction (see TBOO II pp. 64, 136, and 287). Caspar (in KGW 3 p. 462) states plausibly that the large solar refraction is a result of Tycho's acceptance of a large solar parallax, since their effects are in opposite directions.

three minutes at most, which, added to $9\frac{2}{3}'$, makes the refraction-free motion over $10\frac{3}{4}$ hours equal to $12\frac{2}{3}'$, which, if it also were free from parallax, would have been $11\frac{1}{2}'$. Therefore, the excess of $1\frac{1}{3}'$ is the longitudinal parallax for the two observations, clearly minimal, untrustworthy, and entirely negligible.

On 1583 January 16 at $7^h\,30^m$ in the evening, Mars was 23° 29′ from the bright star in the foot of Erichthonius at an altitude of 51°. The next morning at $5^h\,0^m$, it was 43° 58′ from Cor Leonis at an altitude of 15°. And, measured by the straight edge, Mars was perfectly collinear with the two stars. And so, since Mars's motion is carried out along this line, Brahe made a note that, given the diurnal motion of Mars, these observations could give the longitudinal parallax. This is obtained here as follows. On January 16 at $10\frac{1}{2}^h$ it was 23° 27′ from the bright star in the foot of Erichthonius. On January 17 at $10\frac{3}{5}^h$ it was 23° $12\frac{1}{2}'$ from the same star. Therefore, the diurnal motion would be $14\frac{1}{2}'$. Now in order to comply with Brahe's advice, we must set out the distance between the foot of Erichthonius and Cor Leonis, which is found to be 67° 21′. Subtracting Mars's distance from the bright star in the foot of Erichthonius, 23° 29′, leaves Mars's distance from Cor Leonis, 43° 52′, at $7\frac{1}{2}^h$ in the evening, 6 minutes less than the position at $5^h$ in the morning, 43° 58′. The time interval is $9\frac{1}{2}$ hours, to which
66 corresponds $5\frac{5}{8}'$ of the diurnal motion. Here, therefore, the sum of the two parallaxes would be no more than $\frac{3}{8}'$, except that the amount of Mars's longitudinal refraction at 15° is added to it. But this is quite small. For Cancer and Leo are setting extremely obliquely, and Mars's large northern latitude put it at nearly the same altitude as Cor Leonis.

On January 17 at $5^h\,20^m$ in the evening Mars was 23° 16′ from the foot of Erichthonius. On the following day, the 18th, at $3^h\,0^m$ in the morning, this distance was 23° 9′, and at $5^h\,5^m$ in the evening it was 23° $1\frac{1}{2}'$. So the motion over $23^h\,45^m$ is $14\frac{1}{2}'$, but over $9^h\,40^m$ it is 7′. This should have been 6′. We are left with a longitudinal parallax of no more than 1′. Refraction does not affect anything, since in both instances the altitude of Mars was about 30°.

Likewise, at $7^h\,34^m$ its distance from the seventh star of Gemini[7] was 7° 51′. At $4^h\,52^m$ in the morning, it was 7° 59′ from the same star. Therefore, over $9^h\,18^m$ it moved 8′. We thus have one minute more than before. Of this star (at the shoulder of Gemini), Brahe wrote,

[7] Kappa Geminorum.

'Note that I am taking Mars's distance from this star because its course passes through it, as it were, so that the morning and evening distances compared might show the parallax.' I have reproduced this here so that the reader may rest assured that Brahe did not proceed without a purpose.

On January 18 at $8^h\ 52^m$ in the evening there was 44° 22′ between Mars and Cor Leonis. At $4\frac{3}{4}^h$ in the morning the same distance was 44° $27\frac{1}{3}'$. Therefore, the motion over $7^h\ 53^m$ was $5\frac{1}{3}'$. On January 19 following, at $7^h\ 3^m$, this distance was 44° $32\frac{1}{2}'$. Therefore, for $22^h\ 11^m$ the motion is $10\frac{1}{2}'$. So for 8 hours there would be less than 4′ of motion. Our profit is about $1\frac{1}{2}'$ of parallax.

But now let us calculate, for January 17, how much of an increase in the hourly motion should result from a parallax greater than the solar parallax as usually accepted. Since we consider the sun's parallax to be three minutes, let Mars have four.

| 1583 January 17 | $5^h\ 20^m$ | | $15^h\ 0^m$ | |
|---|---|---|---|---|
| | ° | ′ | ° | ′ |
| Position of the sun . . | 7 | 22 Aquarius | 7 | 31 Aquarius |
| Its right ascension . . . | 309 | 47 | 309 | 56 |
| Add the hour angle . . | 79 | 0 | 225 | 0 |
| Right ascension of meridian | 28 | 47 | 174 | 56 |
| Degree of the meridian . | 0 | 56 Taurus | 24 | 29 Virgo |
| Declination . . . . . . | 11 | 50 | 2 | 12 |
| Oblique ascension risen . | 118 | 47 | 264 | 56 |
| Degree rising . . . . . | 19 | 41 Leo | 26 | 0 Scorpio |
| Ninetieth degree from rising | 19 | 41 Taurus | 26 | 0 Leo |
| Between deg. of mer. and 90th . . . . . . | 18 | 45 | 28 | 29 |
| Between deg. of mer. and zenith . . . . . | 44 | 5 | 53 | 43; and hence, |
| Between zenith and ninetieth . . . . . . | 40 | 40 | 47 | 41; that is, |
| Altitude of the ninetieth | 49 | 20 | 42 | 19 |
| Corresponding long. horiz. parall. . . . . . . . | | 2′ 36 sec | | 2′ 58 sec. |

| 1583 January 17 | $5^h\ 20^m$ | $15^h\ 0^m$ |
|---|---|---|
| | ° ′ | ° ′ |
| And because Mars is about therefore, | 10 0 Cancer | 10 0 Cancer. |
| Between Mars and the ninetieth . . . . . | 50 19 | 46 0 |
| Corresponding parallax in longitude | 2 0″ east | 2 8″ west |

67 It follows that the motion of Mars over those hours should have appeared 4′ greater than what follows proportionally from the diurnal motion. Since this is repudiated by the observations, Mars's parallax is not so large.

There exist similar observations from 1585, 1595, and others, from which an exceedingly small parallax, often none at all, is found. There even was an occasional note in Brahe's hand saying, 'It strayed over to the wrong side.' So this is the first way of investigating Mars's parallax.

I shall now add the other way, because of its beauty; I cannot use the Brahean observations in it. Therefore, I am going to give you a clown show, in that I use my own observations, showing by example why Brahe needed such diligence, precision of instruments, assistants, and other equipment.

I have two instruments, which I use through the generosity of the Hon. Mr Friedrich Hoffmann, L. B.: an iron sextant and a brass azimuthal quadrant. The latter is two and a half feet in diameter and the former three and a half, and both are calibrated in one-minute divisions.

Now at this very time, 1604, at which I am considering parallax (whether that of the sun more than Mars's is hard to say, for my *Hipparchus*[8] requires Mars's aid even in the lunar eclipses), a very suitable occasion for observation has arisen, if the latitude or climate had been different and Mars had moved a little higher. For on 19/29

[8] A projected work on the sizes and distances of the heavenly bodies, which Kepler never published although his work on its composition extended over many years.

February of this year 1604, Mars was stationary both in longitude and in latitude at the same time. This occurred in Libra, and therefore from Mars's rising to the sun's rising the angle of the horizon with the ecliptic continually decreased. Consequently, according to Ch. 9 of the *Astronomiae Optica*[9] the latitudinal parallax, if any, continually increases. But from the increment, found through the columns of the parallactic table[10] opposite the initial and final angles of the ecliptic with the horizon, the whole horizontal parallax, at the front of the column, is known.

**There follows the series of my observations**

On the night between Thursday and Friday, which was February 17/27, while Corvus was on the meridian, there was 9° 44′ between Mars and Spica, and between it and the Northern pan[11], 17° 41′; and between Mars and Arcturus, 29° 13′. Also, to test the sextant, we measured the interval between Arcturus and Spica as 32° 57′, which should have been 33° 1′ 45″, as is clear if you calculate it using the right ascensions and declinations, or latitudes and longitudes, which Tycho assigned to these stars in Book I of the *Progymnasmata*. Therefore, my distances are smaller than the true distances by $4\frac{3}{4}$ minutes. I applied this correction to the distances of Mars from the fixed stars, so as to make it 9° 48′ 45″ from Spica, 17° 45′ 45″ from the Pan, and 29° 17′ 43″ [sic] from Arcturus.

I then used the quadrant to obtain the meridian altitude of Mars, 32° 4′, and of Spica, 30° 50′. Since the latter has a declination of 9° 2′, Mars is left with a declination of 7° 48′. However, the altitude of Spica showed that my perpendicular was not well enough set up. For the
68 altitude of the equator, at my location, is 39° 54′. Accordingly, the meridian altitude of Spica is 30° 52′, and of Mars, 32° 6′. Now, from the declination of Mars and its distance from the fixed star, its right ascension comes out:

[9] KGW 2 p. 274.
[10] KGW 2, following p. 240. For the use of this Table, see above, pp. 12–15.
[11] Beta Librae.

| | ° | ′ | ″ |
|---|---|---|---|
| From Spica | 305 | 57 | 36 |
| From the Pan | 306 | 3 | 17 |
| Difference | 0 | 5 | 41 |
| Resultant mean | 306 | 0 | 26[12] |

From the right ascension and declination of a star, to find its longitude and latitude without calculation, with the help of tables.

Term: What is the base of the latitude?

I am not certain whether, when (as happened a few times) the clamps holding the arm loosened and it (being a heavy piece of iron) fell precipitously and hit hard, the sights might not have changed their position, since they are removable and subject to dislocation. But from this right ascension, I first selected the degree of the right sphere that was rising at the same time, 28° 1′ 0″ Libra, from Tycho's table of right ascensions[13]. Its declination, from another of that author's tables[14], is 10° 48′ 30″, and Mars's is 7° 48′. Therefore, it is distant from the ecliptic, the oblique path, by 3° 0′ 30″ on the circle of declination. But the angle which the circle of declination makes with the ecliptic is 68° 59′ from the appropriate table. Its complement is 21° 1′. And in my table of parallax[15], under the heading of 60′, I find, opposite 68° 59′, the entry 56′ 1″. Under 30″, however, I find 28″. But since I have thrice 60′ in this distance of Mars from the ecliptic (which I call the base of the latitude), I multiply what I extracted under 60′ by 3[16]. This gives me a latitude of 2° 48′ 31″. The same operation

[12] In addition to the right ascensions' all being 100° too large, there is a more serious discrepancy in the computation. Spica's right ascension, from Tycho's tables (TBOO 3 p. 376) is 196° 4′ in 1600, or 196° 7′ 10″ in 1604, corrected for precession. The difference between this and the right ascension of Mars given by Kepler as computed from Spica's position is 9° 50′ 26″. Thus the difference in right ascensions is greater than the linear distance between Mars and Spica (9° 48′ 45″), as if one leg of a right triangle could be greater than the hypotenuse.

Recomputation yields the following results:

| | Corrected Right Asc. | Dec. | Distance from Mars | Difference in Right Asc. | Kepler's Value |
|---|---|---|---|---|---|
| Spica | 196° 7′ 10″ | 9° 2′ S | 9° 44′ 12″ | 205° 51′ 22″ | 205° 57′ 36″ |
| Pan | 223° 57′ 46″ | 7° 51′ S | 17° 45′ 45″ | 206° 12′ 1″ | 206° 3′ 17″ |

Clearly, the distances from Mars given by Kepler are still too small, despite Kepler's $4\frac{3}{4}'$ correction. What remains a mystery is how Kepler arrived at his figures for Mars's right ascension.

[13] TBOO 3 p. 74.

[14] TBOO 3 p. 71.

[15] KGW 2 following p. 240.

[16] Relying on the approximation $\sin 3x = 3 \sin x$, for small angles.

opposite 21° 1′ shows me what has to be subtracted from the degree rising at the same time, namely, 1° 5′ 4″. Accordingly, Mars's position will be 26° 56′ Libra.[17] I come within a minute of this using a computation whose fundamentals I am going to be presenting in this work.

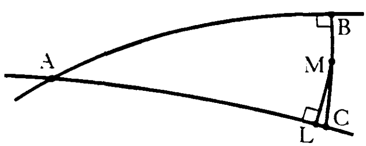
To test the latitude of Mars, I consulted the distance from Arcturus, in comparison with that star's latitude and longitude from Tycho, and Mars's longitudinal position just found, and its reply to me was that Mars was at a latitude of 2° 47′ 48″. Before, it was at 2° 48′ 31″.

On February 19/29, we had moved the sights, and began to observe Mars rising. Its distances from Arcturus were noted, and were these:

| ° | ′ |
|---|---|
| 29 | 22½ |
| | 24 |
| | 20 |
| | 22 |

I think we are ten minutes too high. For the wind was blowing so hard that it was only by a glowing coal that we could cast light upon the scale so as to read it. And the altitude of Mars was then 11°. Later, the back of Leo[18] culminated at altitude 62° 37′, according to a

[17] The preceding computation, more thoroughly delineated, is as follows.




Let AB be the equator, AC the ecliptic, A the autumnal equinox, and M the position of Mars. First, from observation, the arc AB, representing Mars's right ascension, is found. BM, Mars's delination, is likewise given through observation, and BC is obtained from Tycho's table of declinations of the ecliptic. Thus, the difference, MC, is given. The angle BCA may be calculated or found in a table, and if ML be drawn at right angles to AC, another side and angle of triangle MLC may be computed using the law of sines:

$$(\sin MC)/(\sin L) = (\sin LM)/(\sin C);$$

or, since L is a right angle,

$$\sin LM = \sin MC \sin C.$$

But the table of parallax in the *Astronomiae pars optica* is so constructed as to perform this computation; and thus the latitude, LM, is obtained. To find the longitude AL, one obtains AC from Tycho's table of right ascensions, and subtracts CL. CL is found by using the approximation that angles C and M are very nearly complementary, as triangle MCL is very nearly a plane triangle. So:

$$\sin CL = \sin MC \sin (90^\circ - C),$$

and the table of parallax is used as before. (For the use of that table, see the translator's introduction, pp. 12–15 above.)

[18] Delta Leonis.

corrected plumb line. Thus the altitude of the equator was shown to be 39° 55′, almost correct. At that moment, Mars's altitude was 23°. We next reinvestigated the former distance, which came out to be:

| ° | ′ | | |
|---|---|---|---|
| 29 | 14 | Therefore | 12½ |
| | 19 | without doubt | 14 |
| | 13 | the previous | 10 |
| | 18 | distance was | 12. |

69 For first, when Mars was near the horizon, refraction moved it toward Arcturus, later letting it drop down when Mars had acquired some altitude. It was, however, the cold and the extremely biting winds that occasioned so much variety in observations made at the same time. For it was impossible to handle the iron and close the clamps with bare hands, and with gloves the arm was not securely enough clamped to allow a very precise reading. Vindemiatrix[19] showed a meridian altitude of 53° 5′, a little greater than it should have had. But Spica's 30° 54′ was correct within one minute. The altitude of Mars at culmination was 32° 6′, the same as it had been two days before, and of Arcturus, a correct 61° 13′. Hence, by computation, it is concluded that the distance of Mars and Arcturus was 29° 18⅓′. Now since, according to the *Prutenic Tables* and to my computations, Mars was stationary in longitude at this time, there could be no change in meridian altitude resulting from its wandering about on the ecliptic. For this reason, since the meridian altitude remained quite the same (for my instrument allows an uncertainty of one minute), no change occurred in the latitude, either, during this time.

On February 22 or March 3 we tested the sextant, just as we had done above, and found 26° 2′ between Canis Minor and Orion's higher shoulder[20], which is shown by calculation to be 26° 2′ 15″. So also, between Canis Minor and Palilicium[21] was found 46° 22½′, which Tycho, in his letters, indicates is 46° 22′. Therefore, when the fifth star of Leo[22] culminated, it was found, with the arm of the instrument fixed at 29° 17′, that the distance of Mars and Arcturus was less, but with it fixed at 29° 13½′ the distance was now more, while no error could be found at 29° 15′. Then the whole sky unexpectedly clouded over. It became clear again on the morning of March 4, and

19 Epsilon Virginis.
20 Procyon and Betelgeuse, respectively.
21 Aldebaran.
22 Zeta Leonis.

now, when Antares culminated, with the arm fixed at 29° 19′, each star lined up evenly [with the sights]. However, it seemed that something needed to be added, but at 29° 20′ too much had been added. When the observation was finished, Saturn preceded the meridian by less than Jupiter preceded Saturn.

On the night following February 29 or March 10, the instrument having meanwhile been displaced, this distance was, first, between 29° 9′ and 29° 10′, half an hour before the culmination of the Heart of Hydra[23]. Later, it appeared to the investigators to be between 29° 12′ and 29° 13′, as it was now higher and free from refraction. For at the end of this observation it had an altitude of $19\frac{1}{6}'$. But a little afterward – I do not know whether the sights were disturbed – it would not allow that much, for it appeared to be 29° $9\frac{1}{2}'$. The Tail of Leo[24] was about half a degree from the meridian. The altitude of Mars was then $24\frac{3}{4}°$. The Tail of Leo, when culminating, had the correct altitude of 56° 44′, within one minute. When one third of the distance between Mars and Spica had crossed the meridian, it first appeared to us to be 29° $9\frac{1}{2}'$, but the cylinder was not well enough applied, as it was too long. Thus, a little later, this could not be accepted, but 29° $10\frac{1}{4}$ or a little less seemed to be required. And Mars was seen on both sides of the cylinder.

Then, between Mars and Spica, there was 9° 26′, and less than 9° 27′.

Mars culminated at an altitude of 30° $19\frac{1}{2}'$[25].

There was then 18° 25′ between Mars and the Northern Pan [of Libra].

70 For the investigation of the sextant, the distance between Spica and the Pan was taken as 27° 39′, although it should have been 27° 34′. Also, the distance between Spica and the northern star in the head of Scorpius[26] was 39° $32\frac{1}{2}'$, which should have been 39° $26\frac{1}{2}'$. So the sextant reads five minutes high. Furthermore, calculation of Mars's position also provides evidence of this. For unless you diminish Mars's distance from the fixed stars by five minutes, the right ascensions measured from Spica and the Pan will be discrepant by 10 minutes, but if (as required by the examination of the sextant) the five minutes be subtracted, they coincide exactly, and will be 205° 27′

[23] Alpha Hydrae.
[24] Beta Leonis.
[25] This figure appears to be two degrees too small.
[26] Beta Scorpii.

10″[27], with declination 7° 35½′. Therefore, the position is 26° 18′ 48″ Libra, with latitude 2° 47′ 20″[28]. You see that the latitude is manifestly the same, though meanwhile the planet had retrogressed 38′ in longitude. If, from the position of Mars found thus, you find its distance from Arcturus, it comes out to be 29° 9⅙′, while using the maladjusted instrument it was 29° 14′.

Now, with the Heart of Scorpius[29] culminating, our distance was 29° 13½′, the instrument in the meantime having been displaced and later repositioned. We next tested the sextant again, which showed 44° 45′ between Polaris and the Tail of Cygnus[30]. This should have been 44° 39½′. Therefore, the instrument was in its pristine state. But when Saturn had passed the meridian by one degree, the distance could not admit 29° 13½′; nevertheless, it was greater than 29° 12½′, about 29° 13′.

So this is the series of observations. I would be crazy if I tried to use them in an undertaking requiring great precision. Therefore, I am presenting an example to another more diligent and successful observer, rather than an argument. I also hope that the nausea evoked by these uncertain observations will lead readers to cling all the more fervently to the extremely certain Tychonic ones. Now, on with the example.

*The first and second days, in their concurrence, only serve to show the station in latitudinal motion. In both, Mars was 29° 18′ from Arcturus, and in both, its meridian altitude was 32° 7′ or 6′. Those days were busying me in preparation for meeting the following days properly, should the instruments be needed.*

*But on March 3, with the Mouth of Leo[31] culminating, the distance*

[27] Computation does not bear out Kepler's contention. For Spica and the Northern Pan, the data (corrected for precession) are as follows:

| | Right Asc. | Declination |
|---|---|---|
| Spica | 196° 7′ 10″ | 9° 2′ 20″ S. |
| Pan | 223 57 45 | 7 51 S., |

from TBOO 3 p. 376. The right ascensions differ by 27° 50′ 35″. If the five minute correction be applied to each of the distances given by Kepler, their sum is 27° 41½′, ten minutes less than the difference of right ascensions. Further, since these are actual distances and not distances in the right sphere, they are greater than the corresponding distances in the right sphere (that is, the components of the distances parallel to the equator). The amount here turns out to be about five minutes. Therefore, *with the corrections applied*, the right ascensions measured from Spica and the Pan will be discrepant by fifteen minutes, while without corrections the discrepancy will be but five minutes. This last discrepancy disappears if the measured distances are *increased* by two and a half minutes.

[28] If the corrections suggested in the previous note are applied, the position would be 26° 28¼′ Libra, latitude 2° 51¾′ N.

[29] Antares.

[30] Deneb.

[31] Lambda Leonis.

*was 29° 15', and with the Heart of Scorpius*[32] *culminating, 29° 19' plus. Therefore the distance changed about $4\frac{1}{4}$' over the interval. And since Arcturus and Mars have very nearly the same longitude, this change of distance bespeaks a variation of latitudinal parallax. I am not unaware that 29° 19' is hardly different from 29° 18', and that by analogy with the previous day the latter ought to be the distance at about the same time if Mars is standing still. I also know that when the Mouth of Leo is on the meridian Mars is $12\frac{1}{2}$° high and is somewhat affected by refraction. But of these we shall speak afterwards. For now, let us entirely ignore them, so as not to complicate our example. Now the altitude of the nonagesimal was $57\frac{1}{3}$° (about) when the Mouth of Leo was culminating, but later, when the Heart of Scorpius was culminating, it was $20\frac{1}{3}$°. I will therefore look through the parallactic table to find the column in which, from a distance from the zenith of $32\frac{2}{3}$° to a distance of $69\frac{2}{3}$°, the entry in the table would change $4\frac{1}{4}$'. I find that this happens in the column whose heading is 9'*[33]*. Therefore, the maximum parallax of Mars would be 9'. And since, on that day, the distance of Mars and earth was to the distance of Mars and the sun as 28 is to 60 (given approximately by an anticipatory acquaintance with the hypotheses of Tycho and Copernicus), the ratio of the parallaxes will be the inverse of this, and the maximum solar parallax will be 4' 24". It is supposed to be 3' 0".*

71 *But now let us consider that at an altitude of $12\frac{1}{2}$° Mars would be subject to refraction, if the table of fixed star refractions constructed at Hven is valid for Prague. At this altitude, it was 4' 20", 2' 18" of which is attributed to the latitude, by which Mars is caused to be closer to Arcturus. If, however, we apply the sun's refraction to Mars (as it often appears we should), it would be 8' 45" at this altitude, twice as great. Therefore, the latitudinal parallax would also be twice as great, 4' 36". In this way, the whole difference which observation imposes upon itself, at these two different moments, would be due entirely to refraction. Reckoning it in the former way would leave a latitudinal parallax of 2', the difference in parallax in the column whose heading is 5'. This would give the sun only 2' 25" of maximum parallax. So refraction makes our third day, too, suspect, and ultimately quite worthless. I know that since Arcturus and Mars are 9 degrees apart*[34]*, which is one third of the amount by which Arcturus's latitude exceeds*

[32] Antares.

[33] An error. This difference in this column of the table is 3' 33". The difference under 11' is 4' 20", and under 12' is 4' 45". Hence, the correct value would be about $11\frac{1}{2}$'.

[34] In longitude.

*Mars's, it comes about that not all the latitudinal refraction is subtracted from the distances from Mars, and that the parallax changes Mars's latitude more than it changes this distance from Arcturus. But since this is very small, I have, with greater fear, considered it as something to be left buried. Let him observe it who is able to measure more precisely.*

*Now on the fourth day, what seems to have been accomplished is nothing other than the total destruction of Mars's parallax. The meridian distance ought to have been 29° 9½′ with an accurate instrument, and consequently 29° 14′ with the faulty one. But at the end it was found to be 29° 13½′, when the latitudinal parallax (had there been any) ought to have been greater, and hence the distance from Arcturus greater. And after Mars attained an altitude of 19°, the distance was found to be 29° 12½′, and one minute greater at the end. This would be a very small parallax. And what is this ratio? When its altitude was 9 degrees (when Hydra was culminating), the distance was 29° 9′ with a faulty instrument, and still subject to refraction. Later, at an altitude of 25° and near the meridian, it was again 29° 9′, measured twice at different moments. Could the refraction have been zero initially, that the arc thus remain constant? Or should it rather be said that I (though to myself I might have seemed most diligent) erred in observing? This would chiefly be due to the length of the cylinder.*

Still, it is at least established by these observations, whatever their quality, that Mars's parallax is not greater than 4′, which is the amount of uncertainty in the instrument. It is more credible that the parallax is very small. In chapter 64 below, you will find further discussion of this point.

That the parallax of Mars is greater than the sun's, on the other hand, the proportion in the Tychonic and Copernican hypotheses argues, from which Mars's parallax could easily be computed if we were certain of the sun's parallax. Is, then, the procedure for finding the sun's altitude and parallax from eclipses uncertain? It is indeed relatively uncertain quantitatively, but as concerns the thing itself it is perfectly certain. The sun is not nearer than 230 semidiameters of the earth, but it is not an infinite number of semidiameters away. But between 700 and 2000 semidiameters (of which the first quantity is from my *Mysterium cosmographicum*, while the other two are given as the upper and lower limits in observations of eclipses) it does not appear that any particular number has yet been demonstrated, as I shall prove in my *Hipparchus*.

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# 12

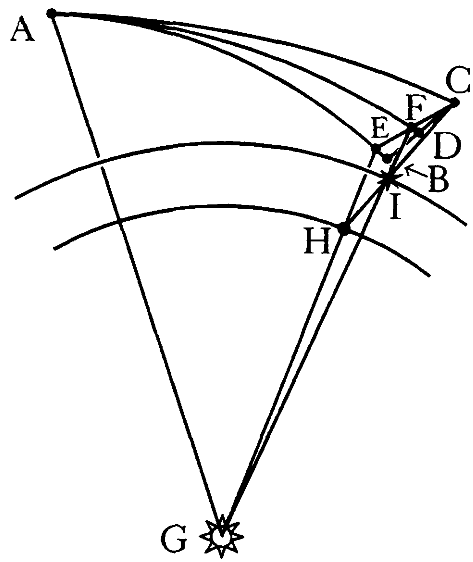
## Investigation of Mars's nodes

Means are not wanting of investigating the planets' first inequality through observations, even when these are entangled in the second inequality. Nevertheless, in this second part, I prefer to follow the footsteps of the authorities and make use of acronychal observations, in order to establish my credibility. For I want to be sure that later, when I bring forth something contrary to accepted opinion, no one can complain that the briar-path of his own method was unexplored.

Furthermore, it is now clear that nothing of any importance is wanting in Mars's diurnal parallax as taken by Tycho. I shall therefore move on, little by little, towards the reduction of the observed positions of Mars to the point opposite the sun's apparent position.

As a first step, we must find the nodes. Tycho Brahe used to investigate them thus:

*In the diagram of chapter 9, let* A *be the position of the node,* E *the planet's position on the ecliptic in 1595,* C *the observed sidereal position of the planet, 17° 56′ 5″ Taurus,* EC *the observed latitude, 0° 5′ 15″ north. Now it is presupposed that the angle* EAC *is about 4° 34½′, the same as the maximum northern latitude observed in the same way in 1585. In the right triangle* CEA *(or isosceles triangle* CBA*; the difference is unimportant in this procedure), from side* CE *and angle* EAC *he sought the length* EA*, the distance of the ecliptic position from the node.* There is nothing wrong with this operation, since EC is small and near the node. However, the need for accuracy in this demonstration commends another. *For it was said in chapter 9 that the angle* EAC *is not constant, whence through their different*




*latitudes, the different oppositions will also show different positions for the node. Nor is* EAC *as great as the maximum apparent latitude, because* AC *is a curved arc, and besides, the planet's path as it would be seen from the centre of the sun is not* AC, *but some path inside (*AF, *say). Therefore,* A *will not necessarily be the node, at least as found through this operation.*

I, therefore, investigated the nodes differently, using observations on the day on which they were at the node. Even though this method depends upon some preconceptions, and the subject is treated more accurately below in part 5, it is previewed here, if only to give confirmation.

My presupposition is that when the planet, in its eccentric motion, is truly at the node, it can by no disposition of the sun or earth be made to appear elsewhere than at the node. For in the Copernican hypothesis, this is *per se* in agreement with the nature of things: that the moving faculty of any star is not bound to observe a star foreign to it (including the earth), but has its own laws governing its circuit. In the Ptolemaic hypothesis, this would be exactly as if one were to say that the epicycle moves, not with respect to a line from the sun
73 through its own centre, but with respect to certain positions beneath the fixed stars, beneath which the planet lies in the plane of the

ecliptic. In the Tychonic system, the same will be said of the eccentric.

Further, what I presupposed, I have found to be true using the following observations:

I. On 1590 March 4 at $7^h$ $10^m$ in the evening, Mars's declination was 9° 26′ N, and the right ascension was 22° 35′ 10″. Hence, its position was 24° 22′ 56″ Aries, and its latitude was 3′ 12″ south. Parallax and refraction were opposite and approximately equal, and are therefore ignored.

II. On 1592 January 23 at $10^h$ $15^m$ in the evening, Mars was at 11° 34′ 30″ Aries with latitude 0° 2′ south. The altitude of Mars was 25°, and therefore (from the table for the fixed stars)[1], there was no refraction. The parallax was about as much as the sun's, because Mars and the sun were sextile[2], and were therefore about equally distant from earth. Nearly all of this was latitudinal. Therefore, about two minutes of latitude must be added northward to Mars in order to free it from parallax, and it thus falls upon the ecliptic. For on February 6 it was already at about 7′ northern latitude.

III. On the evening of 1593 December 10, Mars was observed at the ascending node. For after correction of the horizontal variations it retained no more than 0° 0′ 45″ north latitude.

IV. On 1595 October 27 at $12^h$ $20^m$, Mars's true latitude after the removal of parallax was 0° 2′ 20″ south. On the 28th, when the parallax had similarly been removed, the latitude was 0° 0′ 25″ north. Therefore,* in the meanwhile, it was at the ascending node.

*This approximate argument is sufficient for the present enterprise. In ch. 61 and 67 below, with everything considered more carefully, it is found to have been at the node at $15^h$ on the 29th.

Counting backwards 687 days, the number of days in Mars's revolution on its eccentric, starting from noon on 28 October, one ends on 1593 December 10, and on the preceding night Mars was observed near the node. Count back another 687. This brings you to 1592 January 23, when the planet was observed right at the node. If you do the same a third time, you come out at 1590 March 7. And on the fourth day preceding, Mars had some southern latitude, which it would make up over the four days, so as to fall at the node about the seventh.

From this it is known that it makes no difference where the earth is, either in sidereal position or with respect to Mars. In the Ptolemaic

[1] Tycho's table appears in *Stellarum inerrantium restitutio*, folium 28 v, in TBOO 3 p. 377. Brahe believed that the light from the fixed stars is refracted differently from sunlight; hence, he had two tables of refraction.

[2] That is, about sixty degrees apart.

system, it makes no difference where the sun is with respect to the centre of Mars's epicycle and where Mars is on the epicycle, and in the Tychonic system, it makes no difference where the centre of the eccentric, or the sun, is located with respect to the line from Mars through the earth, so drawn that Mars lies in the plane of the ecliptic. For the diameter of the nodes is always the same in Copernicus and Ptolemy, and in Tycho it is always parallel to itself, except that over the ages the nodes move slightly. This motion was not perceived over these six years.

Now let us find the opposite node.

I. On the morning of 1595 January 4, when Mars was observed at $7^h$ $10^m$ at altitude 8°, with reference to Spica Virginis and Cor Scorpii[3], it was observed at latitude 0° 3′ 46″ N, and it was itself at 13° 36′ 40″ Sagittarius. The parallax was small, because Mars was more than
74 twice as far from the earth as is the sun. Refraction, on the other hand, was large: 6′ 45″ from the table for the fixed stars, or $11\frac{1}{4}$ minutes from the table for the sun[4]. This is nearly all latitudinal, owing to the low altitude of the nonagesimal[5]. Therefore, Mars (reckoning by the refraction displayed by the sun) was really at a few minutes south latitude – about 2 or 3, or even more.

II. On the night of 1589 April 15 Mars's observed latitude was 1° 7′ north. The parallax of the annual orb was drastically increased, owing to the approach of Mars and earth. After 21 days, the latitude decreased to $6\frac{2}{3}'$ north. And, although afterwards, on May 6, it decreased somewhat more slowly, since the star was moving away from the earth, we shall still not be far wrong if we extrapolate proportionally: we make 21 days be to the number of days after which Mars falls upon the ecliptic, as 60 minutes of diminution is to the $6\frac{2}{3}$ minutes remaining. This rule shows it to be two and one third days, so that Mars would be at the node on May 9. Once again, counting thrice 687 days thence, we come out on 1594 December 30. On this day, Mars ought to have been at the node, and for the next 5 days, up to the morning of January 4, it should have been moving off southward. And indeed, from observations of it on January 4, we have given it a few minutes of south latitude. This eccentric position was not observed more frequently. It is enough that we have this 1595 obser-

3 Antares.
4 See footnote 1 in this chapter.
5 The nonagesimal is the point on the ecliptic 90° westward from the point of intersection of the ecliptic and the horizon (the ascendant). See the translator's introduction for an explanation of its role in the measurement of refraction and parallax.

vation, provided that it does not disagree with us. In 1589 there is nothing that we can bring into question. Nor should it disturb you that in 1589 we assigned a latitudinal motion of $6\frac{2}{3}'$ to $2\frac{1}{3}$ days, while we did not allow so much over a 5-day period around 1595 January 4. For, as will appear in the course of this work, the latitude is most attenuated at conjunction with the sun (as in 1595), owing to the parallax of the annual orb, and at opposition (as in 1589) it is augmented. It is therefore fitting that the diurnal motion in latitude appear less in 1595, and greater in 1589.

Now, how are the sidereal positions of the two nodes found? Thus: one finds an approximate value for the mean motion of Mars at each place, using tables for Mars (which, accordingly, we presuppose for this purpose). Whether you do this with the help of the Prutenic or Tychonic tables, taking into account the true precession of the equinoxes, you will find that on the morning of 1594 December 30, the mean position of Mars is 27° $14\frac{1}{2}'$ Scorpio, and on the morning of 1595 October 28 it was at 5° 31′ Taurus. It therefore appears that the diameter of the nodes does not pass through the centre of equable motion, but far beneath it. For from 5° 31′ Taurus to 27° $14\frac{1}{2}'$ Scorpio is more than from the latter to the former.

If, on the other hand, you make use of the Tychonic equations, 11° 30′ must be subtracted from the former figure and 11° 17′ added to the latter. Accordingly, the one comes out to be 15° $44\frac{1}{2}'$ Scorpio, and the other, 16° 48′ Taurus, which are Mars's equated eccentric positions[6]. As you see, the nodes are nearly opposite one another at about $16\frac{1}{8}°$ Taurus and Scorpio, when viewed from the centre of the planetary system, which Ptolemy described as a point very near the earth, and Copernicus and Tycho, a point very near the sun.

Further, it will be seen below, in part 5, how much we have to change these positions of the nodes when we change the equations by transposing the theory of the sun from the sun's mean motion to its apparent motion.

[6] That is, Mars's positions as if observed from the sun's mean position (Ptolemy and Tycho), or the centre of the earth's orbit (Copernicus).

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# 13

## Investigation of the inclination of the planes of the ecliptic and of the orbit of Mars

Terms: Inclination and Latitude are understood differently. 'Inclination' concerns the angle at the sun or centre of the planetary system, which is (for Copernicus) formed by lines drawn from the body of Mars and from its position on the ecliptic. 'Latitude' is the angle under which any inclination appears when viewed from the earth. For Ptolemy, the inclination is the angle between

In the previous chapter, the nodes and limits have been found quite accurately according to the opinions of Tycho Brahe and myself. Now it is to be inquired what exactly the inclination of the plane of Mars's orbit is to the plane of the ecliptic.

It is not so evident how to deduce this from the observations themselves. For the angle of this inclination is set up around the centre of the planetary system, which for Copernicus and Tycho is the sun.

But the eye cannot be placed at the sun so that this angle might thence appear in the sidereal positions and be measured, and the maximum distance of the limit from the ecliptic, seen from another place, will also be seen under another angle. In the Ptolemaic form there might appear to be a more direct procedure, but this is not so. For it will be demonstrated that the plane of the epicycle always remains parallel to the plane of the ecliptic. Therefore, place the centre of the plane of the epicycle at either limit, and let the planet lie on the line of longitude passing from the centre of vision through the centre of the epicycle. The planet will then either be more distant than the centre of the epicycle from the observer, and thus its distance from the ecliptic will appear less than the distance of the centre of the epicycle from the same ecliptic, or it will be nearer to the observer, and will appear greater than what we seek.

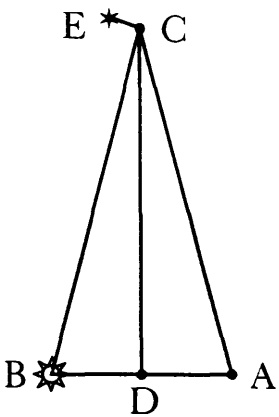
In this difficulty we may take consolation from this one circumstance: that the purpose for which we seek to know the inclination as one of our principles is not such as to require the highest accuracy. It

the straight lines drawn from the earth through the centre of the epicycle and through the epicycle's position on the ecliptic.

The latitude is the angle made by straight lines through the earth, one through the body of the planet, the other through the place on the ecliptic which corresponds to it.

will consequently permit us to use those means which furnish indirect evidence of the quantity of the inclination, of which we shall offer three.

Now it is apparent from what has just been said that it will be most directly helpful to us if we find an observation of the star Mars at that moment at which Mars is equidistant from the sun and the earth and lies on the line drawn from the sun to 16° or 17° Leo or Aquarius (the positions of the limits). In the Ptolemaic form, this is where the centre of the epicycle is at 16° or 17° Leo or Aquarius and Mars is as far from the earth as is the centre of the epicycle. In Mercury alone this problem will not have a [corresponding] position.




*Let* B *be the sun,* A *the earth, and upon* AB *let the isosceles triangle* ABC *be set up, with* C *the planet's position in the plane of the ecliptic, to which* CE *is drawn perpendicularly so as to meet the orbit of Mars, with the body of Mars at* E. EC *will therefore appear the same whether seen from* B *the sun or* A *the earth – this is immediately evident.*

*But in order that it be known at what position Mars is equidistant from the sun and the earth, observe that when the lines from Mars at* C *and the earth at* A *falling upon the sun at* B *make the angle* CBA *right, then* CB *is shorter than* CA. *Consequently,* BA, *the position opposite the sun's, should make with* BC, *the eccentric position of Mars, an angle less than 90°, in order that* CAB *and* CBA *be equal. Therefore, if*

BC *is directed toward 17 Leo, the sun ought to be beyond 17 Taurus and before 17 Scorpio. Or if on the contrary* BC *is directed toward 17*
76 *Aquarius, the sun should be beyond 17 Scorpio and before 17 Taurus.* We use these circumlocutions to denote morning risings or evening settings, with Mars and the sun sextile or quintile[1].

*In the Ptolemaic form, if* C *be the earth,* A *the centre of the epicycle, and* B *Mars,* CAB *will not be able to be right, if* CA *and* CB *are to be equal. So the anomaly of commutation*[2] *ought to be more than 90° and less than 270°.*

If you wish to work a little more precisely, take from Copernicus or from an anticipation of Tycho's rendition the approximate ratio 1525:1000 as (in Copernicus) the ratio of the orbits of Mars and earth, will consequently permit us to use those means which furnish indirect evidence of the quantity of the inclination, of which we shall offer three.

*So the triangle* ACB *is isosceles with sides* AC, CB *equal, and (with* AB = *1000),* BC *is* $1666\frac{2}{3}$*, when directed toward 17° Leo. Therefore, if* CD *be dropped perpendicular to* AB*, and* AD*, which is half* AB*, be made equal to 1000,* AC *will be* $3333\frac{1}{3}$*. Looking this up in a table of secants, we find the angles* CAD *and* CBD *to be 72° 33′. So also, at 16° or 17° Aquarius, with* AB = *1000,* AC *is 1375, so if* AD = *1000,* AC *is 2750, corresponding in the table of secants to 68° 40′.*

*Therefore, with* BC *directed towards 16° or 17° Leo or thereabouts, the apparent position of Mars,* AC*, ought to be* $72\frac{1}{2}$*° from the apparent position of the sun* AB*. And with* BC *at 16° or 17° Aquarius, these ought to be* $68\frac{2}{3}$*° from one another. And since the sum of the two (*CAB, CBA*) at 17° Leo is 145°,* ACB *will be 35° at 17° Leo. Wherefore, Mars, which lies on the line* AC*, ought to be seen at 22° Virgo (the sun being on* AB *at 5° Sagittarius), or at 12° Cancer (the sun being at 30° Aries).*

*Similarly, at 17° Aquarius, since the sum (*CAB, CBA*) is* $137\frac{1}{3}$*°,* ACB *will be* $42\frac{2}{3}$*°, wherefore Mars, lying on the line* AC*, ought to be seen at* $24\frac{1}{3}$*° Sagittarius*[3] *(the sun being on* AB *at 16° Libra) or at 0° Aries (the sun being at 9° Gemini).*

Something approximating this could have happened, first, in

1 That is, the apparent angle between Mars and the sun would be in the region of 60° to 72°.
2 *Anomalia commutationis.* This denotes the motion of the planet upon the epicycle, as measured from the epicycle's apogee; hence, in the heliocentric system, it is the relative motion of the planet and the earth.
3 As Caspar points out (KGW 3 p. 463), this is clearly wrong. Instead of 'at $24\frac{1}{3}$° Sagittarius (the sun being on AB at 16° Libra)', the text should read 'at $4\frac{1}{3}$° Capricorn (the sun being on AB at 26° Libra).'

November of 1586 or 1588; again, in April of 1581, 1583, 1596, and 1598; third, in September or October of 1587 and 1589; and fourth, in May or June of 1580, 1582, 1595, and 1597. In the last instance, suitable observations are lacking, since Mars could hardly be observed, or be seen at all, on account of its small ascension in Aries and the bright evenings caused by the sun in Gemini.

Accordingly, on 1588 November 10 at $6^h$ $30^m$ in the morning the planet Mars was seen at 25° 31′ Virgo, with a latitude of 1° 36′ 45″ north, the sun being at 21[4] Scorpio. The sun was thus only $62\frac{1}{2}$° from Mars, although it should have been 72° from Mars in order to make the triangle isosceles, as the problem requires. Therefore, Mars was then farther from the earth than from the sun. Consequently, its latitude at that place appeared less than was the true inclination.

On December 5 following, at $6^h$ in the morning, Mars was seen at 9° $19\frac{2}{5}$ Libra, with a latitude of 1° $53\frac{1}{2}$′ N., the sun being at 23 Sagittarius. Therefore, since the sun was $73\frac{1}{2}$° from Mars, the digression of the point which Mars then occupied on its orbit was a little less than 1° $53\frac{1}{2}$′ (since there should have been 72° between Mars and the sun).
77 Since the present angle is greater, the distance of Mars from the earth turns out to be less than the distance of Mars from the sun. The apparent magnitude of the inclination – at least, of this point from the plane of the ecliptic – is consequently greater. But since on December 5 the planet in its eccentric position was already several degrees beyond the limit, again diminishing its true digression from the ecliptic, this was consequently greater right at the limit. And since these two effects cancel one another, the maximum inclination of the planes will be about 1° 50′.

Similarly, on 1586 October 22 at $6^h$ in the morning, about dawn, there was about 6° 9′ eastward between Mars and Cor Leonis[5]. The declination of Mars from the equator was 13° 0′ 40″ north. Hence, its apparent longitude was found to be 0° 7′ Virgo[6], latitude 1° 36′ 6″ N[7]. The sun stood at 8° Scorpio, 68° from Mars. It should have been farther. Consequently, the line between Mars and the earth was longer than that between Mars and the sun. And so the apparent latitude was less than the true digression of the planet from the ecliptic, and this was, in fact, long before it reached the limit.

But on November 2 at $4\frac{2}{3}^h$ in the morning, with the sun at $19\frac{2}{5}$°

4 Should be 28.

5 Regulus.

6 0° $2\frac{1}{2}$′ Virgo, by the translator's computation.

7 1° 37′, by the translator's computation.

Scorpio, Mars was seen at 5° 52′ Virgo, with latitude 1° 47′ N. The sun was $73\frac{1}{2}$° from Mars, by a nearly exact measure. But Mars was a few degrees before its northern limit which is at about 16° 17′. Therefore the latitude at this position appeared about right, although exactly at the limit it is reckoned to be greater than 1° 47′; namely, about 1° 50′.

On December 1 following, at $7\frac{1}{2}^h$ in the morning, the equatorial distance between Cor Leonis and Mars was 25° $12\frac{1}{4}$′, and Mars's declination was 6° $2\frac{1}{4}$′. Hence is found its longitude, 20° 4′ 30″ Virgo, and latitude, 2° 16′ 30″, with the sun at 18° Sagittarius, which is 88° from Mars. It should have been only $72\frac{1}{2}$°. Therefore the line between Mars and the earth is made less than that between Mars and the sun, and because of this lesser distance, the digression appeared greater than it really was. At this point, therefore, the digression from the ecliptic was less than 2° $16\frac{1}{2}$′. Indeed, it was much less, but not so as to be much more than 1° 47′. So here the magnitude of the maximum inclination is indirectly confirmed to be 1° 50′.

On the other hand, on 1583 April 22, at $9\frac{3}{4}^h$ in the night, an interval of 20° 58′ was observed between Mars and the Dog[8] and 22° $47\frac{1}{2}$′ between Mars and Cor Leonis. Hence, the position of Mars is found to be 1° 17′ Leo, with latitude 1° $50\frac{2}{3}$′ north. The sun was at 11° Taurus, 80° distant from Mars. This should have been $72\frac{1}{2}$°. Accordingly, Mars is closer than it should be. Therefore, its observed latitude is greater than its true digression from the ecliptic. But Mars is more than twenty-one degrees beyond the northern limit. So at the limit, its digression would again be greater. Therefore, the opposite effects again cancel one another, and the maximum inclination is 1° 50′.

Likewise, at $8^h$ in the evening on 1596 March 9, it was observed at 15° 49′ Gemini, with latitude 1° $49\frac{2}{3}$′ north. The sun was at 30° Pisces, 76 degrees from Mars. It should have been a little closer. Therefore, Mars's true digression from the ecliptic was a little less than the observed latitude. However, this digression was not at its maximum, since Mars had not yet approached within about 25 degrees of the limit. So once again, indirect support is provided for a maximum digression of about 1° 50′ at the limit.

78 Now, at the other limit, at 17° Aquarius, although observations are rarer, there is one available.

On 1589 September 15 at $7\frac{1}{4}^h$ in the evening, Mars was observed at

[8] Procyon.

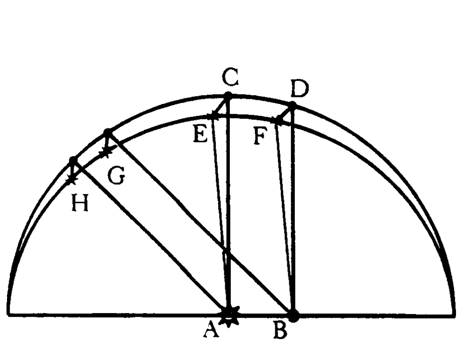
16° $47\frac{1}{3}'$ Sagittarius with 1° $41\frac{2}{3}'$ southern latitude. But when the correction for refraction of light appropriate to the altitude is applied, its position is 16° $45\frac{2}{3}'$, with latitude 1° $52\frac{1}{3}'$ south. The sun was at 2° Libra, $74\frac{1}{3}°$ distant from Mars. It ought to have been only $68\frac{2}{3}°$. Therefore, the observed latitude is a little greater than the digression of its position from the ecliptic. However, that is not the most distant point, as it is several degrees before the limit. Therefore here, too, the effects cancel.

On November 1 following, at $6\frac{1}{6}^{h}$, it was seen at 20° $59\frac{1}{4}$ Capricorn, with latitude 1° 36′ south, the sun being at 19° Scorpio. While it was then no more than 62° from Mars, it should have been $68\frac{2}{3}$. Therefore, the apparent latitude is less than the true digression from the ecliptic. But at the same time, the digression at this point is less than the digression at the limit, because this point is beyond the limit. Therefore, the maximum inclination is much greater than 1° 36′, and by all indications is about as great as the apparent latitude on September 15, namely, 1° 50′, approximately.

I have carried through one method, in which a knowledge of the approximate proportion of the orbs is presupposed. The observations lie within the limits set by this calculation, indicating readily enough the maximum inclination of the planes.

I shall now present another method, for which rarer, more select observations are required. If these are to be had, what we are seeking will be found without prior knowledge of the ratio of the orbs and this without the encumbrance of a laborious computation.

When two planes cut one another, any two lines drawn to the same point on the line of intersection and at right angles to that line always include one and the same angle.



*Let the plane of the ecliptic be* ACDB, *the plane of Mars's orbit* AEFB, *and let them intersect one another in* AB. *Let the sun be at* A, *the earth at* B, *and from* A *and* B, *perpendicular to* AB, *let* AC *and* BD *be drawn in the plane of the ecliptic, and* AE *and* BF *in the orbit of Mars. Let the planet be at* F. *The inclination* EAC *of the limit* E *will be equal to the apparent latitude of the planet at* F, *namely*, FBD. *You will therefore note that if ever there is a perfect quadrature of the sun and Mars, with the line* BA, *that is, the sun, at 16° or 17° Taurus or 16° or 17° Scorpio – where between the line* BA *from earth to sun (which in this instance is also the line of intersection of the planes), and the line* BF *from earth to Mars, there is 90° or one quadrant – then, whatever the apparent latitude of Mars* FBD *will be there, that will also be the maximum inclination of the planes* EAC, *although at that place* F, *Mars is not as far from the ecliptic as at* E.

The first such day fell on 1583 April 22, which only just now I had under consideration. The sun was at 11 Taurus, five or six degrees below the node. The earth, therefore, was above the line of intersec-
79 tion towards Mars. On this account, the apparent latitude is made greater than the truth, since it is seen from nearer. On the other hand, since there are not 90° between the sun and Mars, the apparent latitude will on this account be less than the truth. On the supposition that these opposite deviations cancel one another, the inclination of the planes will approximately equal the observed latitude. The observed latitude was 1° 50⅔′. Therefore, the inclination of the planes is approximately that much.

On 1584 October 30, there was a select occasion, but no observation is available. However, on November 12 following, at $1\frac{1}{2}^h$ in the night, when the sun had already fallen about 14° or 15° below the diameter of intersection, the earth having risen that much (for Copernicus), or the diameter of intersection having fallen that much toward the earth (for Tycho), Mars was seen at 23° 14′ Leo, with latitude 2° 12⅗′ north, while the sun was at 1° Sagittarius. This angle is somewhat diminished owing to the inclination of Mars's line of vision to the line of intersection. But it is greatly augmented by its approach toward the earth. Therefore, the inclination is much less than 2° 12′, namely, 1° 50′.

On 1585 April 26 at $9^h\ 42^m$, Mars was seen at 21° 26′ Leo with latitude 1° 49¾′ north. The sun was at 16° Taurus, right near the node. Mars's line of vision was a little inclined, since Mars was beyond 16°

Leo. Therefore, the angle of maximum inclination of the planes was only a little greater than $1°\ 49\frac{3}{4}'$; that is, $1°\ 50'$ or a little greater.

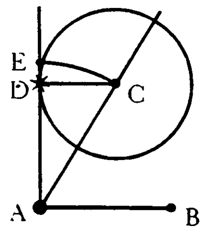
Similarly, near the other limit, on 1591 October 16 at $6^h\ 30^m$ in the evening, Mars was seen at $1°\ 27\frac{1}{3}$ Aquarius with latitude $2°\ 10\frac{5}{6}'$ south, decreasing. (For on October 10 preceding, the latitude was $2°\ 18\frac{2}{3}'$, and on October 2 it was $2°\ 38\frac{1}{2}'$.) The sun was at $2\frac{1}{2}°$ Scorpio, above the node. The earth was therefore below the node towards Mars. So because of this proximity, the observed latitude was greater than the inclination of the plane of the ecliptic. Fourteen days later, when the sun was at the node, if it were again to have decreased 28 minutes (the amount of change in the previous 14 days), there would remain $1°\ 45'$. But the ratio of decrease does not remain the same when the earth departs from a star, or vice versa. For at a greater distance the decrease is less. Therefore nothing can be adduced here against a maximum inclination of $1°\ 50'$. On the contrary: it is indirectly confirmed.

The demonstration can be extended farther. *Let* BA *be a line drawn from the earth through the body of the sun at the place of the node, 17° Scorpio or Taurus, and let the planet be observed at any point whatever on the zodiac. Now the latitude which it appears to have measures the inclination of a point on the plane truly removed from the limit by an amount equal to Mars's apparent removal from the limit. Let Mars be observed on* BG. *Draw* AH *parallel to it. The apparent latitude of point* G *seen from* B *will be the same as the inclination of point* H. *And* BG *and* AH *are directed towards the same sidereal degree, because they are parallel.* For example, in the observation of 1585 April 26, the sun was at 16° Taurus, and Mars was observed at $21°\ 26'$ Leo, with latitude $1°\ 49\frac{3}{4}'$. Therefore, the inclination at the eccentric position of $21°\ 26'$ Leo is $1°\ 49\frac{3}{4}'$. And since $21°\ 26'$ Leo is 5° from the limit, and the sine of 85° is 1/250 less than the whole sine[9], the maximum inclination here is greater by 1/250 of it, that is, about $1°\ 50\frac{1}{2}'$.

80 In the Ptolemaic hypothesis, the demonstration of this theorem proceeds on the following basis.

*Let* A *be the earth,* AB *the line through the sun and the point opposite, at 17° Taurus or Scorpio,* AD *Mars's line of vision,* D *Mars, and* BAD *a right angle.* AD *will accordingly be at 17° Leo or Aquarius. And because* D *is Mars, a line drawn from* D *parallel to* BA

[9] That is, the sine of 90°.

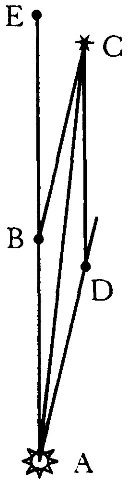



*will pass through the centre of the epicycle* C *(since the motion of Mars on its epicycle follows the motion of the sun on its orbit). Take a point* E *on* AD *such that* AE *equals* AC. *Therefore, since* AC *will not be at 17° Leo or Aquarius, it will not be so far from the ecliptic as the northern limit* E. D *will likewise not stand so far from the ecliptic as* E, *because* CD *and all the points of the epicycle are equally distant from the ecliptic, since the plane of the epicycle, in order to make the hypothesis equivalent, is supposed always to remain parallel to the plane of the ecliptic.* But proportionally as D or C is less distant from the ecliptic than E, D is closer to A than E, with the result that the distance D is proportionally greater [sic], and both may be seen under the same angle seen from A. *Now, according to spherical trigonometry, as the distance of* C *from the ecliptic is to the distance of* E *from the ecliptic, so is the sine of arc* CB *(that is,* AD*) to the whole sine* AE, *because* ECB *is a circle inclined above* AB. *But* C *and* D *are equally distant from the ecliptic, as was just said. Therefore,* AD *is to* AE *as the distance of* D *(or the perpendicular drawn from* D *to the ecliptic) is to the perpendicular from* E. *Therefore, the triangles* ADD *and* AEE *will be similar, since they have right angles at points* D *and* E *on the ecliptic, and the sides are proportional. Also, they will be concurrent, since the sides (*AD, AE*) are drawn in the plane of the ecliptic through the same point* A, *and are directed toward the same point of longitude, 17° Leo or Aquarius. Therefore, the lines* AD, AE *in the orbit are also concurrent; that is, the line drawn from the earth* A *through Mars* D *at this position will hit upon* E, *the centre of the epicycle, when it is at the limit. Thus the angles of maximum inclination and of observed latitude of Mars will be the same at this place.*

A third way depends upon computation and upon prior knowledge of the ratio of the orbs. We shall have a taste of this one only for the

sake of confirmation. A true and accurate treatment is reserved for part V and chapter 63, and is not required here.

In Tycho's table of oppositions, the apparent latitude at 21° 16′ Leo was 4° $32\frac{1}{6}$′.




*Let* A *be the sun,* B *the earth,* C *Mars on the eccentric. Thus the line* AE *running out through the earth* B *among the fixed stars intersects the ecliptic, and* AC *intersects the orbit of Mars. And since Mars is at 21° Leo, near the limit, the angle* EAC *is near its maximum. I find its magnitude as follows. Let* BA *be 1000,* AC *1664, and* EBC *4° $32\frac{1}{6}$′. Therefore, as* AC *is to* EBC, *so is* BA *to* BCA, *2° 43′ 27″*[10]. *This, subtracted from* EBC, leaves the required angle BAC, 1° 48′ 43″; hence, right at the limit, it would be about 1° 49′ and would be somewhat altered if the ratio BA to AC is altered (more on this below). In this manner, from any given acronychal observation with a comparatively great latitude, the inclination is found, first, of that point on the orbit, and then at its maximum, by considering the distance from the node or limit. For example, on 1593 August 24, the
81 apparent latitude at opposition to the sun came out to be 6° 3′ south. Mars was at $12\frac{1}{2}$° Pisces. *Now let* BA *be 1000 and* AC *1389, from our prior knowledge. As* CA *is to the sine of* CBE, BA *is to the sine of* BCA, *4° 21′ 10″. Subtracting from* CBE, *this leaves the required angle*

[10] This is correct if one replaces the angles with their sines.

BAC, *1° 41′ 50″. However, this position is about 26° from the limit, 64° from the node. So, as the sine of 64° is to this digression from the ecliptic of 1° 42′, so is the whole sine to the maximum inclination of the planes,* which comes out to be 1° 53′. We need not be concerned about the three minutes' excess, for they arise from the assumed ratio, for which see part IV below.

In the Ptolemaic form, A *will be the earth,* C *the centre of Mars's epicycle, and* D *the lowest point on the epicycle, since Mars is at or about opposition to the sun. And because the sun's line* EA *is on the ecliptic, while the plane of the epicycle is set up parallel to the plane of the ecliptic,* CD *will be parallel to* EA. *Therefore,* BAC *and* ACD, *the inclinations of the eccentric and the epicycle, are equal. But, owing to the full equivalence of the hypotheses,* CD *is equal to* BA; *that is, as* AB *is to* AC *in Copernicus, so is the semidiameter of the Ptolemaic epicycle* DC *to* CA, *the line from the earth to the centre of the epicycle. Therefore,* CDA *and* CBA *are equal, and* EBC *and* BAD *are equal, which are the apparent latitudes.*

# 14

## The planes of the eccentrics do not librate

The convolutions of Ptolemy's hypothesis forced him to accumulate many monstrosities in the theory of latitudes. For when he decided to make the plane of the epicycle tip every which way (it not being immediately clear, through the mists of his hypothesis, that the plane of the epicycle is parallel to the plane of the ecliptic), he contrived the latitude from three components, and in order that contraries counterbalance one another,* utterly wrenched his epicycle from its parallel position. He did not choose to find average values, whether on the grounds that his observations were not closely spaced, or that he distrusted those that were so, and hence accepted extreme values which were in error.

*See Maestlin's *Epitome astronomica*, on the final page of his account of the outer planets.

Efficient cause of the latitudes.

As a consequence, you may see that in the usual computation (e. g., in Magini's ephemerides) there is no conjunction whatever of Mars and the sun which is not, as they say, 'through the body'. This, if it were true, would render vain nature's latitudinal temperings, which prevent the excessive arousal of the sublunar powers that often repeated physical conjunctions would cause.

Copernicus, ignorant of his own riches, ever took it upon himself to express Ptolemy, not the nature of things, to which, nonetheless, he of all men came closest. (In this regard, see Rheticus's *Narratio prima*). For although he rejoiced to find that when the earth approaches a planet, the latitudes are in general greater, he still did not dare to reject the remaining Ptolemaic increase in the latitudes (which theory would not assent to such an approach of the earth). Instead, in the style of Ptolemy, he fabricated librations of the planes

of the eccentrics, in which the angle of inclination (which for Ptolemy
82 was fixed) would be varied. Moreover, in a manner close to being monstrous, this happens not according to the laws of motion[1] of its own eccentric, but to those, clearly foreign, of the earth's orb. See Copernicus Book VI ch. 1.

Armed by my own skepticism, I always opposed this gratuitous connection of diverse orbs as a cause of motion, even before seeing Tycho's observations. In this I more greatly congratulate myself that the observations were found to stand in agreement with me, as has happened with many other preconceived opinions.

But lest anyone deem me untrustworthy on this very account, claiming that I would treat the observations with prejudice, let him now witness that I have most solidly demonstrated the absence of librations in the inclination of the eccentric. For three ways of investigating the maximum inclination were proposed. In the first, the sun was near Mars's quintiles and sextiles[2], that is, about as close to Mars's conjunction as it could be and still allow Mars to be conveniently seen and observed. In the second, it was near quadrature with Mars, and in the third it was right opposite Mars. But at all three places with respect to the sun, Mars exhibited one and the same maximum inclination (1° 50′, about), northwards, at the same place on its eccentric, and the same amount southwards at the opposite position. Likewise, in ch. 12, it appeared that when Mars was near the nodes of its motion on the eccentric, no matter what position the sun occupied on its orb (whether near Mars or far from it), Mars was never seen to have any latitude. And in the fifth part below it will be proved in many ways that the declination of Mars's orbit from the ecliptic is constant at any particular position on its orbit.

And so let us conclude with great certainty that the inclination to the ecliptic of the planes of the eccentrics does not vary at all. (For why shall I not form a general conclusion where there is no reason why this should be a property of one single planet? Even so, I have demonstrated the same from the observations for both Venus and Mercury.) And a follower of Ptolemy may learn from this that the plane of the epicycle is always parallel to the plane of the ecliptic. For this is already demonstrated where the centre is near the limits, and it

[1] *Leges motuum*. This term was frequently used by astronomers of the sixteenth century to denote the principles governing planetary motion. For more on this topic, see the entry '*lex*' in the Glossary.

[2] That is, in the region of 60°–72°.

was proved above in ch. 12 that when the centre is near the nodes, the epicycle lies entirely on the ecliptic.

Peter Apianus's *Opus Caesarium*

Now who will set me up a fountain of tears, by which, for his deserts, I might bewail the pathetic industry of Apianus, who in his *Opus Caesarium*[3], following his trust in Ptolemy, spent so many hours and wasted so many ingenious meditations trying to express, by means of spirals and corollae and helices and volutes and a vast labyrinth of the most intricate curves, a human figment which the nature of things clearly disowns? But that man shows us that he was easily capable of equalling nature by the divine talents of his most perspicacious wits. For the rest, he entertained his mind with these illusions (in which he rivalled nature herself), which were thoroughly mastered and assembled in his models, and he is consequently worthy of undying fame, whatever fate fortune herself might have in store for his works. But what are we to say of the empty artistry of those who made the devices? For they make six hundred, nay rather twelve hundred little wheels, so they can triumph in the presentation of the latitudes (that is, human figments thereof) in their works, and reap the consequent reward.

[3] Ingolstadt, 1540. A lavishly produced book consisting mainly of movable graduated paper wheels, like circular slide rules, by which one could predict planetary positions.

83

# 15

## Reduction of observed positions at either end of the night to the line of the sun's apparent motion[1]

Now, with that investigation carried through to its conclusion, and the position of the nodes, the inclination of the planes, and its constancy all demonstrated, all of which were necessary for the coming reduction, we shall now define the positions which a planet may occupy on its orbit when the sun itself is diametrically opposite it. The years 1580 and 1597 may be omitted from the argument, as they present no suitable evidence owing to uncertainty of the observations[2].

I. Suppose, however, that on 1580 November 12 at $10^h\ 50^m$ Mars was observed at 8° 37′ Gemini[3], and the motion over five days was 1° 55′. Since at the given time the sun stood at 0° 45′ 36″ Sagittarius, and its motion over five days was 5° 5′, the sum of the two motions comes to 7° 0′. But the sun was 7° 51′ 24″ removed from opposition to Mars. Of this, seven degrees exactly were traversed in 5 days, or 120 hours. So, according to the same ratio, the remaining 51′ 24″ will be traversed in 14 hours 41 minutes. Therefore, the moment of opposition was November 18 at $1^h\ 31^m$. Its position was 6° 28′ Gemini on the ecliptic. Now this is 20° away from 16½° Taurus. I want to know how much longer this makes the arc on the orbit from the node to the

[1] The modern values for Mars's longitudes given in this chapter were kindly provided by Barbara Welther of the Harvard-Smithsonian Center for Astrophysics.

[2] Kepler believed the observations for 1580 to be uncertain because the diurnal motion was obtained from tables rather than from observations. But in fact, the observation itself was off by 20′. And ironically, although the observations for 1597 were made with a crude instrument, they were very nearly correct. For more of Kepler's comments, see ch. 10, observations I and IX.

[3] Modern value: 8° 57′.

Lansberg's trigonometry.

arc of latitude drawn through 6° 28′ Gemini. So I turn to Philip Lansberg's trigonometry[4]. I mention him out of honour and gratitude, for he has supplied me in quantity with the finest adzes, best adapted for building astronomical foundations, ready at hand, and at little expense of time. Without him, these would have had to be sought from afar with a great deal of trouble and toil, and the handles wouldn't have been so well fitted. From Lansberg, then, the tangent of the 20° side multiplied by the secant of the angle of inclination, 1° 50′, the last five digits being dropped[5], gives an increase of only 18½ of the smallest units, to which corresponds about 35 seconds. Mars therefore, standing opposite 6° 28′ Gemini, was 35″ further along on its orbit. It should therefore be placed at 6° 28′ 35″ Gemini, a tiny and quite unnecessary correction. The latitude was 1° 40′ north.

II. On the night following 1582 December 28, at $11^h\ 30^m$, Mars was observed at 16° 47′ Cancer[6], while the true position of the sun was 17° 13′ 45″ Capricorn. The moment of opposition had therefore passed. Now the sun's diurnal motion was 61′ 18″, that of Mars 24′, and their sum, 85′ 18″. At this moment, the distance between the stars was 26′ 45″. Therefore, as 1° 25′ 18″ is to 24 hours, so is 26′ 45″ to 7 hours 32 minutes. Subtracting this from 11 hours 30 minutes gives December 28 at $3^h\ 58^m$ after noon as the moment of true opposition. Its position on the ecliptic was 16° 54′ 32″ Cancer, and by reduction to the orbit (a 50″ correction), 16° 55½′ Cancer. The latitude was 4° 6′ north, as given by Brahe's table of oppositions. For among the observations I find various latitudes: on the night following December 26, 4° 6′ or 4° 2′, while on the night following December 29, 4° 8′ or 4° 6½′.

84 III. On 1585 January 31 at $12^h\ 0^m$, Mars was observed at 21° 18′ 11″ Leo[7]. The sun was at 22° 21′ 31″ Aquarius. The true opposition had therefore passed. The distance was 1° 3′ 20″. The sun's diurnal motion was 61′ 16″, that of Mars 24′ 15″, and their sum 85′ 31″. Now as 1° 25′ 31″ is to 24 hours so is 1° 3′ 20″ to 17 hours 46 minutes, to which correspond about 18′ of Mars's motion. Therefore the time was January 30 at $19^h\ 14^m$ and Mars's ecliptic position was 21° 36′ 10″ Leo. For reduction, some very small quantity is subtracted, because Mars

[4] Philip Lansberg, *Triangulorum geometriae libri IV* Leyden 1591.

[5] Since decimal notation had not yet come into use, multiplication of trigonometric functions yielded numbers with many extra digits, which had to be removed. This was done, in effect, by dividing by the number of units in the radius. See the translator's introduction for a more complete explanation.

[6] Modern value: 16° 48′ 35″.

[7] Modern value: 21° 19′ 23″.

was then beyond the limit. Therefore, the extension of the arc on the orbit from the following node is directed westward. But because Mars was only 4 or 5 degrees from the node, the subtraction is rendered quite imperceptible. The latitude, as given by the Tychonic table, was 4° 32′ 10″ north. For the observation on January 31 at $12^h$ gives 4° 31′. They added the remainder to the Tychonic figure, on account of diurnal parallax.

IV. On the night following 1587 March 4, at $1^h$ $16^m$ past midnight, the position of Mars was found to be 26° 26′ 17″ Virgo[8], from Cor Leonis[9] and Spica Virginis, with an observed latitude of 3° 38′ 16″ north. But because Mars was elevated 37½° above the horizon, diurnal parallax comes into the reckoning, and subtracts some small quantity from the longitude, thus making it 26° 26′ Virgo, with a slightly greater latitude. For since the sun is nearly twice as far away from earth as is Mars, Mars's parallax will consequently be nearly twice the sun's. On the supposition that the sun's is 3′, Mars's will be about 5′. But when 9° Sagittarius is rising, the nonagesimal[10] is 55° from the zenith, opposite which number in our parallactic table[11] under the column headed 5′ the latitudinal parallax, 4′, is shown. Therefore, the latitude observed from the center of the earth would be 3° 42′ 22″ north. In part V below, this will be useful to us in a more accurate examination of Mars's parallax, where the precise inclination and an absolutely certain distance of Mars for this position will have been established. The sun's true position was 23° 59′ 11″ Pisces. The true opposition was therefore still to come. The stars were 2° 26′ 49″ apart. The sun's diurnal motion was 59′ 35″, that of Mars 24′, and their sum 1° 23′ 35″. As this is to 24 hours, so is 2° 26′ 49″ to 1 day 18 hours 7 minutes, to which corresponds 42′ 7″ of Mars's motion. Therefore, the time of true opposition was March 6 at $7^h$ $23^m$. Mars's position on the ecliptic was 25° 43′ 53″ Virgo. For the reduction to the orbit, 55″ must be subtracted. Therefore, on the orbit it was at 25° 43′ Virgo.

[8] Modern value: 26° 27′ 43″.

[9] Regulus.

[10] The nonagesimal is the point on the ecliptic 90° in antecedence from the rising point (the intersection of the ecliptic with the eastern horizon). The great circle drawn through the zenith and the nonagesimal is perpendicular to the ecliptic; hence, it represents the shortest distance from the zenith to the ecliptic, and is of use in determining parallax. In particular, at the nonagesimal the parallax is entirely latitudinal, longitudinal parallax being reckoned by finding the distance from the nonagesimal. Since at this time (unlike the next observation) Mars is close to the nonagesimal, no longitudinal correction is necessary. For an account of Kepler's treatment of parallax, see the translator's introduction.

[11] The parallactic table, which is essentially a multiplication table, is contained in Kepler, *Astronomiae pars optica*, Frankfurt, 1604, bound opposite p. 275 (KGW 2 p. 240). For its use, see the translator's introduction.

The latitude was decreasing. It was therefore somewhat less than 3° 38′ N., or 3° 42′ corrected for parallax.

V. On the night following 1589 April 15 at $12^h\ 5^m$, the planet was found at 3° 58′ 20″ Scorpio[12], with latitude 1° 4′ 20″ north, decreasing. Mars's altitude was $22\frac{1}{5}$°, where refraction from the table for the fixed stars was zero, and from the table for the sun, $3\frac{1}{2}$′. But the parallax was about twice as great as the sun's, that is, 6 minutes at the horizon. The degree rising was 24° Sagittarius. Therefore, the nonagesimal was 64° from the zenith, giving a diurnal latitudinal parallax of 5′ 24″. Whether it really was that much will become apparent below, through a careful consideration of latitudes. For there, the northern latitude, free from diurnal parallax (and if there is no refraction), comes out to be 1° 9′ 45″ north. And because the altitude of the nonagesimal was 26°, the longitudinal parallax[13] at the horizon is 2′ 38″. But Mars was 40° from the nonagesimal, counting from 4°
85 Scorpio to 24° Virgo, which, under the column headed 2′ 38″ shows a true longitudinal parallax of 1′ 42″, by which Mars is moved further forward than when viewed from the center of the earth, and that on the assumption that it underwent no refraction. But to me it is more probable that it undergoes the same refraction as the sun, greater, that is, than that of the fixed stars, because the opposition of the sun and Mars stirs up the air, while the fixed stars are observed when the air is as calm as possible. Still, let there be no refraction at all, and let Mars be placed at 3° 57′ Scorpio. At that moment the sun was at 5° 36′ 20″ Taurus. At this time, therefore, Mars was 1° 39′ 20″ past opposition to the sun. Mars's diurnal motion, as is clear from comparison with April 13, is 22′ 8″, the sun's, 58′ 10″; the sum, 1° 20′ 18″. As this is to 24 hours, so is 1° 39′ 20″ to 1 day 5 hours 42 minutes. Therefore, the moment of opposition was April 14 at $6^h\ 23^m$ PM. Its position, 4° 24′ 30″ Scorpio, or a little past, if it was subject to refraction or if the previously assumed value for the diurnal parallax was too great. For reduction to the orbit, some imperceptible quantity must be subtracted, since it is barely 12 degrees from the

[12] Modern value: 4° 0′ 21″.

[13] The longitudinal parallax ($P_{long}$) is expressed by the following equation:

$$P_{long} = P \sin A \sin d.$$

where P is the total parallax at the horizon;
A is the altitude of the nonagesimal;
d is the distance of the planet from the nonagesimal.

The column mentioned in the following sentence is found in the parallax table in Kepler's *Astronomiae pars optica* (KGW 2 p. 240).

node. This would be about 24 seconds, which are of no importance, and Mars would be at 4° 24′ Scorpio, with latitude three minutes greater than before. For from the eighth of March up to that time the latitude was decreasing, nor was it a maximum at opposition.

VI. On the night following 1591 June 6 at $12^h\,20^m$, Mars was found at 27° 14′ 42″ Sagittarius[14], with latitude 3° 55½′ south. Refraction was of course provided for, from the table for the fixed stars, since it was large, Mars having no more than 6° altitude on the meridian. There was, however, no mention of parallax. But at that time Mars was distant from the earth by half the solar distance. Therefore, the horizontal parallax is greater than 6 minutes, on the supposition that the sun's parallax is 3′. I omit it nonetheless, partly because the refraction from the table for the sun (which, as I said, is the more probable) exceeds that which Brahe took here by 4½′, counteracting all the parallax, and partly because Mars was on the meridian and near the winter solstice point, and thus had no longitudinal parallax. Of the latitude, on the other hand, it will have to be seen below in part IV whether it might not be a few minutes less, since the parallax projects the planet too far south.

The sun was at 24° 58′ 10″ Gemini. The difference in position between the stars was 2° 16′ 10″. The sun's diurnal motion was 57′ 8″ and Mars's (for four days) was 1° 12′ 24″, since on June 10 at $11^h\,50^m$ it was at 26° 2′ 18″ Sagittarius. For one day, therefore, 18′ 12″. The sum of the diurnal motions is 1° 15′ 20″. This corresponds to 1 day 19 hours 24 minutes, which, added to the 6th at $12^h\,20^m$ (because opposition was yet to come), shows [the moment of opposition to be] the 8th at $7^h\,43^m$. Mars's position was 26° 41′ 48″ Sagittarius, to which are added 52″ for reduction to the orbit, so as to make it about 26° 43′ Sagittarius. The latitude was six minutes greater than on 6 June, because, by the observations, the latitude here was increasing until the fortieth day after opposition, and increased nearly thirteen minutes between the 6th and the 10th of June. Therefore, ignoring parallax and keeping the same refraction it would be 4° 1½′.[15]

[14] Modern value: 27° 14′ 23″.

[15] The computations for this reduction are among the Kepler manuscripts in the USSR Academy of Sciences in Leningrad (vol. XIV 213v–214v). They contrast dramatically with what is presented here: he actually used a dozen observations between May 13 and July 16 in a much more laborious procedure than the present one. Here, as often elsewhere, Kepler has spared the reader much of the agony of his search for the true orbit. (On this volume of the Leningrad Kepler manuscripts, see Gingerich, 'Kepler's Treatment of Redundant Observations', *Internationales Kepler-Symposium Weil der Stadt 1971* (Hildesheim, 1973), pp. 307–314.)

VII. On 1593 August 24 at $10^h\ 30^m$, the ecliptic position of Mars was found to be 12° 38′ Pisces,[16] with latitude 6° 5′ 30″ south. The
86 altitude was great enough that horizontal variations cancelled one another. On the August 29th following, at $10^h\ 20^m$, Mars was observed at 11° 15′ 24″ Pisces, with latitude 5° 52′ 15″ south. It was decreasing precipitously. For before August 10 it was maximum, fourteen days before opposition. The motion for the five days was 1° 22′ 36″, and for one day, 16′ 31″. The sun's position on August 24 at $10\frac{1}{2}^h$ was 11° 2′ 31″ Virgo. The stars were 1° 35′ 30″ apart. The sun's diurnal motion was 58′ 20″. The sum of the diurnal motions was 1° 14′ 51″. This requires 1 day 6 hours 57 min. to opposition, so as to make it August 26 at $5^h\ 27^m$ in the morning. Mars's position was 12° 16′ Pisces. Its latitude was 6° 2′ south, approximately, if the horizontal variations do indeed cancel.

VIII. On 1595 October 30 at $8^h\ 20^m$, the planet was found at 17° 47′ 15″ Taurus[17], not far from the nonagesimal. We may thus be sure of the parallax, although we must take it into account. The latitude was 0° 5′ 10″ north. The sun's position was 16° 50′ 30″ Scorpio. The distance between the stars was 56′ 45″. The sun's diurnal motion was 1° 0′ 35″; that of Mars, 22′ 54″, as appears by comparing the nearby observations. The sum of the diurnal motions was 1° 23′ 29″. If the distance between the stars be divided by this, it comes out to 40′ 47″ of a day, or 16 hours 19 min. Therefore, the true opposition was $0^h\ 39^m$ PM on October 31. Mars's position was 17° 31′ 40″ Taurus. This needs no reduction to the orbit, as it is nearly at the node. The latitude was about 0° 8′ north. But comparison with the preceding and following days shows it should be about 5′ north.

IX. Let us at least suppose (as above) that on 1597 December 10 at $8^h\ 30^m$ Mars's position was 3° $45\frac{1}{2}$′ Cancer[18]. The sun's position was 29° 4′ 53″ Sagittarius. The distance between the stars was 4° 40′ 37″. The sun's diurnal motion was 61′ 20″, that of Mars 23′ 40″ (for in 1580 the diurnal motion in Gemini was 23′, and in 1582, at 17° Cancer, it was 24′). Therefore, the sum of the diurnal motions was 1° 25′ 0″. These data show that the time of true opposition followed 3 days 7 hours 14 min. after, on December 14 at $3^h\ 44^m$ in the morning. Mars's position was 2° $27\frac{1}{3}$′ Cancer. The reduction to the orbit (quite ridiculous here, since the observation itself has an uncertainty of

16 Modern value: 12° 38′ 20″.
17 Modern value: 17° 48′ 32″.
18 Modern value: 3° 44′ 42″.

several minutes) requires the addition of about 52″. Therefore, the corrected position was 2° 28′ Cancer. The latitude, from the table, was 3° 33′ north.

On the same night, the one following December 10, at $12\frac{1}{6}^{h}$, Fabricius in East Frisia found Mars's position to be 3° $40\frac{1}{4}$′ Cancer with latitude 3° 23′ N. In this observation the longitude comes out nearly the same. For the motion over 3h $40^{m}$ is $3\frac{1}{2}$′, so that in the Brahean observation, at $12\frac{1}{6}^{h}$ Mars would have been at 3° 42′ Cancer, two minutes beyond the Fabrician position.

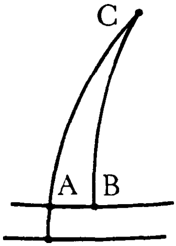
X. On 1600 January 13/23 at $11^{h}$ $40^{m}$, the time being adjusted to Uraniborg time, the planet was observed at 10° 38′ 46″ Leo[19]. The sun's position was 3° 26′ 30″ Aquarius. The stars were 7° 12′ 16″ apart. The sun's diurnal motion for the next few days was 1° 1′ 3″; that of Mars, 23′ 44″. The sum: 1° 24′ 47″. The opposition therefore followed 5 days $2^{h}$ $22^{m}$ after; that is, on January 19/29 at $2^{h}$ $2^{m}$ in the morning, before dawn. Mars was at 8° 38′ Leo. There is no need for reduction, since it is near the limit. The latitude, from the table, was 4° 30′ 50″ N.

87 XI. On the evening of 1602 Febr. 18/28 at $10^{h}$ $30^{m}$, using the Tychonic instruments (with the help of the learned Matthias Sieffard, bequeathed us by Tycho), I took the distance of Mars from the middle star of the tail of Ursa Major[20] to be 52° 22′. And since the distance between Cor Leonis and Procyon was 37° 22′ 20″, which should have been 37° 19′ 50″, we know that the sextant reads $2\frac{1}{2}$ minutes high[21]. Therefore, the corrected distance of Mars from the tail of Ursa was 52° $19\frac{1}{2}$′. And since the latitude of the fixed star is 56° 22′, the remainder, by subtraction is 4° $2\frac{1}{2}$′ – supposing that Mars was at precisely the same longitude as the fixed star. But because there was a difference of $3\frac{3}{4}$ degrees between them (as will appear from the following observations), a slight correction is required.

The reasoning behind this approximation [Ατεχνια] is given in the book on the star in Serpentarius.

For let AB be 3° 43′ 30″ on a parallel close to the ecliptic, B Mars, C the fixed star, and BC 52° 19′ 30″. Dividing the secant of BC by the secant of AB gives the secant of CA, 52° 14′, which, subtracted from 56° 22′ (the latitude of the fixed star) leaves 4° 8′ north as Mars's observed latitude. At the same time, we found 19° 23′ between Mars and Cor Leonis (19° $20\frac{1}{2}$′ corrected), and 21° 20′ between Mars and

[19] Modern value: 10° 40′ 35″.
[20] Zeta Ursae Majoris.
[21] Computation of this distance using Brahe's star tables yields a distance of 37° 17′ 25″, not 37° 19′ 50″ as Kepler claims. Thus, the sextant apparently read five minutes high instead of Kepler's two. The resulting distance of Mars from Zeta Ursae Majoris would be 52° 17′, and the uncorrected latitude, 4° 5′.




the bright star in the wing of Virgo[22] (21° 17½′ corrected). From these two distances (using the latitudes of the star and Mars), Mars's longitude is found to be 13° 19′ 6″ Virgo, by consensus of all measurements[23].

Alternatively, the meridian altitude of Mars was found at $12^h\ 40^m$ using two quadrants. It came out to be 50° 19′, while the tail of Leo[24] was 56° 45′. Therefore, from the declinations and right ascensions of the fixed stars and our distances, Mars's position is determined as 13° 19′ 30″ Virgo, latitude 4° 7′ 55″.[25] This is the Tychonic procedure. I have included the other for the sake of showing a consensus, and also that it might be evident that despite the lack of absolute perfection in the demonstration there do under certain circumstances exist short cuts either in computation or in our understanding. For in that previous procedure there is less in the actual work than in the reporting of it. At Prague, 5° Scorpio was rising[26]. Therefore, the

[22] Gamma Virginis

[23] Actually, no such consensus is found. If one uses the five minute correction suggested by the observations, Mars's position comes out as follows:

| | |
|---|---|
| From Cor Leonis: | 13° 15′ 25″ Virgo |
| From Gamma Virginis: | 13° 23′ 43″ Virgo |
| | (both corrected for precession). |

Since Cor comes before Mars on the zodiac, and Gamma after it, these results show that our correction was excessive. Even Kepler's smaller correction yields results indicating that the sextant was not as far off as Kepler thought. A correction of one minute would be nearer the truth. This would produce the following results:

| | |
|---|---|
| From Cor Leonis: | 13° 19′ 29″ Virgo |
| From Gamma Virginis: | 13° 19′ 46″ Virgo. |

This is near to both Kepler's value (13° 19′ 6″) and the modern value (13° 20′ 6″).

[24] Beta Leonis, or Denebola.

[25] Again, Kepler's corrections prove to be too great. Recomputation yields the following:

| | | |
|---|---|---|
| From Cor: | 12° 56½′ Virgo, lat. 3° 57′ N. | |
| From Gamma: | 13° 16′ Virgo, lat. 4° 7′ 40″ N. | |

For these computations, longitude and latitude of the stars were obtained from Tycho's tables, were corrected for precession, and were converted to right ascension and declination.

[26] This was at the time of the previous observation, namely, $10^h\ 30^m$.

nonagesimal was about $32\frac{1}{2}°$ from the zenith. And since Mars's distance from the earth was rather more than half the sun's distance, the resulting parallax of about 5′ opposite $32\frac{1}{2}°$ in our parallactic table, shows a latitudinal parallax of 2′ 41″. Thus the latitude as seen from the middle of the earth would be 4° $10\frac{2}{3}$′ north. And because the altitude of the nonagesimal was $57\frac{1}{2}°$, the longitudinal parallax at the horizon was 4′ 13″. But since Mars was 38° from the nonagesimal, the longitudinal parallax corresponding to this position is 2′ 36″, and if this be eliminated, Mars would be at about 13° 18′ Virgo[27]. At that moment, the sun's position was 10° 16′ 42″ Pisces. The distance between the bodies was 3° 1′ 18″. The sun's diurnal motion was 1° 0′ 4″ and that of Mars, 24′ 5″ (for it was 24′ 18″ at 21° Leo in 1585, and 24′ at 26 Virgo in 1587). The sum of the diurnal motions was 1° 24′ 9″. The true opposition therefore followed 2 days 3 hours 43 minutes later, on 21 February/3 March before dawn at $2^h$ $13^m$, Mars being at 12° 27′ 35″ Virgo. Forty seconds must be subtracted to reduce the position to the orbit, putting Mars at 12° 27′ Virgo, with slightly less latitude than before, since the latitude was then decreasing. It was therefore about 4° 10′, or 4° $7\frac{1}{3}$′ if the parallax be neglected.

88 But because, since Tycho's death, we have not made frequent observations nor continued them over several days, it would be best, for certainty's sake, also to make use of those observations which David Fabricius of East Frisia, a sedulous practitioner of astronomy, has communicated to me.

On February 16, old style, at $5^h$ in the morning, he took the planet's distances from the tail of Leo (for the latitude), from the neck of Leo[28], and, on the other hand, from the bright star in the southern wing of Virgo, so as to check its longitude by working it out twice .

I could make use of Tycho's line of reasoning, which he routinely adopted in volume I of the *Progymnasmata* when (as here) the declination of the planet was not known. But because that method extends to ten operations, I prefer for the sake of brevity to proceed as I did before with my own observations. Here there are no hidden dangers.

[27] This procedure is vitiated by Kepler's excessively small estimate of the sun's distance, resulting in too large a value for the total horizontal parallax. However, Kepler's practice – exemplified at the end of this paragraph – of giving alternative data with the parallax correction omitted, shows that he suspected the correction to be too large (see also ch. 64). Oddly enough, when he applied the correction to the longitude, he appears to have subtracted only one minute rather than $2\frac{1}{2}$′, thus improving the accuracy of this figure.

[28] Gamma Leonis.

First, the star in the wing of Virgo, adjusted to our time, is at 4° 36′ 30″ Libra with latitude 2° 50′ north. Fabricius found that Mars is 20° 18′ westward from it on the zodiac, which puts Mars at about 14° 18′ 30″ Virgo. This is a preliminary, approximate figure: the longitude will shortly be corrected. Now the tail of Leo is at 16° 4′ Virgo, with latitude 12° 18′ N., and Mars was found to be 8° 17′ distant from the Tail. What is sought is the distance of its parallel from the Tail, since the difference in longitude is 1° 45′. Dividing the secant of 8° 17′ by the secant of 1° 45′ gives the secant of 8° 6′, the arc sought. This, subtracted from the fixed star's northern latitude of 12° 18′, leaves Mars's latitude, 4° 12′ north. This I now take as determined, and compare it with the latitudes of the fixed stars according to the laws of trigonometry. From the Wing of Virgo, I find Mars's longitude to be 14° 19′ Virgo[29]; from the Neck of Leo, 14° 23′ 36″.[30] The mean of these two is 14° 21′ 18″ Virgo[31]. And since the sextant gave distances larger than the truth, the latitude comes out at 4° 14′ north.

On the night following February 23, at $12^h$, he observed Mars in relation to five fixed stars: the Tail of Leo and Arcturus for latitude, and for the longitude, in the first instance, Spica (which followed it), and, in the second, the Neck of Leo and Cor Leonis (which preceded it).

By a rough estimate, I foresee that Mars will fall at $11\frac{1}{4}$° Virgo. It was found to be 9° 24′ from the Tail of Leo, and hence, its latitude comes out at 4° 6′.[32] And now, through this and the latitudes of the fixed stars, together with their distances (17° 26′ from Regulus, 17° 51′ from the Neck of Leo, 37° 28′ from Spica, 44° 15′ from Arcturus), Mars's position comes out at 11° 21′ 23″ Virgo (from Regulus), 11° 20′ 52″ Virgo (from the Neck of Leo), and 11° 17′ 40″ (from Spica)[33]. Again (as you see) the distances err in being too great. For by the distances from the Heart and the Neck [of Leo], Mars is pushed a little eastward and by those from Spica and Arcturus a little westward and more by those from Arcturus, owing to its greater northern

[29] 14° 21′, by the translator's reckoning.
[30] As Kepler neglects to give Fabricius's figure for the distance of Mars from this star, his calculation cannot be checked.
[31] Modern value: 14° 26′ 3″.
[32] According to the translator's computations, the correct value would be 4° 13′.
[33] Modern value: 11° 18′ 24″.

latitude[34]. The mean (ignoring Arcturus), 11° 19′ 20″ Virgo, is very nearly true. Also, the latitude is greater, namely, 4° 7′ 40″ north[35]. Now from February 15 at $17^h$ to 23 February at $12^h$, a period of 7 days 19 hours, Mars moved 3° 0′, 180 minutes in 187 hours. That is, about one minute per hour. And if you wish to take it into account, the parallax (if any) is subtracted from the longitude on February 16 and a little is added on February 23.

89 Because the time of the last observation follows the time of observation found by me by 2 days 21 hours 47 min., the motion corresponding to this time, 1° 7′, is added, giving 12° 26′ Virgo as its position. The agreement is thus very good – it could not be better – so we may be happy with both and avoid checking them against the convenient procedures used by Tycho Brahe.

As for the latitude, on the 16th it was 4° 12′, and on the 23rd it was $4° 7\frac{2}{3}'$. It is therefore about right to set it at 4° 9′ on the 21st, which comes between the other two dates. Correction for parallax makes it somewhat greater. And I, too, was putting it at a little less than $4° 10\frac{2}{3}'$, or 4° 10′.[36]

XII. Finally, in 1604, when I had published my previously written Ephemerides, in which on the night between March 29 and 30/April 8 and 9 the planet was placed on a line from Arcturus to Spica, that very thing appeared. For on the evening of April 8 it was slightly east of that line, but already on April 9 it was to the west. At that time, with the help of Johann Schuler and using Hofmann's sextant, I found 33° 4′ between Arcturus and Spica, which ought to have been $33° 1\frac{1}{2}'$. Therefore, the reading was $2\frac{1}{2}'$ too great. At the same time, between Arcturus and Mars there was $29° 43\frac{1}{2}'$, or 29° 41′ corrected. And since the latitude of Arcturus is $31° 2\frac{1}{2}'$ north, this leaves $2° 21\frac{1}{2}'$ as the

[34] Kepler's argument here is of dubious soundness. First, the figures he gives appear to be in error. Figures obtained by the translator (using Kepler's incorrect latitude) are:

| | |
|---|---|
| From Regulus: | 11° 21′ 25″ Virgo |
| From the Neck: | 11° 14′ 38″ Virgo |
| From Spica: | 11° 14′ 32″ Virgo |

These longitudes do not fall into the pattern described by Kepler. Further, Kepler does not provide the figure computed from Arcturus, which, again obtained through Kepler's latitude, would be 12° 8′ 29″ Virgo. The effect of this is opposite that claimed by Kepler: Mars is placed too far to the east. The mean resulting from these figures (I follow Kepler in omitting Arcturus) is 11° 16′ 52″, considerably different from the figure given by Kepler. Further, if one uses the translator's computed latitude of 4° 13′ 6″, the resulting mean is 11° 17′ 19″, not much better than the figure Kepler gives.

[35] This latitude, derived as it is from the incorrect figure introduced earlier in this paragraph, is smaller than the latitude resulting from the observations Kepler gives.

[36] The observations would suggest a somewhat greater figure than this, perhaps 4° 13′.

latitude of Mars.[37] There was then 54° 8½′ between Cor Leonis and Mars, and at the same time, the same amount between Cor Leonis and Spica. This should, however, have been 54° 2′. There were therefore 6½ minutes too much, while before the excess was only 2½′. The origin of this uncertainty of four minutes cannot be ascribed to the intervention of obstacles, as we were unable to eliminate it while observing. But let us suppose that, as before, the excess was 2½′, making the distance between Mars and Cor Leonis 54° 6′. The error could then be in Spica's position, possibly because Mars was mistaken for Spica, since they were close to one another. Hence, Mars's latitude comes out to 2° 21½′, longitude 18° 25′ Libra. The hour is known since, at the time of observation, the Back of Leo was culminating, whose right ascension was 163° 13′. Now the sun's position at noon was 18° 56′ 24″ Aries, right ascension 17° 27′ 55″. Hence, the difference of ascensions was 145° 45′, which resolves into 9 hours 43 min. The rising point was 22½° Scorpio.[38] Therefore, the nonagesimal was 39° from the zenith, and the distance of Mars from the earth was a little greater than half that of the sun from the earth. So the parallax was about 5½′, and its latitudinal part, 3′ 28″. Therefore the latitude without parallax was 2° 25′ (whether this correction was rightly made, we shall consider below). And because the altitude of the nonagesimal was 51°, and Mars's distance from the nonagesimal was 56°, the longitudinal parallax was 3′ 32″. Therefore, Mars would be at 18° 21½′ Libra.[39] At our chosen moment, the sun's position was 19° 20′ 8″ Aries. The two celestial bodies were 58½′ apart. The sun's diurnal motion was 58′ 38″, and that of Mars, 22′ 36″. (For in 1587 in Virgo it was 24′, and in 1589 at 4° Scorpio it was 22′ 8″). The sum of the diurnal motions was 1° 21′ 14″. From all these data it follows that the true opposition preceded the observation by 17 hours 20 min., namely, on 29 March/8 April at $4^h$ $23^m$ in the morning. Mars's position was 18° 37′ 50″ Libra. For reduction to the orbit, subtract about 39 sec., making Mars's position 18° 37′ 10″ Libra. The latitude was slightly greater than 2° 25′, but, when parallax is ignored, it is 2° 22′ north.[40]

90 Now these twelve eccentric positions of Mars (so called because the

[37] This is clearly 1° too great.

[38] Recomputation shows the rising degree to be more nearly 28° Scorpio. However, this has no significant effect upon Kepler's results.

[39] Modern value: 18° 23′ 3″.

[40] As was noted previously, the data given by Kepler indicate a latitude 1° less than the values given here.

longitudes are freed from the effects of the second inequality) have been established with all possible care. However, it has happened that for a period of as much as eighteen months I built upon the weak foundation of a badly applied observation, and all that work was in vain. Still, if something has escaped me somewhere in this prickly business, I am entirely at a loss to imagine what it could be.

I shall therefore set out all the positions in the following table, with the addition of the mean longitudes from Tycho. I could have gotten these from the Prutenic tables, or from the computation upon which Ptolemy based his demonstrations and which he designed for that purpose, but this would be unnecessary. For if the mean motion needs correction, it will be corrected later. For the present, it will serve to measure the time intervals without any appreciable error.

| | Date, Old Style | | | | | Longitude | | | | Latitude | | | Mean Longitude | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | Year | Day | Month | H[1] | M | D | M | S | Sign | D | M | | S | D | M | S |
| I | 1580 | 18 | November | 1 | 31 | 6 | 28 | 35 | Gemini | 1 | 40 | N. | 1 | 25 | 49 | 31 |
| II | 1582 | 28 | December | 3 | 58 | 16 | 55 | 30 | Cancer | 4 | 6 | N. | 3 | 9 | 24 | 55 |
| III | 1585 | 30 | January | 19 | 14 | 21 | 36 | 10 | Leo | 4 | $32\frac{1}{6}$ | N. | 4 | 20 | 8 | 9 |
| IV | 1587 | 6 | March | 7 | 23 | 25 | 43 | 0 | Virgo | 3 | 41 | N. | 6 | 0 | 47 | 40 |
| V | 1589 | 14 | April | 6 | 23 | 4 | 23 | 0 | Scorpio | 1 | $12\frac{3}{4}$ | N. | 7 | 14 | 18 | 26 |
| VI | 1591 | 8 | June | 7 | 43 | 26 | 43 | 0 | Sagitt. | 4 | 0 | S. | 9 | 5 | 43 | 55 |
| VII | 1593 | 25 | August | 17 | 27 | 12 | 16 | 0 | Pisces | 6 | 2 | S. | 11 | 9 | 55 | 4 |
| VIII | 1595 | 31 | October | 0 | 39 | 17 | 31 | 40 | Taurus | 0 | 8 | N. | 1 | 7 | 14 | 9 |
| IX | 1597 | 13 | December | 15 | 44 | 2 | 28 | 0 | Cancer | 3 | 33 | N. | 2 | 23 | 11 | 56 |
| X | 1600 | 18 | January | 14 | 2 | 8 | 38 | 0 | Leo | 4 | $30\frac{5}{6}$ | N. | 4 | 4 | 35 | 50 |
| XI | 1602 | 20 | February | 14 | 13 | 12 | 27 | 0 | Virgo | 4 | 10 | N. | 5 | 14 | 59 | 37 |
| XII | 1604 | 28 | March | 16 | 23 | 18 | 37 | 10 | Libra | 2 | 26 | N. | 6 | 27 | 0 | 12 |

1. Hours are reckoned from noon: hence, $19^h\ 14^m$ on January 30 is the same as 7h $14^m$ on the morning of January 31.

# 16

## A method of finding a hypothesis to account for the first inequality

Ptolemy, in book 9 chapter 4[1] of the Great Work, where he is about to take up the first inequality, made by way of preface a somewhat cursory declaration of the suppositions of which he wished to make use. It is, in summary, as follows: We see that a planet requires unequal times to traverse opposite semicircles. As, although from $2\frac{2}{3}°$ Cancer through Leo to 26° Sagittarius is less than a semicircle, and from 26° Sagittarius through Aquarius to Cancer is more than a semicircle, nonetheless the planet is found to spend longer on the former than on the latter, although a law of uniformity[2] would require the contrary. For from a mean longitude of $2^s\,23°\,18'$ to $9^s\,5°$
91 $44'$ is $6^s\,12°\,26'$, more than a semicircle, that is, more than half of the planet's periodic time[3]. So from 12° 16′ Pisces through Leo to 12° 27′ Virgo is about a semicircle and 11 minutes. But if the mean longitude of the former position ($11^s\,9°\,55'$) be subtracted from the longitude of the latter ($5^s\,14°\,59'$), the difference is seen to be $6^s\,5°\,5'$, which is $5°\,5'$ more than half. The planet consequently takes a proportionally

[1] Book IX chapter 5, in modern editions.

[2] *Lex aequalitatis.* The phrase is odd enough to suggest the use of an awkward translation. It would be clearer to say 'pattern of uniformity', but the Latin equivalent for that would be *ratio aequalitatis* or perhaps *species aequalitatis*. Kepler's use of the word *lex*, 'law', should be noted, for while it is commonplace to speak of 'laws' in a scientific context today, the term was then only beginning to acquire its modern signification.

[3] Kepler's point here depends upon the understanding that the mean longitude is a measure of time: the planet's periodic time is divided into twelve equal 'signs' (that is what the 's' stands for), each containing 30°. Although the mean longitude also has an angular significance, the angle is not in question here. Kepler is simply taking positions from the table at the end of ch. 15 (of 1591 and 1597) and comparing the difference in longitudes (representing positions) with the difference in mean longitudes (representing times).

shorter time from Virgo through Aquarius to Pisces. Now if you examine adjacent positions one at a time and compare the intervening arcs with the times or with the arcs of mean longitude, you will see that the planet is slowest at one fixed point on the zodiac, and swiftest at the opposite point, and that at the intermediate points its motion gradually increases or decreases, according to its proximity to one or the other.

These things argue first of all that the motion of a planet (however irregular it may appear) is governed according to cycles, and that the present cycle is the successive modification of motion and a return to its previous state. For if the planet moved in straight lines joined by angles (such as motion around a pentangle, an idea which once occurred to me), its motion would suddenly change from swifter to slower in an evident manner, according to the relationship of the lines, and this would happen not in one but in many places on the zodiac, according to the number of lines. However, the inequality that still remains in the planet's motion, after the removal of the inequality that depends upon the sun, is so great that it will be incapable of being either governed or demonstrated by the supposition of a simple circle (one set up at the centre of observation). This can, however, be done by composition (real or imagined) of several circles (as Ptolemy said in his preliminaries to book 3). The simplest ways of doing this are two: by using either an eccentric circle or a concentric with an epicycle.

Thus Ptolemy chose an eccentric for the first inequality, for the sake of distinguishing between the two and providing an aid to comprehension, since an epicycle would be required for the second inequality. Then, thinking over this general description, he denied that a mere eccentric will suffice the planets. For he considered closely what would naturally follow from the simultaneous revolution of an epicycle (to account for the second inequality) and an eccentric (for the first inequality), and realized, by comparing observations, that the centre of the epicycle would apparently approach much nearer to the earth at apogee, and flee farther from it at perigee, than the simple eccentric which accounts for the first inequality would allow. From this discovery, by a continuous train of thought, he was led to find the measure of this approach, and made the important discovery that the centre of the eccentric that carries the centre of the epicycle is at the midpoint between the centre of observation, the earth, and the centre of uniformity or of the eccentric accounting for

the first inequality. And, without a single demonstration, he nevertheless relied upon this principle for the three superior planets.

On this point, see the marginal note to chapter 19.

Copernicus, as he frequently did on other occasions, here too followed his master religiously, his form of hypothesis being accommodated to this criterion.

Not without reason, astronomers have wondered about this, and I among them (using Maestlin's voice), as you see in the *Mysterium cosmographicum* ch. 22 p. 79. Despite my having opined, in that passage of the said book, that Ptolemy used blind guesswork to
92 establish this, the truth is the opposite. For he was able to prove it with a perfectly good demonstration given a suitable observation, as I shall demonstrate below. One finds fault with the theorist only in that he did not transmit to posterity those observations along with the demonstration.

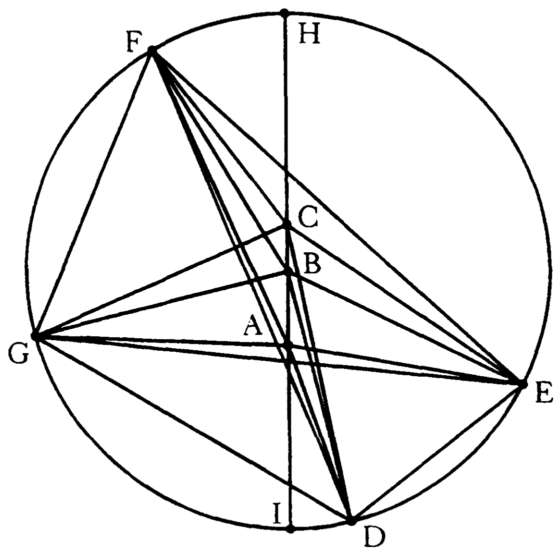
And so, since I thought then that this was altogether too much to assume, and also saw it pointedly called into question by Copernicus when he argued for a change in Mars's eccentricity on the basis of his figures which were not in accord with this bisection of the eccentricity, I envisaged a method which would lead me to a knowledge of the ratio of the two eccentricities (which, as I said, is not indubitably 2:1). And since Ptolemy used three acronychal observations and this preconceived opinion of the ratio of the eccentricities to find the position of the apogee, the correction of the mean longitude, and finally, the magnitude of the eccentricity, I saw that in order to weaken the sinews of the problem (once the axiom of the ratio of the eccentricity is taken away), resulting in a general solution applicable to more than one case, I would need in addition the support of a fourth acronychal observation. And so, in the year 1600, having acquired knowledge of this art, I came to Tycho, and was happy to learn that he too did not assume this ratio, but made an investigation, as his figures indicate. For he made the centre of the (Copernican*) eccentric distant from the centre of vision by 13,680 units, while the point of equality in turn was another 3780 of these units beyond that. In the Ptolemaic form, this would be as if he were to make the distance of the centres of vision and of the eccentric 9900, and the remaining distance between the centre of the eccentric and the point of equality 7560.

*Whose definition is at the beginning of ch. 5 of this book.

Now I myself could also have taken the bisection of the eccentricity as certainly established, and with better reason than Ptolemy, because in ch. 22 of my *Mysterium* I had brought forward a physical

On the *Prodromus*, or *Mysterium Cosmographicum*.

cause for the bisection. Indeed, it was for this very reason that I had come to Tycho, that I might use his observations to inquire further into my opinions expressed in that book. I of course did this without prejudice, and continue to do so. And if I survive to see astronomy achieve its purity and perfection, so that a verdict can be given in the case which I have brought before her tribunal in that book, I promise the reader that I shall retract that book and confirm what is seen as true, faithfully revealing the remainder which has turned out not to be so.[4]




But back to the argument. *About centre* B *let the eccentric* BG *be described, and on it, through* B, *the diameter of the apsides* HI, *taken as if immutable over any number of years. If there be danger of error in this assumption, we are not lacking ways of taking it into account. On this line below* B *let* A *be the observer, and above* B *let* C *be that centre about which angles are made proportional to amounts of time, since (as was just said above) these are not proportional about* A. *Now let* F, G, D, *and* E *be four observations distributed about the circumference*

[4] Kepler kept this promise by publishing a second edition of the *Mysterium cosmographicum* in 1621, with extensive notes in which he frequently repudiated or modified his earlier opinions.

*of the circle, so situated that the planet, stripped of the second inequality, would appear there as if viewed from point* A. *For indeed,*
93 *according to Ptolemy,* A *is the true centre of vision or the centre of the earth, while according to Tycho and Copernicus vision takes place along the lines* FA, GA, DA, *and* EA, *and* A *is the sun.*[5] *But it was said above that in either way the planet is shorn of its second inequality. Now let each point be connected with each of the others, and let* AF *be at 25° 43′ Virgo,* AG *at 26° 43′ Sagittarius,* AD *at 12° 16′ Pisces, and* AE *at 17° 31$\frac{2}{3}$′ Taurus. Hence, the four angles about* A *are given:* FAG *is 91° 0′,* GAD *is 75° 33′,* DAE *is 65° 15$\frac{2}{3}$′, and* EAF *is 128° 11$\frac{1}{3}$′. These must be corrected somewhat on account of the precession of the equinoxes. For in relation to the fixed stars the planet is not so far forward at* E *(the last observation) as is indicated by these numbers. Wherefore* FAE *is a little greater, and the others smaller by the same amount. In the same way, by subtraction of the [mean] longitudes, the angles about* C *are also obtained.*

Proposition. It is now required to select values for angles FAH and FCH such that the points F, G, D, E stand on one circle, and that centre B of that circle lie between the points C and A on the line CA.

The solution is not geometrical, at least if algebra is not geometrical, but proceeds by a double iteration. For algebra, too, forsakes us here, because the categories of art united by [their being concerned with] straight lines do not extend beyond straight lines to angles, unless perchance one would wish to cram the entire theory of sines into this one operation.

But behold what we would be required to do. If we were to assume a value for the angle FAH, then since the line AF has a certain sidereal position, the other leg AH would also be assumed to have a certain sidereal position. But AH shall be the line of the apogee, or the line of the aphelion in the Copernican and Tychonic notion. We are thus required to assume and posit that which is sought. For it was in order to learn the position of this aphelion that we embarked upon this path. In the same way, since the sidereal position of AH (that is, CH) was arrived at through this assumption of ours, and it passes through C the centre of our equant circle (and therefore also through the starting point from which the numbering of its parts has its beginning, namely, the apsis which is conceived as being above H);

[5] In the first edition, the diagram was printed with the letters E and G exchanged, and all subsequent editions and translations (except the present one) have repeated the error. (This error was discovered independently by D.T. Whiteside and the translator.)

and since we are also required to assume the angle FCH, the line CF therefore also acquires its position on the circumference of the equant. And indeed, this is the mean longitude corresponding to the observed position of the planet at F. And we were seeking to know what this mean longitude is. Therefore, in addition to the apogee, we are assuming yet another thing among those which we were seeking.

At the same time, however, it is not unusual, whether in geometry, or arithmetic, or dialectic, to use a form of argument which leads to an absurdity, so that if something absurd is seen to follow from the assumptions, they are rejected as false; and this is carried out until the consequent removal of excesses and defects unveils the exact truth (which in the mathematical disciplines is a mean between the two). In the present case this comes about in the following manner.

*Let the line* CA *be taken as the nominal standard (and thus, let it be given). Since the angles* FCH *and* FAH *are assumed, and hence also the inclinations of the remaining lines to* HCA, *and* AC *is the common side of the four triangles* CFA, CGA, CDA, CEA, *whose angles are given, therefore the four lines* AF, AG, AD, *and* AE *will be given in relation to the length* AC. *And since in the four new triangles* FAG, GAD, DAE, *and* EAF, *the sides are already given with the angles at A between two sides, the individual angles at the bases of the several*
94 *triangles (that is,* AFG, ADG, ADE, *and* AFE*), will not be a matter of ignorance. But* AFG *and* AFE *are parts of the angle* GFE. *And in the quadrilateral* DEFG (if, indeed, it is inscribed in a circle, which is one of the hypotheses here), *it is a consequence that two opposite angles (as* GFE, GDE*) taken together are equal to the sum of two right angles. Therefore, if the sum of the four angles which we have just found differs from the measure of two right angles, we shall pronounce the assumptions false, whether the falsehood be in one or the other of the assumed values, or both.*

*Then, one angle,* FCH, *being kept the same, and the other,* FAH, *being changed,*[6] *a return is made to the beginning, and the sum of the four angles is once more sought. If this sum differs more than the previous sum from two right angles, it suggests that* FAH *was altered in*

[6] This is not what Kepler does in the example that follows. Instead, he changes only the position of the aphelion, letting the mean longitudes remain fixed. That is, he changes FAH and FCH by the same amount. Then later, when the centre of the circle does not fall upon the line of apsides, he changes the mean longitudes and then repeats the process of adjusting the aphelion until the four points lie upon one circle. Again, the centre of the circle is found, and its new position will show how to adjust the mean longitudes so as to bring the centre back to the line of apsides.

*the wrong direction. Therefore, the opposite must be done: if you had added something to it, you now would diminish it, or vice versa. But if, on the other hand, you have come closer to the correct measure, then you will be sure that you are on the way. And then by comparing the original discrepancy with that which remains, you will carry on in that same proportion by increasing or decreasing the angle* FAH.

*But it is still not certain that this second correction will directly reconcile your four angles with the exact measure. For the rate of increase of circular variables is not the same as that of straight ones. Your labour will have to be repeated again and again, until your sum for the angles in question is 180° or very nearly as much (you may safely ignore very small discrepancies).*

*When you have carried this out until the angles* F *and* D *(and therefore the remaining angles* G *and* E*) truly stand upon the same circumference, now, in turn, an enquiry must be made into the other matter which was among our requirements, and that is, whether the centre of that circle* B *lies between* C *and* A *on the same line. For on this point it was said above that Ptolemy assumed it outright, and physical considerations demand that the slowest motion occur where the star is at its greatest distance from* A, *the sun, as at* H. *This can happen in no other way than if* A, B, *and* C *are on the same line. To find this out, let the known angles* GAD, DAE, *be taken as one, so that the angle* GAE *may be known, and in* GAE *from this angle and sides* GA, AE, *let the side* GE *be sought. Now in the triangle* GFE *the angle* GFE *stands upon the circumference. Therefore,* GBE, *the angle at the centre, is its double. But the angle* GFE *was previously found through its parts* GFA, AFE. *So, again, in the isosceles triangle* GBE *the angle* GBE *and the side* GE *are given. Consequently, the angles at the base will not be unknown, as well as* GB *the radius of the circle, in proportion to* AC, *the eccentricity taken at the beginning. And because* BG *and* BGE *are now had, and* AG *and* AGE *were had before, therefore, by subtracting* AGE *from* BGE *(or vice versa, as the case requires), the remainder is* AGB. *Next, in triangle* AGB, AG, BG, *and the included angle* AGB *are given. [Hence, the other sides and angles will not be unknown, and thus the angle* BAG *will be given].*[7] *If this differs from* CAG, *which was taken at the beginning, it shows that* B *itself, contrary to what should happen, does not fall upon the line* CA. *So again we shall pronounce the assumed magnitudes of angles* FCH *and* FAH *to*

[7] This sentence has been supplied by the translator to fill an obvious lacuna in the text.

95 *be false. But if we keep* FCH *fixed and change* FAH *we fall into another absurdity, namely, that the positions* D, E, F, G, *do not fall upon a circle (just as they did not above, before we had finally established the magnitude of* FAH*). Therefore, it is obvious that* FCH, *too, has to be changed. Let it be changed, then; that is, let another quantity be taken at will for the angle* FCH, *and keeping that constant, let the angle* FAH *be adjusted four, five, or six times, or until once again the four angles at* F *and* D *add up to two right angles. Then let an attempt be made at a second enquiry, using the triangles* GAE, GFE, GBE, *and* BGA, *to find* BAG, *in comparison with* CAG *as it has now most recently been established. Here you will again see whether you have departed farther from the truth, or have in truth come closer, and according to the qualities of excess or defect and ratios of the additions you will thence return to the beginning until* BAG *is seen to be equal to the value you had assumed during that trial for* CAG *or* HAG. *When you have arrived at this point, then finally, in triangle* BGA, *you will assign a round number (100,000) to* BG *as a standard, and will find, in the same ratio (through the mediation of the angles),* BA *the eccentricity of the eccentric and* CA *the eccentricity of the equant. Whence, by subtracting* BA, CB *remains. Then you will issue the pronouncement, concerning both the position of the apogee and the correction of the mean motion (which you had assumed in the final operation), that they are well established, at least as far as pertains to this form of hypothesis.*

If this wearisome method has filled you with loathing, it should more properly fill you with compassion for me, as I have gone through it at least seventy times at the expense of a great deal of time, and you will cease to wonder that the fifth year has now gone by since I took up Mars,[8] although the year 1603 was nearly all taken up by optical investigations.

There exist subtle geometers such as Vieta who will think it something great to show up the contrived nature of this method. For in this matter, Vieta did object to Ptolemy, Copernicus, and Regiomontanus.[9] Let them therefore go forth themselves and solve the figure geometrically, and they will be to me great Apolloes. For me it is enough to draw four or five conclusions from a single argument (which includes four observations and two hypotheses); that is, in

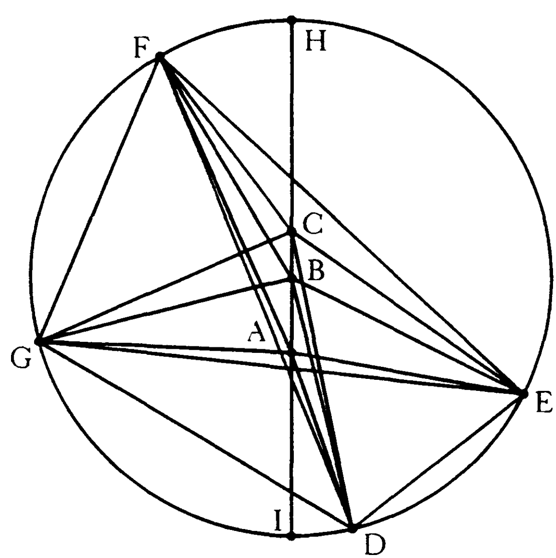
[8] Kepler bet Longomontanus at the time of taking up the problem that he would have the orbit within a week!

[9] For the relevant passage, Caspar refers the reader to *Francisci Vietae Apollonius Gallus seu exsuscitata Apollonii Pergaei* περι επαφων *geometria*, Paris 1600, p. 11.

getting from the labyrinth back to the highway, to show, instead of a geometrical light, a contrived thread, which nonetheless will lead you to the exit. If this method is difficult to grasp, the subject is much more difficult to investigate with no method at all.

There now follows an example of these instructions based upon the four proposed observations.

Because of precession, all positions are reduced to [the time of] the first observation. Here, the apparent longitude was 25° 43′ Virgo, the mean longitude $6^s$ 0° 47′ 40″, and the annual motion of the fixed stars is 51 seconds, as Brahe has demonstrated in the *Progymnasmata*. Therefore, from 1587 March 6 to 1591 June 8 is 4 years 3 months, to which corresponds a motion of precession of 3′ 37″. Therefore, we must set the apparent position in 1591 at 26° 39′ 23″ Sagittarius, and the mean longitude at $9^s$ 5° 40′ 18″. Similarly, from 1587 March 6 to 1593 August 25 is 6 years $5\frac{1}{2}$ months, to which belongs a motion of
96 precession of 5′ 30″. And so Mars is to be placed at 12° 10′ 30″ Pisces, with mean longitude $11^s$ 9° 49′ 34″. Finally, from 1587 March 6 to 1595 October 31 is nearly 8 years 7 months, to which corresponds a motion of 7′ 18″. And so Mars is to be placed at 17° 24′ 22″ Taurus, with mean longitude $1^s$ 7° 6′ 51″.



*Now first, we shall place the apogee or aphelion in 1587 at 28° 44′ 0″ Leo, and second, we shall suppose that the mean longitudes should be increased by 3′ 16″*[10] *so that the mean longitudes are 6ˢ 0° 50′ 56″, 9ˢ 5° 43′ 34″, 11ˢ 9° 52′ 50″, and 1ˢ 7° 10′ 7″.*

| | ° | ′ | ″ | |
|---|---|---|---|---|
| And because CH is | 28 | 44 | 0 | Leo |
| and CF | 0 | 50 | 56 | Libra |
| FCH will be | 32 | 6 | 56. | |
| Also, because CH is | 28 | 44 | 0 | Leo |
| and CD | 9 | 49 | 34 | Pisces[11] |
| HCD will be | 168 | 54 | 26; | |
| its supplement, | 11 | 5 | 34. | |
| Also, because CH is | 28 | 44 | 0 | Leo |
| and CG | 5 | 40 | 18 | Capricorn |
| HCG will be | 126 | 56 | 18; | |
| its supplement, | 53 | 3 | 42. | |
| Also, because CH is | 28 | 44 | 0 | Leo |
| and CE | 7 | 6 | 51 | Taurus |
| HCE will be | 111 | 37 | 9; | |
| its supplement, | 68 | 22 | 51. | |

[10] Kepler makes this adjustment with the benefit of hindsight. He was aware that a small adjustment in the mean longitudes requires a large change in the position of the aphelion. Thus, although his iterative process could begin with nearly any arbitrary aphelion and mean longitudes, a large error, especially in the mean longitudes, could result in more iterations.

One might wonder why the mean longitudes need to be changed, if they are merely arbitrary and relative indications of the time between observations, expressed in terms of the planet's periodic time. The answer is that the mean longitudes also represent actual angles about the equant point C, measured from the vernal equinox. This is convenient in that the mean longitude and the observed longitude are the same when the planet is at either apsis.

[11] It is striking that, immediately after giving the corrected mean longitudes, Kepler puts CD, CG, and CE at the previous, uncorrected mean longitudes, and then, in finding the angles of the equations, returns to the corrected forms. Such errors make it seem hardly remarkable that it took him many trials to establish a usable line of apsides and eccentricity.

It should also be remarked that there are many errors in the following computations. These have not been noted in the translation because the chief value of Kepler's example lies not so much in the numbers he uses as in the procedures he demonstrates. Repetition of the procedure will, of course, yield different values (the translator, for example, would put the aphelion at 28° 50′ 37″ Leo, having increased the mean longitudes 3′ 26″). Those who wish to find errors will find many of them noted by Caspar in KGW 3 pp. 157–167.

For the angles of the equations.

| | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| CF | 0° | 50′ | 56″ | Libra | CG | 5° | 43′ | 34″ | Capricorn |
| AF | 25 | 43 | 0 | Virgo | AG | 26 | 39 | 23 | Sagittarius |
| CFA | 5 | 7 | 56 | | CGA | 9 | 4 | 11 | |

| | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| CD | 9° | 52′ | 50″ | Pisces | CE | 7° | 10′ | 7″ | Taurus |
| AD | 12 | 10 | 30 | Pisces | AE | 17 | 24 | 22 | Taurus |
| CDA | 2 | 17 | 40 | | CEA | 10 | 14 | 15 | |

For the lines from A

*Let* AC *be taken as the standard, with magnitude 10,000. Now as the [sines of the] angles of the equations are to* AC, *so are the [sines of the] angles at* C *to the lines from* A. *Therefore, the sines of the angles at* C, *multiplied by 10,000, are to be divided by the sines of the angles of the equations.*

| | | |
|---|---|---|
| *Sin.* FCH | 53163 | AF |
| *Sin.* CFA | 8945 | |
| | 44725 | 5 |
| | 84380 | |
| | 80505 | 9 |
| | 3875 | |
| | 3578 | 4 |
| | 297 | |
| | 268 | 3 |
| | 29 | 3 |

| | | |
|---|---|---|
| *Sin.* GCH | 79928 | AG |
| *Sin.* CGA | 15764 | |
| | 78820 | 50 |
| | 11080 | |
| | 11035 | 70 |
| | 45 | 3 |

| | | |
|---|---|---|
| *Sin.* DCH | 19240 | AD |
| *Sin.* CDA | 4004 | |
| | 16016 | 4 |
| | 3224 | |
| | 3203 | 80 |
| | 208 | |
| | 200 | 5 |
| | 8 | 2 |

| | | |
|---|---|---|
| *Sin.* ECH | 92966 | AE |
| *Sin.* CEA | 17773 | |
| | 88875 | 5 |
| | 40910 | |
| | 35546 | 2 |
| | 5364 | |
| | 5333 | 30 |
| | 31 | 2 |

97 *For the angles at A*

| | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| AF | 25° | 43′ | 0″ | Virgo | AG | 26° | 39′ | 23″ | Sagittarius |
| AG | 26 | 39 | 23 | Sagittarius | AD | 12 | 10 | 30 | Pisces |
| FAG | 90 | 56 | 23 | | GAD | 75 | 31 | 7 | |
| Supplement: | 89 | 3 | 37 | | | 104 | 28 | 53 | |

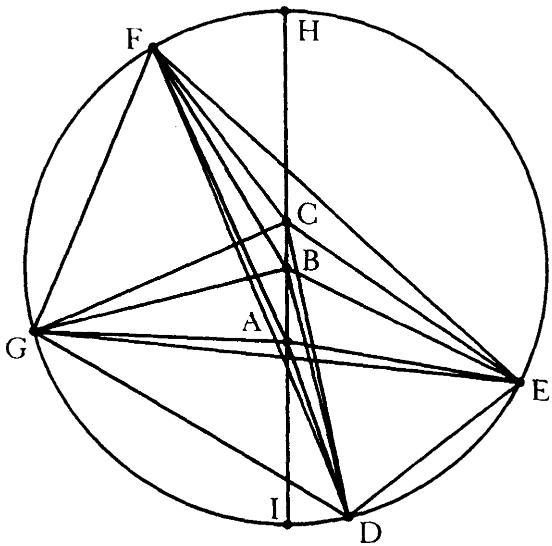
| | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| AD | 12° | 10′ | 30″ | Pisces | AE | 17° | 24′ | 22″ | Taurus |
| AE | 17 | 24 | 22 | Taurus | AF | 25 | 43 | 0 | Virgo |
| DAE | 65 | 13 | 52 | | EAF | 128 | 18 | 38 | |
| Supplement: | 114 | 46 | 8 | | | 51 | 41 | 22* | |

For the angles at F and D

*The angles* AFG, AFE, ADG, ADE, *are approximately the halves of the supplements of the angles at* A[12]. *Those at* F, *however, are smaller, because the lines* AG *(50,703) and* AE *(52,302) have been found to be smaller than* AF *(59,433); and those at* D *are greater, because those lines* AG *and* AE *are longer than* AD *(48,052). And since those four angles about* A *are equal to four right angles, the sum of their supplements consequently will also equal four right angles, because four semicircles are equal to eight right angles. Therefore, half of the sum of the supplements is equal to two right angles, which is what we wish the sum of* GFE *and* GDE *to be. Consequently, the angles at* D *ought to exceed the halves of their supplements by the same amount that those at* F *fall short of theirs. But the tangents of the differences of the angles at the bases in this kind of triangle may be found if you divide the differences of the sides by the sums of the sides, and multiply the quotient by the tangents of the halves of the supplements.*[13] *Therefore, if the [sum of the] two differences of the angles at* F *equal the sum [of the two differences] at* D, *the sum of the angles at* F *and* D *will equal two right angles.*[14]

| | FAG | | GAD | | DAE | | EAF | |
|---|---|---|---|---|---|---|---|---|
| *Halves* | 44° 31′ 48″ | | 52° 14′ 27″ | | 57° 23′ 4″ | | 25° 50′ 41″ * | |
| *Tangents* | 98373 | | 129093 | | 156271 | | 48438 | |
| | AF 59433 | | AG 50703 | | AD 48052 | | AE 52302 | |
| | AG 50703 | | AD 48052 | | AE 52302 | | AF 59433 | |
| *Differences* | 8730 | | 2651 | | 4250 | | 7131 | |
| *Sums* | 110136 | | 98755 | | 100354 | | 111735 | |
| | 770952 | 7 | 197510 | 2 | 401416 | 4 | 670410 | 6 |
| | 102048 | | 67590 | | 23584 | | 42690 | |
| | 99123 | 9 | 59253 | 6 | 20071 | 2 | 33520 | 3 |
| | 2925 | | 8337 | | 3513 | | 9170 | |
| | 2203 | 2 | 7900 | 8 | 3016 | 3 | 8938 | 8 |
| | 722 | 6 | 437 | 4 | 497 | 5 | 232 | 2 |
| *Quotients* | 7926 | | 2684 | | 4235 | | 6382 | |
| *Tangents* | 98373 | | 129093 | | 156271 | | 48438 | |
| | 6886 | 11 | 2581 | 86 | 6250 | 84 | 2906 | 86 |
| | 885 | 33 | 774 | 54 | 312 | 54 | 145 | 34 |
| | 19 | 66 | 103 | 20 | 46 | 86 | 38 | 72 |
| | 5 | 88 | 5 | 16 | 7 | 81 | | 96 |
| *Tangents* | 7797 | | 3465 | | 6618 | | 3142 | |
| *Differences* | F 4° 27′ 30″ | | D 1° 59′ 4″ | | D 3° 47′ 10″ | | F 1° 47′ 59″ | |
| | | | 3 47 10 | | | | 4 27 30 | |
| | | | *Sum of the two at* D 5 46 14 | | | | *Sum of the two at* F 6 15 29 | |

98 *From this it appears, therefore, that the sum of* F *and* D *is less than two right angles, because the difference to be subtracted exceeds that to be added.*




*The amount wanting is 24′ 15″. Now I know from many repetitions of this task that by adding 3′ 20″ to the aphelion, the sums come together. This I shall prove.*

*The angles of the equations and their sines will stay the same, as well as the tangents of the halves of the supplements of the angles at* A.

| But HCF 32° 3′ 36″ | | GCI 53° 7′ 2″ | | DCI 11° 2′ 14″ | | ECI 68° 19′ 31″ | |
|---|---|---|---|---|---|---|---|
| *Sines* 53081 | AF | 79986 | AG | 19145 | AD | 92929 | AE |
| *Sin.* CFA 8945 | | *Si.* CGA 15764 | | *Si.* CDA 4004 | | *Si.* CEA 17773 | |
| 44725 | 5 | 78820 | 5 | 16016 | 4 | 88875 | 5 |
| 83560 | | 11660 | 0 | 3129 | | 40540 | |
| 80505 | 9 | 11035 | 7 | 28028 | 7 | 35546 | 2 |
| 3055 | | 625 | | 3262 | | 4994 | |
| 2683 | 3 | 630 | 4 | 2803 | 8 | 3555 | 2 |
| 372 | | 5 | 0 | 459 | 1 | 1439 | |
| 358 | 4 | | | 10 | 5 | 1244 | 8 |
| 14 | ½ | | | | | 195 | 1 |
| | | | | | | 8 | |

[12] Because the triangles are nearly isosceles.

[13] This is, of course, the 'law of tangents', which, stated algebraically, is:

$$\frac{(a-b)}{(a+b)} \tan \tfrac{1}{2}(180 - C) = \tan \tfrac{1}{2}(A - B).$$

Kepler neglected to state that it is the tangent of *half* the difference that is found in this way.

[14] That is,

(GDA − DGA) + (EDA − DEA) should equal (FGA − GFA) + (FEA − EFA)

| AF 59341 / AG 50740 | |
|---|---|
| 8601 | |
| 110081 | |
| 770567 | 7 |
| 89533 | |
| 88065 | 8 |
| 1468 | |
| 1101 | 1 |
| 367 | |
| 330 | 3 |
| 37 | 3 |

| AG 50740 / AD 47815 | |
|---|---|
| 2925 | |
| 98555 | |
| 197110 | 2 |
| 95390 | |
| 88700 | 9 |
| 6690 | |
| 5913 | 6 |
| 777 | 8 |

| AD 47815 / AE 52281 | |
|---|---|
| 4466 | |
| 100096 | 4 |
| | 4 |
| | 6 |
| | 2 |

| AE 52281 / AF 59341 | |
|---|---|
| 7060 | |
| 111622 | |
| 669732 | 6 |
| 36268 | |
| 33486 | 3 |
| 2782 | |
| 2232 | 2 |
| 550 | 5 |

| *Tangents* 98373 / 7813 | | 129093 / 2968 | | 156271 / 4462 | | 48438 / 6325 | |
|---|---|---|---|---|---|---|---|
| 6886 | 11 | 2581 | 86 | 6250 | 84 | 2906 | 28 |
| 786 | 96 | 1161 | 81 | 625 | 08 | 145 | 29 |
| 9 | 83 | 77 | 40 | 93 | 72 | 9 | 68 |
| 2 | 94 | 10 | 32 | 3 | 12 | 2 | 40 |
| 7686 | | 3831 | | 6973 | | 3064 | |
| F 4° 23′ 41″ | | D 2° 11′ 37″ | | D 3° 59′ 10″ | | F 1° 45′ 18″ | |
| | | | | 2 11 37 | | 4 23 41 | |
| | | | | *Sum at* D 6° 10′ 47″ | | *Sum at* F 6° 8′ 59″ | |

99 *Here the sums differ by no more than 1′ 48″. So we have now moved the apogee too far forward, and must move it back by another 12″. But not much care is needed with such a small difference. We shall make it up 'from the equal and the good' [that is, by interpolation], in order to be able to carry on with our method. Before, when we had erred in defect by 29′ 15″, the sum of the differences at* F *and* D *was 12° 1′ 44″. Now, when we have erred in excess by 1′ 48″, this sum comes out to be 12° 19′ 46″. And so, since 31′ produces 18′ in the sum of the differences, therefore, $1\frac{4}{5}'$ produces about 1′, so that the exact sum is 12° 18′ 44″, whose half, 6° 9′ 22″, is the sum at either* F *or* D.

For the Triangles GFE, GBE.

| | |
|---|---|
| *In* FAG *half the supplement was* | 44° 31′ 48″. |
| *In* FAE | 25 50 41. |
| *The sum* | 70 22 29. |
| *From this subtract the sum of the differences* | 6 9 22. |
| *The remainder is* GFE | 64 13 7. |
| *Therefore, its double, at* GBE, *will be* | 128 26 14, |
| *The supplement of which is* | 51 33 46; |
| *Its half,* | 25 46 53. |

| | | | |
|---|---|---|---|
| *Also, in the first trial,* GA *was* | 50703; | *and* AE, | 52302; |
| *In the second,* | 50740; | | 52281; |
| *Difference* | 37 | | 21. |
| *Therefore it is now* | 50739 | | 52282. |

| | | | |
|---|---|---|---|
| *And because* GAD *is* | 75° | 31′ | 7″, |
| *and* DAE | 65 | 13 | 52, |
| *Therefore,* GAE *is* | 140 | 44 | 59; |
| *its supplement,* | 39 | 15 | 1. |

*Next,* GE *is sought, from the sides* GA, AE, *and the angle* GAE.

| | | |
|---|---|---|
| GA | 50739 | |
| AE | 52282 | |
| *Difference* | 154300 | |
| *Sum* | 103021 | 1 |
| | 51279 | |
| | 41208 | 4 |
| | 10071 | |
| | 9272 | 9 |
| | 799 | 7 |

*Half the supplement of* GAE 19° 37′ 30″

| | | | | |
|---|---|---|---|---|
| *Tangent* | 35658 | | | |
| | 1497 | | | |
| | 356 | 58 | | |
| | 142 | 63 | | |
| | 32 | 08 | | |
| | 2 | 49 | | |
| | 534 | 0° | 18′ | 21″ |
| *Half the supplement* | | 19 | 37 | 30 |
| AGE | | 19 | 55 | 51 |

*As the sine of* AGE *is to* AE, *so is the sine of* GAE *to* GE.[15]

| | | | | |
|---|---|---|---|---|
| *Sine* GAE | 63271 | | *3307935 | GE |
| AE | 52282 | *Sine* AGE | 34088 | |
| | 3163550 | | 306792 | 9 |
| | 126542 | | 240015 | |
| | 12654 | | 238616 | 70 |
| | 5062 | | 1399 | |
| | 127 | | 1363 | 4 |
| | 3307935* | | 36 | 1 |

[15] When Kepler marks a number with an asterisk, he is alerting the reader that the number is carried over to or from somewhere else nearby.

100 *Therefore, in GBE, as GBE is to GE, BGE is to BE.*

| | | | | |
|---|---|---|---|---|
| 43494 *Sine* GBE. | | 4218701 | | |
| 97041 GE. | | 78327 | | *Sine* GBE |
| 3912460 | | 391635 | 5 | |
| 304458 | | 302351 | | |
| 1740 | | 234981 | 3 | |
| 43 | | 67370 | | |
| 4218701 | | 62662 | 8 | |
| | | 4708 | | |
| | | 4699 | 6 | |
| | | 9 | 0 | |

| | | |
|---|---|---|
| BG | 53860 | |
| AG | 50739 | |
| *Difference* | 312100 | |
| *Sum* | 104599 | |
| | 209198 | 2 |
| | 102902 | |
| | 94140 | 9 |
| | 8762 | |
| | 8368 | 8 |
| | 394 | 4* |

| | | | |
|---|---|---|---|
| *And because* AGE *was* | 19° | 55′ | 51″ |
| *But now* BGE *is* | 25 | 46 | 53 |
| BGA *will be* | 5 | 51 | 2 |
| *Supplement* | 174 | 8 | 58 |
| *Half* | 87 | 4 | 29 |

| | | | | | |
|---|---|---|---|---|---|
| *Tangent* | 1957200 | | | | |
| | 2984 * | | | | |
| | 39144 | | | | |
| | 17615 | | | | |
| | 1564 | | | | |
| | 78 | | | | |
| | 58401 | 30° | 17′ | 8″ | |
| | | 87 | 4 | 29 | |
| | | 117 | 21 | 37 | BAG. |

*In the last trial we had moved the aphelion forward a total of 3′ 8″.*

| | | | | |
|---|---|---|---|---|
| *Therefore, because* AH *was* | 28° | 47′ | 8″ | Leo, |
| And AG *was* | 26 | 39 | 23 | *Sagittarius,* |
| HAG *or* CAG *was* | 117 | 52 | 15 | |

*Therefore* B *is a little off the line* CA *on the side of* G, *since* CAG *is greater than* BAG *by 30′ 38″. But I know from many trials that by the addition of one half minute to the mean longitude,* B *is made to lie upon the line* CA. *But at the same time, to keep the quadrangle on the circle, the aphelion has to be moved forward 2′. It would be worthwhile seeing this through, at the same time demonstrating the eccentricity. So since 30″ are added to* CF *and its counterparts, and 2′ to* CH, HCF *is diminished by 1′ 30″. Therefore,*

HCF is 32° 2′ 6″; GCI 53° 8′ 32″; DCI 11° 0′ 44″; and ECI 68° 18′ 1″.

*But the angles of the equations are increased and decreased by 30″.*

*Therefore,*

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| CFA 5° 8′ 26″ | | CGA 9° 4′ 32″ | | CDA 2° 17′ 10″ | | CEA 10° 13′ 46″ | |
| *Sines* HCF 53044 | AF | 80012 | AG | 19102 | AD | 92913 | AE |
| *Sines* CFA 8960 | | 15758 | | 3989 | | 17758 | |
| 44800 | 5 | 78790 | 50 | 15956 | 4 | 88790 | 5 |
| 8244 | | 12220 | | 3146 | | 4123 | |
| 8064 | 9 | 11030 | 7 | 27923 | 7 | 35516 | 2 |
| 180 | | 1190 | | 3537 | | 5714 | |
| 179 | 2 | 1103 | 7 | 3191 | 8 | 5327 | 3 |
| 01 | 0 | 87 | 5 | 346 | | 387 | |
| | 1 | | | 319 | 8 | 355 | 2 |
| | | | | 27 | 7 | 32 | 2 |

| | | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| **101** | AF 59201 | | AG 50775 | | AD 47887 | | AE 52322 | |
| | AG 50775 | | AD 47887 | | AE 52322 | | AF 59201 | |
| | 8426 | | 2888 | | 4435 | | 6879 | |
| | 109976 | | 98662 | | 100209 | | 111523 | |
| | 769832 | 7 | 197324 | 2 | 400836 | 4 | 669138 | 6 |
| | 72768 | | 91476 | | 42664 | | 18762 | |
| | 65986 | 6 | 88796 | 9 | 40084 | 4 | 11152 | 1 |
| | 6782 | | 2680 | | 2580 | 2 | 7610 | |
| | 6599 | 6 | 1973 | 2 | | 6 | 6691 | 6 |
| | 183 | | 707 | | | | 919 | |
| | 110 | 1 | 690 | 7 | | | 892 | 8 |

*The tangents remain:*

| | | | |
|---|---|---|---|
| 98373 | 129093 | 156272 | 48438 |
| 7661 | 2927 | 4426 | 6168 |
| 6886 11 | 2581 86 | 6250 84 | 2906 28 |
| 590 22 | 1161 81 | 625 08 | 48 44 |
| 59 02 | 25 82 | 31 25 | 29 06 |
| 98 | 9 03 | 9 36 | 3 87 |
| 7536 | 3779 | 6917 | 2988 |
| 4° 18′ 36″ | 2° 9′ 52″ | 3° 57′ 24″ | 1° 42′ 41″ |
| | 3 57 24 | | 4 18 36 |

*The one sum 6 7 16.* *The other sum 6 1 17.*

*We have six minutes left over, which are removed by moving the aphelion back 38″. So, since it was at 28° 49′ 8″ Leo, it will now be at 28° 48′ 30″ Leo.*

## Test

102

| HCF 32° 2′ 44″ | | GCI 53° 7′ 54″ | | DCI 11° 1′ 22″ | | ECI 68° 17′ 23″ | |
|---|---|---|---|---|---|---|---|
| 53060 | | 80001 | | 19120 | | 92905 | |
| 8960 | | 15758 | | 3989 | | 17758 | |
| 4480 | 5 | 78790 | 50 | 15956 | 4 | 88790 | 5 |
| 8260 | | 12110 | | 3164 | | 4115 | |
| 8064 | 9 | 11031 | 7 | 27923 | 7 | 35516 | 2 |
| 196 | | 1080 | | 3717 | | 5634 | |
| 179 | 2 | 945 | 6 | 3591 | 9 | 5327 | 3 |
| 170 | 1 | 135 | 9 | 126 | | 307 | |
| 79 | 9 | | | 120 | 3 | 178 | 1 |
| | | | | 6 | 1 | 129 | 7 |

The numbers have the same labels as immediately above.

| | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|
| 59219 | | | 50769 | | | 47931 | | | 52317 | | |
| 50769 | | | 47931 | | | 52317 | | | 59219 | | |
| 8450 | | | 2838 | | | 4386 | | | 6902 | | |
| 109988 | | | 98700 | | | 100248 | | | 111536 | | |
| 769916 | 7 *was* | 7 | 1974 | 2 *was* | 2 | 400992 | 4 *was* | 4 | 669216 | 6 *was* | 6 |
| 75084 | | | 864 | | | 37608 | | | 20984 | | |
| 65993 | 6 | 6 | 7896 | 8 | 9 | 30074 | 3 | 4 | 11154 | 1 | 1 |
| 9091 | | | 744 | | | 7534 | | | 9830 | | |
| 8799 | 8 | 6 | 691 | 7 | 2 | 7017 | 7 | 2 | 8922 | 8 | 6 |
| 292 | 3 | 2 | 53 | 5 | 7 | 517 | 5 | 6 | 908 | 8 | 8 |
| | *Diff.* 21. | | | *Diff.* 52. | | | *Diff.* 51. | | | *Diff.* 20. | |

| | | | | |
|---|---|---|---|---|
| | 98373 | 129093 | 156271 | |
| | 21 | 52 | 51 | 48438 |
| | 98373 | 2 \| 58168 | 1 \| 56271 | 20 |
| | 19 \| 6746 | 64 \| 5465 | 78 \| 1355 | 9 \| 68760 |
| *Increase in tangent* | 21 | 67 | 80 | 10 |
| *Increase in arc* | 41″ | 2′ 14″ | 2′ 39″ | 19″ |
| | | 2 39 | | 41 |
| | | *was* 6° 7′ 16 | *Note the* | *was* 6° 1′ 17″ |
| | | *now* 6 2 23 | *equality.* | *now* 6 2 17 |

103 *And so, with the quadrangle contained in the circle, it is once more enquired whether* B *lies upon the line* CA. *And from the sum of 70° 22′ 29″ established above, subtract the difference of 6° 2′ 20″ just found. The remainder is*

| | | | |
|---|---|---|---|
| GFE | 64° | 20′ | 19″ |
| *Its double,* GBE | 128 | 40 | 18 |
| *Supplement* | 51 | 19 | 42 |
| BGE | 25 | 39 | 51 |
| *In the last trial,* GA *was* | 50769 | | |
| AE *was* | 52317 | | |

| | |
|---|---|
| 154800 | |
| 103086 | 1 |
| 51714 | |
| 51543 | 50 |
| 171 | 2** |

*The tangent of half the supplement of* GAE *remains* 35658

| | | | |
|---|---|---|---|
| 1502 ** | | | |
| 178 | 29 | | |
| | 71 | | |
| 535½ | | 18′ | 24″ |
| | 19° | 37 | 30 |
| AGE | 19 | 55 | 54 |

| | | |
|---|---|---|
| *Sin* GAE 63271 | 3310148* | |
| AE 52317 | 34089 | *Sin* AGE |
| 3163550 | 306801 | 9 |
| 126542 | 242138 | |
| 18981 | 238623 | 7 |
| 633 | 3515 | |
| 442 | 3409 | 10 |
| 3310148* | 106 | 3 GE. |

| | | |
|---|---|---|
| BG 53866 | | |
| GA 50769 | | |
| 309700 | | |
| 104635 | 2 | |
| 209270 | | |
| 100430 | 9 | |
| 94172 | 6 | |
| 6258 | 0 | |

| | | | |
|---|---|---|---|
| AGE | 19° | 55′ | 54″ |
| BGE | 25 | 39 | 51 |
| BGA | 5 | 43 | 57 |
| *Suppl.* | 174 | 16 | 3 |
| *Half* | 87 | 8 | 1½ |
| *Tangent* | 1997100 | | |
| | 2960 | | |
| | 1198 | 26000 | |
| | 17973 | 9 | |
| | 39942 | | |
| | 59114 | | |

| | | | | |
|---|---|---|---|---|
| *Aphelion* | 28° | 48′ | 30″ | *Leo* |
| AG | 26 | 39 | 23 | *Sagit.* |
| | 117 | 50 | 43 | CAG |

| | | | |
|---|---|---|---|
| 30° | 35′ | 22″ | |
| 87 | 8 | 1 | |
| 117 | 43 | 23 | BAG |
| 62 | 16 | 37 | |

B *still lies 7′ 20″ from the line* CA *towards* G.

*Whence we see that because before, by adding 30″ to the mean motion and 82″ to the aphelion, we moved forward 23′ 18″, we will take up the remaining 7′ 20″ by adding 9″ to the mean motion and 25″ to the aphelion. Therefore, the total addition to Tycho's longitude is 3′ 55″,*[16] *and the aphelion is placed at 28° 48′ 55″ Leo.*

*And with such a small error, one incurs no disadvantage by finding* BA *from the given angles and sides of triangle* CAG *as if* B *lay exactly on the line* CA.

| | | |
|---|---|---|
| 104 | *Sine* BGA 998800000 | |
| | *Sine* BAG 8852 | 1 |
| | 11360 | |
| | 8852 | 1 |
| | 2508 | 2 |
| | 17704 | |
| | 7376 | 8 |
| | 7082 | |
| | 294 | 3 |

[16] That is, he began by adding 3′ 16″ to the mean longitudes, then added another 30″, and finally, 9″, the total being 3′ 55″.

*Therefore* BA *is 11,283 where* BG *is 100,000.*
*But as* BG, *53,866, is to 100,000, so is 100,000 to* AC.

| | |
|---|---|
| BG 53866 | 1 |
| 46134 | |
| 430928 | 8 |
| 30412 | 5 |
| 26933 | |
| 3479 | 6 |
| 3232 | 4 |

*Therefore,* AC *is 18564*
*And* BC *7281 where* BG *is 100,000.*

*But to exclude all error, let us interpolate.*

| | | | | |
|---|---|---|---|---|
| *At first,* BG *was* | 53860 | AG 50739 | BGA 5° 51′ 2″ | BAG 62° 38′ 23″ |
| *Now it is* | 53866 | 50769 | 5 43 57 | 62 16 37 |
| *Difference* | 6 | 30 | 7 5 | 21 46 |
| *Further, in part 3,* BAG *is* | | | | 62 8 37 |
| *To be removed* | 2 | 11 | 2 25 | 5 41 32 |
| BG *corrected* | 53868 | AG 50780 | BGA 5° 41′ 32″ | 67 50 9 |

| | |
|---|---|
| 100000 | |
| BG 53868 | 1 |
| 46132 | |
| 430428 | 8 |
| 30392 | |
| 26933 | 5 |
| 3459 | |
| 3232 | 6 |
| 227 | 4 |

| | |
|---|---|
| *Sine* BGA 99190 | |
| *Sine* BAG 88414 | 1 |
| 11776 | |
| 8841 | 1 |
| 2935 | |
| 2652 | 3 |
| 283 | |
| 265 | 3 |
| 18 | 2 |

So the whole eccentricity remains 18.564
but that of the eccentric 11,332
and of the equant 7,232

In the Copernican and Tychonic form, the diameter of the small epicycle would be 3616, and of the greater, 14,948. Or, following what was said at the end of the fourth chapter, the tangent may be used instead of the sine, in this manner.

105 *The maximum equation shall be investigated at the ninetieth degree. Let* HCG *be 90.* BC, *the sine of the angle* BGC, *will be 4° 8′ 51″. And* GBC *will be 85° 51′ 9″. And* GC *99,738. But in the Copernican form, with* C *at the centre of the concentric,* GC *will be 100,000. Therefore, in order that* CGA, *the angle of the equation, remain unchanged, the same 18,564 is to be increased in the same ratio for Tycho and Copernicus:*

| | |
|---|---|
| 1856400000 | |
| 99738 | 1 |
| 85902 | |
| 79790 | 8 |
| 6112 | |
| 5984 | 6 |
| 128 | 1 |
| 99 | 3 |

*The composite Copernican-Tychonic eccentricity. And in the tangents, this shows 10° 32′ 38″ as the common angle of the equation at 90° of anomaly.*

Therefore the corrected diameter of the smaller epicycle is 3,628.
of the greater 14,988.

Compare all this with ch. 5, where I transposed the Tychonic rendition from the mean to the apparent motion of the sun, and see how slight the difference is.

So this is the method by which the hypothesis of the first inequality was investigated using four acronychal positions of Mars. In this, with Ptolemy, I have supposed that all positions of the planet throughout the heavens are so arranged as to be on the circumference of one circle; that the planet moves most slowly where it is at its greatest distance from the centre of the earth (according to Ptolemy) or of the sun (according to Tycho and Copernicus); and that the point about which this retardation is measured is fixed. Everything else I have demonstrated, although the demonstration is indirect in form. But whether the things I assumed in the demonstration are in fact so, or the opposite, will become clear in what follows.

106 I shall now test the remaining eight positions against this hypothesis, for the sake of consensus, But in order that the test be universal and legitimate, I shall also throw in the motion of the apogee. So I shall take this up first.

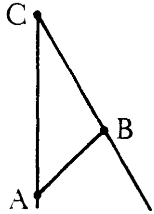
# 17

## A preliminary investigation of the motion of the apogee and nodes

This investigation will be of the same degree of certainty as the observations (or rather, the Ptolemaic tradition). If that theorist had not existed, we would know even less today about those very slow motions. For besides him there is no one at all to be found, from the earliest records of civilisation to the present, who could help us here.

We lay down here those suppositions found in Ptolemy, which are not in all respects perfectly certain. First, that the fixed stars have remained exactly in the zodiacal positions in which Ptolemy placed them (Ptolemy book 7). Second, that Ptolemy's figure for the sun's eccentricity was correct: 4153, where the semidiameter of the orbit is 100,000 (Ptolemy book 3 ch. 4). Third, that the sun's apogee was fixed at $5\frac{1}{2}$° Gemini (in the same chapter). Fourth, that Mars's apogee (when its motion is referred to the sun's mean motion) was found to be at $25\frac{1}{2}$° Cancer (Ptol. book 10 ch. 7). Fifth, that the eccentricity of Mars was 20,000 where its semidiameter is 100,000 (in the same chapter). Sixth, that the ratio of the epicycle (in Ptolemy) or of the annual orb (in Tycho and Copernicus) to the orb of Mars was as 100,000 to 151,900. Hence, where the semidiameter of the sun's orb (or the earth's) is 100,000, Mars's eccentricity will be 30,380 (Ptolemy book 10 ch. 8).

We shall proceed as in chapter 5. *Let* A *be the point about which the earth's orb is described,* C *Mars's equalizing point,* B *the centre of the sun's orb. And because* AB *is at* $5\frac{1}{2}$° *Gemini, while* AC *is at* $25\frac{1}{2}$° *Cancer,* CAB *is 50°. By supposition,* AB *is 4153, while* AC *is 30,380 of the same units. Therefore, since two sides and the included angle are*




*given, the angle* CBA *is given, and is 123° 27′. And because* BA *is directed towards 5½° Sagittarius, the direction of* BC *(by subtraction of 123° 27′) will be about 2° 3′ Leo, for Ptolemy's time. Also at that time, the eccentricity of the equant, after transposition to the sun's true motion, was 18,353. I discovered this above by transposition of the Tychonic value, 18,342, with one change: for the size of Mars's orbit, instead of 151,386 I took 152,500, which is nearer the truth. But this is tangential to the matter at hand,* to which we return.

107 *On the motion of the aphelia*

Because the precession of the equinoxes was exceedingly high around Ptolemy's time, while before and after there remains not the least suspicion of any such thing, I shall separate this out, and relate the position of the apsis to the fixed stars. At that time, Cor Leonis was at 2° 30′ Leo. The apsis or aphelion of Mars therefore preceded this star by 27′, in about A.D. 140. In our times, in 1587, Tycho Brahe found this star at 24° 5′ Leo. Since the aphelion progressed to 28° 49′ Leo, it was 4° 44′ east of Cor Leonis. If you add the above mentioned 27′ to this, the sum (5° 11′) is the motion over the 1447 years between 140 and 1587. Therefore, the annual motion is very nearly 13″: 6′ 29″ every 30 years. If, in turn, you add to this the Tychonic value for the motion of the fixed stars or of precession, which is quite uniform and is the same for all times (Ptolemy's alone excepted), namely, 25′ 30″ in 30 years, the sum comes to 31′ 59″. Therefore, the annual motion of Mars's aphelion with respect to the equinox is 1′ 4″ in our time.

*On the motion of the nodes*

Although it is not really necessary, we shall consider this subject here because it is related to the motion of the aphelia. And because Ptolemy says in book 13 ch. 1 that Mars's northern limit is 'near the

end of Cancer, and almost at its apogee', it would therefore have been at 29 Cancer, that is, $3\frac{1}{2}°$ before Cor Leonis. Despite this, in book 3 ch. 6 Ptolemy put the northern limit exactly at the position of the apogee ($25\frac{1}{2}°$ Cancer) because it made calculations easier. And today it is at about 16° 20′ Leo, about 7° 45′ before Cor Leonis. By subtraction of 3° 30′, the northern limit, and consequently the nodes, are seen to have retrogressed 4° 15′ from Cor Leonis. This accords well with the motions of the moon, whose apogee likewise has a progressive motion with respect to the fixed stars, while its nodes retrogress. So the annual westward motion is 10″ 34‴, or 5′ 17″ in 30 years. Subtract this from the motion of precession, 25′ 30″. The remainder is 20′ 13″. And this is the present motion of Mars's nodes with respect to the equinoctial point in 30 years, likewise eastward.

Table of the motion of the aphelia and nodes

| Years | Aphelion | | Limits & nodes | |
|---|---|---|---|---|
| | M | S | M | S |
| 1 | 1 | 4 | 0 | 40 |
| 2 | 2 | 8 | 1 | 21 |
| 3 | 3 | 12 | 2 | 1 |
| 4 | 4 | 16 | 2 | 42 |
| 5 | 5 | 20 | 3 | 22 |
| 6 | 6 | 2 | 4 | 3 |
| 7 | 7 | 28 | 4 | 43 |
| 8 | 8 | 32 | 5 | 24 |
| 9 | 9 | 36 | 6 | 4 |
| 10 | 10 | 40 | 6 | 45 |
| 11 | 11 | 44 | 7 | 25 |
| 12 | 12 | 47 | 8 | 6 |
| 13 | 13 | 51 | 8 | 46 |
| 14 | 14 | 55 | 9 | 27 |
| 15 | 15 | 59 | 10 | 7 |
| 16 | 17 | 3 | 10 | 48 |
| 17 | 18 | 7 | 11 | 28 |
| 18 | 19 | 11 | 12 | 9 |
| 19 | 20 | 15 | 12 | 49 |
| 20 | 21 | 19 | 13 | 30 |
| 21 | 22 | 23 | 14 | 10 |
| 22 | 23 | 27 | 14 | 50 |
| 23 | 24 | 31 | 15 | 31 |
| 24 | 25 | 35 | 16 | 11 |
| 25 | 26 | 39 | 16 | 52 |
| 26 | 27 | 43 | 17 | 32 |
| 27 | 28 | 47 | 18 | 12 |
| 28 | 29 | 51 | 18 | 53 |
| 29 | 30 | 55 | 19 | 33 |
| 30 | 31 | 59 | 20 | 13 |

| | Aphelion | Limits & nodes |
|---|---|---|
| Months | Seconds | Seconds |
| 1 | 5 | 3 |
| 2 | 11 | 7 |
| 3 | 16 | 10 |
| 4 | 21 | 13 |
| 5 | 27 | 17 |
| 6 | 32 | 20 |
| 7 | 37 | 23 |
| 8 | 43 | 27 |
| 9 | 48 | 30 |
| 10 | 54 | 33 |
| 11 | 59 | 37 |
| 12 | 14 | 40 |

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# 18

## Examination of the twelve acronychal positions using the hypothesis we have found

I shall use that form[1] of calculation which I explained above in chapter 4, because it is easier to use. Also, it is indubitable that not half a minute (actually somewhat less) would be gained or lost by using the Copernican or Tychonic form, as I noted in the same place.

You see, then, O studious reader, that the hypothesis found by the method developed above, is able in its calculations not only to account, in turn, for the four observations upon which it was founded, but also to comprehend the other observations within two minutes – a magnitude which this star, when in its acronychal position, always fills and even exceeds[2]. This shows that if anyone
110 were to repeat the above method, taking in turn various sets of four observations, the same eccentricity with the same division, an identical aphelion, and very nearly the same mean motion, would always result. I therefore proclaim that the acronychal positions displayed by this calculation are as certain as the observations made with the Tychonic sextants can be. As I have said before, these observations are subject to some degree of uncertainty (at least two minutes), owing to Mars's small but appreciable diameter, and refraction and parallax, which are not yet known with complete certainty[3].

Finally, you see how nothing prevents the transposition of acronychal observations from the mean to the apparent motion of the sun,

[1] Reading '*forma*' instead of '*firma*'.
[2] Mars actually subtends only about $18''$ at opposition.
[3] This hypothesis theoretically gives results accurate within one minute, provided that its constants are correctly established.

| | 1580 s | ° | ′ | ″ | 1582 s | ° | ′ | ″ | 1585 s | ° | ′ | ″ | 1587 s | ° | ′ | ″ | 1589 s | ° | ′ | ″ | 1591 s | ° | ′ | ″ |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Aphelion for 1587 | Leo | 28 | 48 | 55 | 4 | 28 | 48 | 55 | 4 | 28 | 48 | 55 | 4 | 28 | 48 | 55 | 4 | 28 | 48 | 55 | 4 | 28 | 48 | 55 |
| Motion over the intervening years | | | 6 | 42 | | | 4 | 28 | | | 2 | 14 | | | | 0 | | | 2 | 15 | | | 4 | 32 |
| Aphelion in the year written above | 4 | 28 | 42 | 13 | 4 | 28 | 44 | 27 | 4 | 28 | 46 | 41 | 4 | 28 | 48 | 55 | 4 | 28 | 51 | 10 | 4 | 28 | 53 | 27 |
| Mean longitude | 1 | 25 | 49 | 31 | 3 | 9 | 24 | 55 | 4 | 20 | 8 | 19 | 6 | 0 | 47 | 40 | 7 | 14 | 18 | 26 | 9 | 5 | 43 | 55 |
| Add | | | 3 | 55 | | | 3 | 55 | | | 3 | 55 | | | 3 | 55 | | | 3 | 55 | | | 3 | 55 |
| Corrected mean longitude | 1 | 25 | 53 | 26 | 3 | 9 | 28 | 50 | 4 | 20 | 12 | 14 | 6 | 0 | 51 | 35 | 7 | 14 | 22 | 21 | 9 | 5 | 47 | 50 |
| Therefore, the angle C is | | 87 | 11 | 13 | | 49 | 15 | 37 | | 8 | 34 | 27 | | 32 | 2 | 40 | | 75 | 31 | 11 | | 126 | 54 | 23 |
| Its sine | | 99880 | | | | 75767 | | | | 14909 | | | | 53058 | | | | 96823 | | | | 79961 | | |
| Eccentricity of the equant | | 7232 | | | | 7232 | | | | 7232 | | | | 7232 | | | | 7232 | | | | 7232 | | |
| | | 65088 | | | | 50624 | | | | 07232 | | | | 36160 | | | | 65088 | | | | 50624 | | |
| | | 6509 | | | | 3616 | | | | 2893 | | | | 2169 | | | | 4339 | | | | 6509 | | |
| | | 579 | | | | 506 | | | | 651 | | | | 36 | | | | 578 | | | | 651 | | |
| | | 58 | | | | 43 | | | | 6 | | | | 6 | | | | 14 | | | | 43 | | |
| | | | | | | 5 | | | | | | | | | | | | 2 | | | | 1 | | |
| | | 7223 | | | | 5479 | | | | 1078 | | | | 3837 | | | | 7002 | | | | 5783 | | |
| Part of the equation | | 4 | 8 | 33 | | 3 | 8 | 26 | | 0 | 37 | 4 | | 2 | 11 | 57 | | 4 | 0 | 55 | | 3 | 18 | 55 |
| | | 91 | 19 | 46[4] | | | | | | | | | | | | | | | | | | | | |
| The angle B | | 88 | 40 | 14 | | 46 | 7 | 11 | | 7 | 57 | 23 | | 29 | 50 | 43 | | 71 | 30 | 16 | | 123 | 35 | 28 |
| Its half | | 44 | 20 | 7 | | 23 | 3 | 36 | | 3 | 58 | 42 | | 14 | 55 | 21 | | 35 | 45 | 8 | | 61 | 47 | 44 |
| Tangent | | 97706 | | | | 42572 | | | | 6955 | | | | 26650 | | | | 72002 | | | | 186464 | | |
| Quotient resulting from the division of the sum of the sides by the difference | | 79643 | | | | 79643 | | | | 79643 | | | | 79643 | | | | 79643 | | | | 79643 | | |
| | | 716787 | | | | 318572 | | | | 47786 | | | | 159286 | | | | 557501 | | | | 796430 | | |
| | | 55750 | | | | 15929 | | | | 7168 | | | | 47786 | | | | 15929 | | | | 637144 | | |
| | | 5575 | | | | 3982 | | | | 398 | | | | 4779 | | | | 16 | | | | 47786 | | |
| | | 48 | | | | 557 | | | | 40 | | | | 398 | | | | | | | | 3186 | | |
| | | | | | | 16 | | | | | | | | | | | | | | | | 478 | | |
| | | | | | | | | | | | | | | | | | | | | | | 32 | | |
| Tangent | | 778160 | | | | 33906 | | | | 5539 | | | | 21225 | | | | 57344 | | | | 148506 | | |
| | | 37 | 53 | 22 | | 18 | 43 | 47 | | 3 | 10 | 13 | | 11 | 59 | 0 | | 29 | 49 | 54 | | 56 | 2 | 40 |
| | | 44 | 20 | 7 | | 23 | 3 | 36 | | 3 | 58 | 42 | | 14 | 55 | 21 | | 35 | 45 | 8 | | 61 | 47 | 44 |
| Angle at A | | 82 | 13 | 29 | | 41 | 47 | 23 | | 7 | 8 | 55 | | 26 | 54 | 21 | | 65 | 35 | 2 | | 117 | 50 | 24 |
| Aphelion | | 148 | 42 | 13 | | 148 | 44 | 27 | | 148 | 46 | 41 | | 148 | 48 | 55 | | 148 | 51 | 10 | | 148 | 53 | 27 |
| Mars's position is | | Gemini | | | | Cancer | | | | Leo | | | | Virgo | | | | Scorpio | | | | Sagitt. | | |
| | | 6 | 28 | 44 | | 16 | 57 | 4 | | 21 | 37 | 46 | | 25 | 43 | 16 | | 4 | 26 | 12 | | 26 | 43 | 51 |
| Should be | | 6 | 28 | 35 | | 16 | 55 | 30 | | 21 | 36 | 10 | | 25 | 43 | 0 | | 4 | 24 | 0 | | 26 | 43 | 0 |
| Difference | | | 0 | 9 | | | 1 | 34 | | | 1 | 36 | | | 0 | 16 | | | 2 | 12 | | | 0 | 51 |

4. This is the sum of angle C and the part of the equation. The angle at B is its supplement.

| | 1593 | | | | 1595 | | | | 1597 | | | | 1600 | | | | 1602 | | | | 1604 | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | s | ° | ′ | ″ | s | ° | ′ | ″ | s | ° | ′ | ″ | s | ° | ′ | ″ | s | ° | ′ | ″ | s | ° | ′ | ″ |
| Aphelion for 1587 | Leo | 28 | 48 | 55 | 4 | 28 | 48 | 55 | 4 | 28 | 48 | 55 | 4 | 28 | 48 | 55 | 4 | 28 | 48 | 55 | 4 | 28 | 48 | 55 |
| Motion over the intervening years | | | 6 | 48 | | | 9 | 14 | | | 11 | 30 | | | 13 | 43 | | | 15 | 56 | | | 18 | 11 |
| Aphelion in the year written above | 4 | 28 | 55 | 43 | 4 | 28 | 58 | 9 | 4 | 29 | 0 | 25 | 4 | 29 | 2 | 38 | 4 | 29 | 4 | 51 | 4 | 29 | 7 | 6 |
| Mean longitude | 11 | 9 | 55 | 4 | 1 | 7 | 14 | 9 | 2 | 23 | 11 | 56 | 4 | 4 | 35 | 50 | 5 | 14 | 59 | 37 | 6 | 27 | 0 | 12 |
| Add | | | 3 | 55 | | | 3 | 55 | | | 3 | 55 | | | 3 | 55 | | | 3 | 55 | | | 3 | 55 |
| Corrected mean longitude | 11 | 9 | 58 | 59 | 1 | 7 | 18 | 4 | 2 | 23 | 15 | 51 | 4 | 4 | 39 | 45 | 5 | 15 | 3 | 32 | 6 | 27 | 4 | 7 |
| Therefore, the angle C is | | 11 | 3 | 16 | | 111 | 40 | 5 | | 65 | 44 | 34 | | 24 | 22 | 53 | | 15 | 58 | 41 | | 57 | 57 | 1 |
| Its sine | | 19174 | | | | 92934 | | | | 91171 | | | | 41280 | | | | 27528 | | | | 84759 | | |
| Eccentricity of the equant | | 7232 | | | | 7232 | | | | 7232 | | | | 7232 | | | | 7232 | | | | 7232 | | |
| | | 07232 | | | | 65088 | | | | 65088 | | | | 28928 | | | | 14464 | | | | 57856 | | |
| | | 6509 | | | | 1446 | | | | 0723 | | | | 0723 | | | | 5062 | | | | 2893 | | |
| | | 072 | | | | 651 | | | | 072 | | | | 145 | | | | 362 | | | | 506 | | |
| | | 51 | | | | 22 | | | | 51 | | | | 58 | | | | 14 | | | | 36 | | |
| | | 3 | | | | 3 | | | | 1 | | | | | | | | 6 | | | | 6½ | | |
| | | 1387 | | | | 6721 | | | | 6593 | | | | 2985 | | | | 1991 | | | | 6130 | | |
| Part of the equation | | 0 | 47 | 42 | | 3 | 51 | 14 | | 3 | 46 | 50 | | 1 | 42 | 40 | | 1 | 8 | 26 | | 3 | 30 | 52 |
| The angle B | | 11 | 50 | 58 | | 107 | 48 | 51 | | 61 | 57 | 44 | | 22 | 40 | 13 | | 14 | 50 | 15 | | 54 | 26 | 9 |
| Its half | | 168 | 9 | 2[4] | | 53 | 54 | 26 | | 30 | 58 | 52 | | 11 | 20 | 6 | | 7 | 25 | 8 | | 27 | 13 | 5 |
| Tangent | | 963600 | | | | 137171 | | | | 60045 | | | | 20046 | | | | 13021 | | | | 51433 | | |
| Quotient resulting from the division of the sum of the sides by the difference | | 79643 | | | | 79643 | | | | 79643 | | | | 79643 | | | | 79643 | | | | 79643 | | |
| | | 7167870 | | | | 796430 | | | | 477858 | | | | 159286 | | | | 079643 | | | | 398215 | | |
| | | 477858 | | | | 238929 | | | | 00318 | | | | 319 | | | | 23893 | | | | 07964 | | |
| | | 23893 | | | | 55750 | | | | 40 | | | | 48 | | | | 159 | | | | 3186 | | |
| | | 4779 | | | | 0796 | | | | | | | | | | | | 8 | | | | 239 | | |
| | | | | | | 557 | | | | | | | | | | | | | | | | 24 | | |
| | | | | | | 08 | | | | | | | | | | | | | | | | | | |
| Tangent | | 767440 | | | | 109247 | | | | 47822 | | | | 15965 | | | | 10370 | | | | 409628 | | |
| | | 82 | 34 | 30 | | 47 | 31 | 49 | | 25 | 33 | 30 | | 9 | 4 | 14 | | 5 | 55 | 14 | | 22 | 16 | 32 |
| | | 84 | 4 | 31 | | 53 | 54 | 26 | | 30 | 58 | 52 | | 11 | 20 | 6 | | 7 | 25 | 8 | | 27 | 13 | 5 |
| Angle at A | | 166 | 39 | 1 | | 101 | 26 | 15 | | 56 | 32 | 22 | | 20 | 24 | 20 | | 13 | 20 | 22 | | 49 | 29 | 37 |
| Aphelion | | 148 | 55 | 43 | | 148 | 58 | 9 | | 149 | 0 | 25 | | 149 | 2 | 38 | | 149 | 4 | 51 | | 149 | 7 | 6 |
| Mars's position is | | Pisces | | | | Taurus | | | | Cancer | | | | Leo | | | | Virgo | | | | Libra | | |
| | | 12 | 16 | 42 | | 17 | 31 | 54 | | 2 | 28 | 3 | | 8 | 38 | 18 | | 12 | 25 | 13 | | 18 | 36 | 43 |
| Should be | | 12 | 16 | 0 | | 17 | 31 | 40 | | 2 | 28 | 0 | | 8 | 38 | 0 | | 12 | 27 | 0 | | 18 | 37 | 10 |
| Difference | | | 0 | 42 | | | 0 | 14 | | | 0 | 3 | | | 0 | 18 | | | 1 | 47 | | | 0 | 27 |

4. This is obviously the supplent, not the half.

so as to keep me from, not just imitating, but even surpassing, the certitude of the Tychonic calculation, which has been raised as an objection against my abandoning the sun's mean motion.

*Translator's note on the Table*

The computations presented in the table are quite straightforward and, except for the changed ratio of the eccentricities, follow the standard Ptolemaic form. Nevertheless, for readers who are not familiar with that form, the following brief explanation may be helpful.

As before, A is the sun, B the centre of the eccentric, C the centre of the equant, H and I aphelion and perihelion. Let P represent the planet.

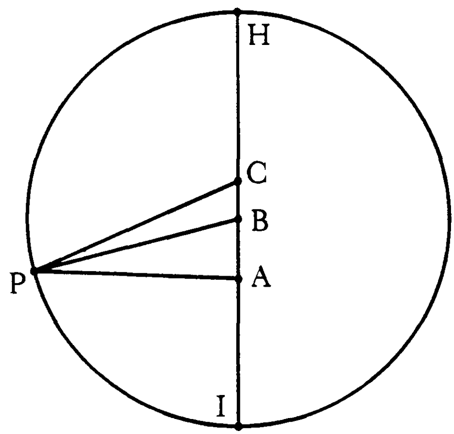
The angle at C is the angle between the aphelion and the mean longitude (the direction of the line CP). Or rather, it is the supplement of that angle, namely the angle BCP. So in triangle CBP, the sides BC and BP are known (7232 and 100,000, respectively), as well as the angle BCP, so the angle CBP may be found using the law of sines. Kepler's actual procedure is to find the angle BPC (which he labels 'part of the equation'), add it to angle BCP, and find the supplement of the sum, which is the angle BCP (entered into the table as 'Angle B').

The angle at A is found using the law of tangents, and here (as usual) Kepler uses a short cut. According to that law,

$$((BP - AB)/(BP + AB)) \tan \tfrac{1}{2} CBP = \tan \tfrac{1}{2} (BAP - BPA)$$

and

$$BAP = \tfrac{1}{2}(BAP + BPA) + \tfrac{1}{2}(BAP - BPA) = \tfrac{1}{2} CBP + \tfrac{1}{2}(BAP - BPA).$$



Thus Kepler enters half the angle CBP twice: once on the line labelled 'half', and again on the line preceding the 'Angle at A'. The angle at A is then the sum of the two preceding lines.

Mars's position is then found by adding or subtracting the angle at A to or from the position of the aphelion.

# 19

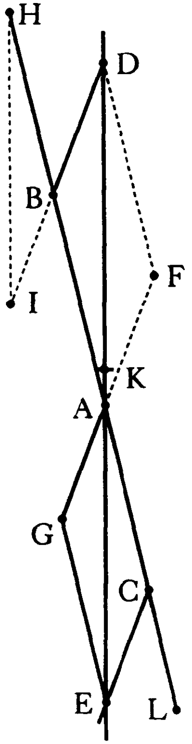
## A refutation, using acronychal latitudes, of this hypothesis constructed according to the opinion of the authorities and confirmed by all the acronychal positions

Who would have thought it possible? This hypothesis, so closely in agreement with the acronychal observations, is nonetheless false, whether the observations be considered in relation to the sun's mean position or to its apparent position. Ptolemy indicated this to us when he taught that the eccentricity of the equalizing point is to be bisected by the centre of the eccentric bearing the planet. For here neither Tycho Brahe nor I have bisected the eccentricity of the equalizing point. Now for Copernicus* it was a matter of religion not to neglect this anywhere. For he made very little use of observations, perhaps thinking that Ptolemy used no more than are referred to in his Great Work. Tycho Brahe balked at this. For in imitating Copernicus, he set up this ratio of the eccentricities, which the acronychal observations require. But when this was gainsaid not only by the acronychal latitudes (for these still underwent some increase arising from the second inequality) but also, and much more forcefully, by observations of other positions with respect to the sun which are affected by the second inequality, he stopped here and turned to the lunar theory, and I meanwhile stepped in.

*In Saturn and Jupiter he bisected it simply; that is, the Copernican form attributes the quadrant to the semidiameter of the epicycle. In Mars, however, since he had attributed to the epicycle the quadrant of the Ptolemaic eccentricity, he argued that in our time the whole Ptolemaic eccentricity must be diminished, but left to the epicycle its original quantity. And so he moved the centre of the eccentric (to speak Ptolemaically) 40 units closer to the centre of the annual orb than to the centre of the circle of the equant. Book 5 ch. 16. See also ch. 16 of this book.

Now the method by which the whole theory of Mars could easily be absolved of error, if the premises were correct, and by which it is demonstrated to be incorrect, is this.

111 First, through the latitudes at acronychal positions. *In the Copernican form, let the line* DE *be set out in the plane of Mars's eccentric, upon which let* A *be the sun,* D *the northern limit,* E *the southern limit, or the point nearest it. And through* A *let the straight line* HL *be drawn,*




*lying in the plane of the earth's eccentric orb. Now let* AH *and* AD *be conceived as lying in the one plane of a circle of latitude, and likewise* AL *and* AE. *And let the earth in 1585 lie at* B *on* AH, *and in 1593 let it be at the point* C *on* AL. *Now* AB *and* AD *are directed toward 21° Leo, so that the sun* A *seen from* B *appears at 21° Aquarius. And on the other hand,* E *and* C *are at 12° Pisces, so that the sun* A *seen from the earth* C *appears at 12° Virgo. But 12° Virgo is nearer to the sun's apogee than is 21° Aquarius. Therefore,* BA *is shorter than* AC. I have taken these lines from vol. I p. 98 of Tycho Brahe's *Progymnasmata*, and shall suppose them to be correct, although below (by a method developed for the purpose) I am going to be showing them to be slightly different. In Tycho BA is shown as 97,500, and AC as 101,400, while in the correction which is to follow, BA turns out a little longer and AC a little shorter; they are not equal, however. *Now because in ch. 13 above, through two procedures independent of the*

*present enquiry, the angle* BAD *was found to be about 1° 50′ at the limit (about 16° Leo), therefore, at four or five degrees from the limit it will be 1° 49½′. But* HBD, *the apparent latitude in 1585, was 4° 32′ 10″. Hence, given the angles* HBD *and* BAD, *their difference* BDA *of 2° 42′ 40″ is also given. Now as the sine of* BDA *is to the known length* BA, *so is the sine of* DBA *to* DA. *So that if* BA *is taken to be 97,500,* DA *comes out to be 163,000. But if* BA *be 100,000,* DA *will be 167,200.*

*Likewise, since in 1593* C *and* E *are in Pisces, and Mars is 26° from its limit, 64° from the node, consequently, the whole sine is to the sine of the maximum inclination 1° 50′ as the sine of 64° is to the sine of* CAE, *the inclination at this position.* CAE *is therefore 1° 39′. But the apparent latitude* LCE *was 6° 3′. Therefore the angle* AEC *is 4° 24′. And now, as before, as the sine of* AEC *is to the known length* AC, *so is the sine of* ACE *to* AE. *So that if* AC *is taken to be 101,400,* AE *comes out out to be about 139,300. But if* AC *be 100,000,* AE *comes out to be about 137,380. Now since 21° Leo is about 8° from the aphelion, the line* AD *will be about 150 parts longer at aphelion (as will be clear to anyone who computes the distances from the hypothesis we have found and substitutes these numbers); that is, either 163,150 or 167,350. And since 12° Pisces is about 13° from the perihelion,* AE *when right at the perihelion will be about 300 parts shorter; that is, either 139,000 or 137,080. So we have the lengths of the lines* AD *and* AE *when right at the apsides, when they are parts of the same straight line* DE. *So let them be added:*

| | | | |
|---|---|---|---|
| DA | *163150* | *or* | *167350* |
| AE | *139000* | *or* | *137080* |
| *So the whole,* DE, *is* | *302150* | *or* | *304430* |
| *the half,* DK, | *151075* | *or* | *152215* |
| *Resulting eccentricity* AK | *12075* | *or* | *15135* |

*Let these numbers be substituted for the original ones, where the radius of the eccentric was 100,000. So, as 151,075 is to 100,000, so is 12,075 to 8000, and as 152,215 is to 100,000, so is 15,135 to 9943.*

112 Therefore, the exact eccentricity of the eccentric, as indicated by the acronychal latitudes, is somewhere between 8000 and 9943, where the radius of the eccentric orb is 100,000. But our hypothesis based upon the acronychal observations of longitude resulted in an eccentricity of the eccentric of 11,332, far different from that which is near the mean between 8000 and 9943. Therefore, something among

those things we have assumed must be false. But what was assumed was: that the orbit upon which the planet moves is a perfect circle; and that there exists some unique point on the line of apsides at a fixed and constant distance from the centre of the eccentric about which point Mars describes equal angles in equal times. Therefore, of these, one or the other or perhaps both are false, for the observations used are not false.

The same demonstration also holds against that hypothesis established by observations related to opposition to the sun's mean position, because the latitudes remain about the same over the time interval between the two moments. Thus they show an eccentricity of the eccentric of 9943, while in ch. 5 above, 12,600 was taken from the Brahean rendition, and 12,352 in the Ptolemaic equant, where the whole eccentricity of the equalizing point was 20,160 or 19,763.

For the transformation of our diagram to the Ptolemaic form, *let* DE *be the line of apsides,* A *the earth,* D *and* E *the centre of the epicycle at the highest and lowest apsis, and from both points* D *and* E *let straight lines be drawn toward the earth* A *parallel to the plane of the ecliptic* BC. *On these, let* DF, EG *be taken as radii of the epicycle, equal to* BA, AC, *and let the planet be at* F *and* G. *Therefore, the inclination* FDA *will be equal to the inclination* BAD, *and the line of vision* AF *will be parallel to the original line* BD. *Therefore, the observed latitude,* HAF *and* HBD, *is the same. The same must be said of the congruent triangles* ACE *and* EGA. *And thus the demonstration and the magnitudes of the corresponding lines are the same.*

It may occur to the reader to question why I make the semidiameter of Mars's epicycle unequal to itself, namely, DF equal to BA, which is longer, and EG equal to CA, which is shorter. I answer from the first part, that this happens because of the transposition of observations from opposition to the sun's mean position to opposition to the sun's apparent position. If we were to stay with the sun's mean motion – and the present line of argument prevails even then – DF and EG will remain equal at least up to this point. But for this see Part I ch. 6.

For the Brahean form, *leaving one of the triangles (*DBA, *say) the same, so that* B *may be the motionless earth and* A *the sun in 1585, let* AB *be extended so that* BH *may be equal to* AC, *and let* H *be the sun in 1593, at 12° Virgo, and let* HI *be made equal and parallel to* AE *in the same direction, so that Mars at perigee may be at* I, *and at apogee at* D; HBA *the ecliptic,* BHI, BAD *the inclination;* IBA *the latitude at*

*perigee, and* DBH *at apogee. So again the sum of* DA *and* HI *will come out the same, whose half is* DK*; and the eccentricity will be* KA.
113 The only difference is this, that for Ptolemy the plane of the epicycle, and for Tycho the plane of the eccentric, is moved from north to south and back, remaining parallel to itself, while in Copernicus both stay in the same place.

Meanwhile, consider this also. In chapter 16 I had found the combined eccentricity to be 18,564, whose half, 9282, is just about the mean between 8000 and 9943. And Ptolemy too, as was remarked above, had taught us that half of the eccentricity found by acronychal observations is to be assigned to the eccentricity of the eccentric. So it was not without reason that he did so, and we should not rashly reject this bisection, since the observed latitudes support it.

Now if, on the other hand, we bisect the eccentricity of 18,564 that we found, we shall indeed represent the acronychal positions near the middle longitudes on the eccentric accurately enough, but not so the positions around the octants and towards the apsides.

*Take, for example, the opposition in 1593. From the preceding chapter, the simple anomaly was $6^s$ 11° 3′ 16″. I multiply the sine of 11° 3′ 16″, namely, 19,174, by 9282 (before, it was to be multiplied by 7232). This gives 1780, the sine of 1° 1′ 12″, which is part of the equation. When this is added to 11° 3′ 16″, the sum is the semi-equated anomaly, $6^s$ 12° 4′ 28″. The supplement of this is 167° 55′ 32″, whose half is 83° 57′ 46″. The tangent of this is about 945,500, which, multiplied by the perihelial distance of 90,718 and in turn divided by the aphelial distance of 109,282, gives a tangent of 784,880, whose arc is 82° 44′ 20″. This, subtracted from the 83° 57′ 46″ found earlier, leaves 1° 13′ 26″, which is the other part of the equation. If this be added to the semi-equated anomaly, and the sum be added to the aphelion, it puts the planet at 12° 13′ 37″ Pisces, which differs from the former hypothesis by three minutes, and is more distant from the observed position. For it should have been 12° 16′ Pisces.*

*This appears more clearly at 17° Cancer in 1582. For with the eccentricity bisected, Mars falls at 17° $4\frac{3}{4}$′ Cancer, and this calculated value differs from ours by $7\frac{2}{3}$′ at about 45° from aphelion, but by about 9′ from the observation.*

And from this difference of eight minutes, so small that it is, the reason is clear why Ptolemy, when he made use of bisection, was satisfied with a fixed equalizing point. For if the eccentricity of the equant, whose magnitude the very large equations in the middle

longitudes fix indubitably, be bisected, you see that the very greatest error from the observations reaches 8′, and this in Mars, which has the greatest eccentricity; it is therefore less for the rest. Now Ptolemy professed not to go below 10′, or the sixth part of a degree, in his observation. The uncertainty or (as they say) the 'latitude' of the observations exceeds the error in this Ptolemaic computation.

* Nevertheless, in the equations of the centre for the annual orb, these 8 minutes in some places increase to as much as 30 minutes.

Since the divine benevolence has vouchsafed us Tycho Brahe, a most diligent observer, from whose observations the 8′ error in this Ptolemaic computation is shown, it is fitting that we with thankful mind both acknowledge and honour this benefit of God. For it is in this that we shall carry on, to find at length the true form of the celestial motions, supported as we are by these arguments showing our suppositions to be fallacious. In what follows, I shall myself, to the best of my ability, lead the way for others on this road. For if I had thought I could ignore eight minutes of longitude, in bisecting the
114 eccentricity I would already have made enough of a correction in the hypothesis found in ch. 16. Now, because they could not have been ignored, these eight minutes alone will have led the way to the reformation of all of astronomy, and have constituted the material for a great part of the present work.

# 20

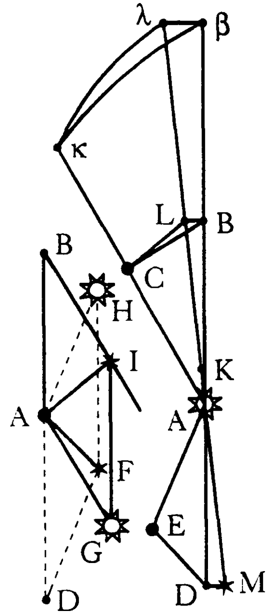
## Refutation of the same hypothesis through observations in positions other than acronychal

I shall now proceed to the other argument whereby the eccentricity of the eccentric as found in chapter 16[1] is proven false, despite its true rendering of the longitudinal motion. This argument is based upon observations of Mars at positions with respect to the sun other than opposition, when the planet was observed in the region of the eccentric's apsides.

On 1600 March 5/15 about midnight, Mars was observed at 29° 12½′ Cancer with latitude 3° 23′ north. Its mean longitude, corrected by our addendum[2], was $4^s$ 29° 14′ 58″, while the aphelion was at $4^s$ 29° 2′ 45″. Therefore, the anomaly was $0^s$ 0° 12′ 13″, requiring an equation of 2′, to be subtracted, according to the hypothesis of eccentric positions established above. Therefore, Mars's eccentric position was 29° 13′ Leo, and the sun's position, 25° 45′ 51″ Pisces.

*In the diagram, let* A *be the sun,* B *Mars, and* C *the earth. By subtraction of* CB *(29° 12½′ Cancer) from* AB *(29° 13′ Leo) the angle* CBA *will be 30° 0′ 30″, while by subtraction of* CA *(25° 45′ 51″ Pisces) from* CB *(29° 12′ 30″ Cancer),* BCA *will be 123° 26′ 39″. But as [sin]* CBA *is to* CA*, so is [sin]* BCA *to* BA*. But* CA*, the distance of the sun from the earth, is 99,302 from Tycho's table. (Although this is incorrect, the true value is nevertheless between this and 100,000, as we shall learn below in chapter 30). Therefore,* AB *is between 165,680 and 166,846.*

[1] Incorrectly given as 17 in the first edition and in KGW.
[2] This is the 3′ 55″ that Kepler said should be added to the mean longitudes (ch. 16 p. 268).




At perihelion, let the observation be taken that was made on the night following 1593 July 30 at $1^h\ 45^m$ [3]. Mars was found to be at 17° $39\frac{1}{2}'$ Pisces with latitude 6° $6\frac{1}{4}'$ south. Mars's mean longitude was $10^s$ 26° 16′ 38″, aphelion $4^s$ 28° 55′ 43″, so Mars was 2° 39′ 5″ from perihelion, to which corresponds an equation of 32′, to be subtracted, in accordance with the above hypothesis, making Mars's eccentric position $10^s$ 25° 44′ 30″, while the sun's apparent position was 17° 3′ 0″ Leo.

*In the diagram let* BA *be extended to* D, *and let* AD *be at 25° 44′ 30″ Aquarius and* ED *at 17° 39′ 30″ Pisces. Therefore,* EDA *is 21° 55′ 0″. And because* ED *is at 17° 39′ 30″ Pisces and* EA *at 17° 3′ Leo* AED *is therefore 149° 23′ 30″. Now as [sin]* EDA *is to* EA *so is [sin]* AED *to* AD. *But* EA, *the sun's distance from the earth, is 102,689 from Tycho's table, an erroneous figure, to be sure, but it is surely greater*
115 *than 100,000. Therefore,* AD *is between 140,080 and 136,409. But since the star Mars is $2\frac{2}{3}$ degrees from perihelion,* AD *will be shorter by*

[3] That is, July 31 at 1.45 am.

*about 15 at the perihelion itself, that is, between 140,065 and 136,394. Figures for both apogee and perigee must be increased, because they were computed using observations related to the ecliptic. Thus* AD *and* AB *are lines in the plane of the ecliptic.* On which point, take this

**Protheorem to be used frequently below.**

BY OBSERVATIONS OF THE STAR MARS RELATED TO THE ECLIPTIC, AND BY LINES IN THE PLANE OF THE ECLIPTIC FOUND THROUGH THOSE OBSERVATIONS TO SHOW THE LENGTH OF LINES CORRESPONDING TO THEM AND NEXT TO THEM IN THE PLANE OF MARS'S OWN ORBIT.

*Let the line* BAD *be set out in the plane of the ecliptic and through* A, *which denotes the sun or centre of the world, let the straight line* LAM *be so drawn in the plane of the orbit that the star be at* L *and* M. *Now let the earth be at* C, *and the triangle* CAB *be part of the plane of the ecliptic, to which the plane of the triangle* LBA *is to be understood to be perpendicular. Let the points* C, L *and* B *be joined, and lines be extended to the surface of the sphere of the fixed stars:* AB *to* β, AL *to* λ AC *to* κ; *and let* κβ *be an arc of the ecliptic,* βλ *an arc of the circle of latitude, and* κλ *a transverse arc. Thus the observation of the star's position beneath the fixed stars is referred to the ecliptic, by means of an arc of the circle of latitude drawn at right angles to the ecliptic* κβ *through the observed position of the star, and the triangle* CLB *is part of the plane of that circle. But* λβ *is also supposed to be the circle of latitude, perpendicular to the ecliptic* κβ. *Therefore, the planes* CLB *and* LBA *of the two circles perpendicular to the ecliptic intersect at the line* LB. *Therefore, by Euclid XI. 19, the line of intersection* LB *will be perpendicular to the plane of the ecliptic* CBA *and to the line* BA *contained in it; that is,* LBA *will be a right angle. Therefore, once the length of* BA *on the ecliptic is found, and the angle* LAB *is known, it will be impossible to lack knowledge of the length of* LA *which is sought. Q. E. F.*

Now in the matter at hand, since the inclination, or angle LAB is 1° 48′ at this position, LA is 82 parts longer than BA (in the same units as before), and AM 72 parts longer than AD .

| | |
|---|---|
| So the corrected apogees AL come out to be | 165,762 or 166,928 |
| Perigees AM | 140,137 or 136,466 |
| Sums LM | 305,899 or 303,394 |
| Halves KL | 152,950 or 151,697 |
| Eccentricity KA | 12,812 or 15,371[4] |

In new units, taking KL or KM as 100,000, the eccentricity of the eccentric is between 8377 and 10,106. But our hypothesis postulated 11,332, which exceeds both of these. Therefore, it postulated something false.

You should not let it disturb you that the second number, 10,106, which was arrived at through the assumption that AC and AE are equal, comes rather close to 11,332. For since I have related these observations to the sun's apparent position, constructing the eccen-
116 tricity from the centre of the sun's body, AC and AE will therefore not be equal. Consequently, this eccentricity will be much less than 10,106, and would in fact be 8377 if the sun's distances were correctly given as 99,302 and 102,689, which the requirements of this demonstration led us to take as 100,000 and 100,000. But since these Tychonic distances will be corrected below, and will be brought closer to the average radius, the eccentricity being sought here certainly lies between these two limits (8377 and 10,106). In fact, it is approximately half the total eccentricity found previously (18,564); that is, 9282.

To go through the same demonstration in the Ptolemaic hypothesis of the second inequality, proceed as in the previous chapter. *Draw* AI, BI, AF, DF *parallel to* CB, CA, ED, EA *in the larger diagram, and fix the earth at* A, *and the centre of the epicycle (or, more correctly, the point about which the epicycle is rotated, distant from the centre of the epicycle by the whole of the sun's eccentricity) at* D *and* B; *the sun at* H *and* G; *in such a manner that* AH *is equal and parallel to* EA *and* AG *to* CA, *so that the angles of equated anomaly of commutation are* HAD *and* GAB; *let Mars be at* I *instead of* B *or* L *and at* F *instead of* D *or* M; *and the lines* AG, AH *parallel to* BI *and* DF *(the lines of the planet's position on the epicycle) will be the sun's position.* The rest is obvious.

For the Tychonic form and hypothesis of the second inequality, *let* A *remain the earth,* H *and* G *the sun; and let* HF, GI *be drawn parallel*

[4] This should be 15,231, or 10,040 where KL is 100,000.

*and equal to* AD, AB, *so that Mars is again at* F *and* I. *The lines of vision,* AF *and* AI, *will therefore also be the same as in Ptolemy, and will be parallel to* ED, CB, *the lines of vision in the larger diagram. Therefore, they point in the same direction from the sun, and the sum of the lines* HF, GI *will equal the prior* BD. *Because the lines are parallel, the proof will be the same as the one at the beginning of the chapter.*

Now, as in the previous chapter, I shall accommodate the proof that the eccentric's eccentricity has been falsely determined to the Brahean rendition as well, which depends upon the sun's mean motion. This is done so that no one will think the discrepancy is a result of my having wrongheadedly transposed the observations from the sun's mean motion to its apparent motion.

On 1600 March 5, Mars's mean longitude was, in Tycho's reckoning, $4^s$ 29° 11′ 3″, apogee 23° 41′ Leo. Therefore, the simple anomaly was 5° 30′, which, in his reckoning, requires an equation of 1° 7′ 11″, to be subtracted, so as to give Mars an eccentric position of $4^s$ 28° 3′ 52″, with the sun's mean position 23° 44′ 31″ Pisces. *In the above diagram let* A *be the point of the sun's mean motion, distant from the centre of the sun by the whole of the sun's eccentricity. Therefore, the angle* CBA *is 28° 51′ 22″, and* BCA *is 125° 28′ 0″.* Also, this demonstration requires that AE and AC be assumed equal, namely, 100,000, retaining those suppositions made by Tycho and the ancients. These will be considered in Part III below, where it will be shown that the distance of the earth from the point of the sun's mean position is somewhat less; that is, that the Ptolemaic epicycle or the Copernican-Tychonic annual orb is not centred upon that point about
117 which equal angles are traversed in equal times. But for now let us hold to the fundamentals as given: *and let* CA *be 100,000; therefore,* AB *will be 168,760.*

At perigee, on 1593 July 30, since (in Brahe's reckoning) Mars's [mean] longitude was $10^s$ 26° 12′ 43″ and the apogee was at 23° 34′ Leo, the simple anomaly was 182° 38′ 43″ which requires an equation of 35′ 52″, to be added. Therefore, Mars's eccentric position was $10^s$ 26° 48′ 35″, and the sun's mean position 18° 24′ 31″ [Leo]. *Therefore, in the diagram,* EDA *will be 20° 50′ 55″, and* AED *will be 158° 45′ 0″*[5]. *Let* EA *again be 100,000, although below (as has just been remarked) it will turn out to be somewhat greater.* AD *is therefore*

[5] This should be 150° 45′.

*137,300. This you shall diminish by 15 so as to fit right at perigee: let it be 137,285. The other you shall increase by 100, so as to fit exactly at the apogee, so it will be 168,860. But we shall increase both (as before) because of the inclination of the planes, 82 being added at apogee and 72 at perigee. The corrected values will be:*

| | |
|---|---|
| AB | 168942 |
| AD | 137357 |
| BD | 306299 |
| BK | 153150 |
| KA | 15792, |

*the eccentricity from the point of the sun's mean motion or (in, the Ptolemaic form) on the line of apsides drawn through the centre of the epicycle.*

*Now where* BK *is 100,000,* KA *is 10,312, However, the Tychonic rendition based upon acronychal observations and presented in chapter 8 required* BK *to be greater, namely, 12,352.*

It has therefore been shown that the Tychonic rendition is also subject to the same incongruity, that the eccentric has one eccentricity when computed from acronychal observations, and a different one when computed from the other observations.

And meanwhile, the observations in this Tychonic rendition lead the way to bisection. For Tycho's figure for the whole eccentricity of the equalizing point is 20,160, half of which is 10,080, or in the form of the Ptolemaic equant, 9882. And here we have found it to be 10,312, which closely approximates half the Tychonic value. Indeed, it will approach much nearer, decreasing to a value less than the Tychonic (that is, to a very exact 9282), when AC in the greater diagram (BI in the smaller, on the left) is diminished, and along with it AB or GI (the distance at apogee); and, in turn, when AE of the right diagram (and its equal and equivalent DF of the left) is increased, and along with it AD or HF (the distance at perigee). For when the lesser part is increased, and the greater diminished, the difference between the two is decreased.

The blame for this discrepancy among the different ways of finding the eccentricity (I am repeating this over and over so that it will be remembered) falls entirely upon the faulty assumption studiously entertained by me, in common with Tycho and all who have ever devised hypotheses. For the necessary consequence of this enquiry is

that there is no single fixed point on the planet's eccentric about which the planet always sweeps out equal angles in equal times. We would instead have to make such a point reciprocate up and down along the line of apsides – if, indeed, we can keep the other assumption of a circular orbit. And how such a reciprocation could be reconciled with natural principles, I do not see.

118 But in fact the other assumption will be demolished, in chapter 44 below; that is, the orbit of the star is not a perfect circle, but an oval, and its greatest diameter is the line of apsides, while its least is that passing through the centre to the middle longitudes. No wonder, then, that the other observations at points not at opposition to the sun do not accord with the hypothesis constructed in chapter 16, since in it we have made two false assumptions.

# 21

## Why, and to what extent, may a false hypothesis yield the truth?

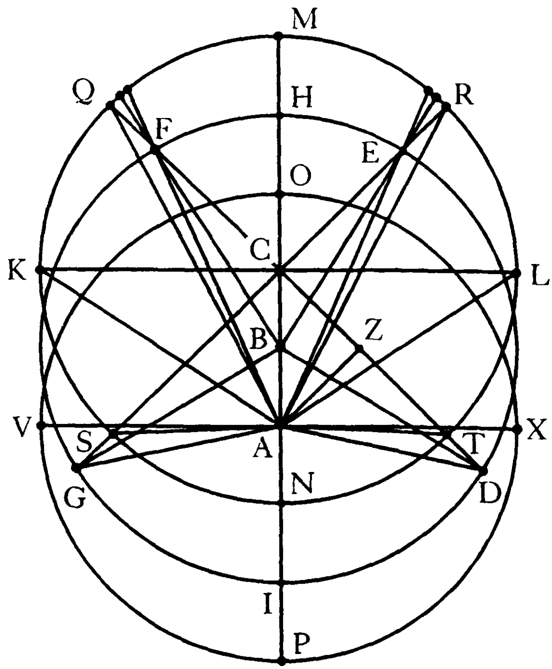
I particularly abhor that axiom of the logicians, that the true follows from the false, because people have used it to go for Copernicus's jugular, while I am his disciple in the more general hypotheses concerning the system of the world. I therefore considered it particularly worth while now to show the reader how it does happen here that the true follows from the false.

First, you have already seen that what has followed is not exactly the truth. For the path of the planet through the one plane of the ecliptic was considered in two ways: first, in respect to its longitude beneath certain degrees and minutes of the circle of the zodiac, and second, in respect to its altitude or distance from the centre of the world about which it moves, which comes out differently from those determined by the zodiacal positions. Therefore, our false supposition, although it does put the planet in the right longitudinal position at the right time, does not give it the right altitude. So it is not exactly the truth that follows from this false hypothesis.

Further, even concerning the longitude alone, the lack of any perceptible difference in effects between the as yet unknown true hypothesis and the false one assumed by us does not make the effect identical. For there can be a small discrepancy which the senses do not perceive.

There are, however, occasions upon which a false hypothesis can simulate truth, within the limits of observational precision, with respect to the longitude. These I shall now demonstrate.

*Through the centre of the world* A *let the straight line* MP *be set out,*




*falling upon opposite parts of the zodiac (29° Leo and Aquarius, say). And let it so be that according to some true hypothesis the planet spends half its time between lines* AM *and* AP *on the left and the other half on the right, so that after successive halves of its periodic time it is alternately on the lines* AM *and* AP. *And let it be assumed that this*
119 *particular effect of the true hypothesis is expressed by some other hypothesis that has been discovered. And so let a circle or any other curvy line (whatever seems appropriate) be described about a centre taken on the line* MP, *with the sole provision that it go around the centre of the world* A *and that it be cut into two equal parts by the line* MP. *What is proposed will happen if the planet traverse the circle with a uniform motion (one which is regular about any one particular point on the line* MP, *whether fixed or movable); as, for instance, if the circle* OP *were described about centre* A *and moved uniformly about it.* So all these circles and other figures have something in common through

which what was proposed occurs, namely, that they move around the centre of the world, and move regularly around some point on the line MP. Now this or that figure or circle, and one or another point of uniform motion, out of all those which are comprehended by these same specifications, can be false. But we have brought about what was proposed, not through this false kind, but through the true one which is contained within the generally false class we have chosen.

Now let us continue, *and let it happen that after successive quarter periods the planet lies on the lines* AM, AK, AP, AL, *the angles* MAK, MAL, *being less than right angles. Here, then, the former circle* OP *will be in error at the sides. For since the motion was supposed regular about* A, *a line drawn through* A *perpendicular to* MP *(namely,* VX*) will make the angles* MAV, MAX *measures of quarter periods. And accordingly, this hypothesis would put the planet on the lines* AV, AX, *when it should have been on* AK, AL.

Now experience testifies that the planet's motion closely emulates circularity (although it may perchance not exactly attain it), and it is the nature of motions of this kind to undergo gradual intensification and remission, admitting nothing sudden. *Therefore, the error of this hypothesis of the circle* OP *will begin little by little from the line* AM, *will grow continually greater, becoming a maximum at* AK, *and will again gradually decrease and vanish at* AP. *Therefore, the uniform and concentric hypothesis* OP *will never be more in error than it is at* AK, AL, *where it errs by the angles* KAV, LAX, *which, for Mars, are* $10\frac{1}{2}°$.

So let there now be another hypothesis which, in addition, also shows us the lines AK, AL. But again, there can be several hypotheses that do this. *For we might connect the points where* AK *and* AL *intersect the circle* OP, *and where this straight line intersects the straight line* MP *we may place the centre of uniform motion of the circle* OP, *so that the motion of the circle* OP *becomes nonuniform. We would then also have [the planet on] the lines* AK, AL *[at the appropriate times].* But since we have a certain inclination towards choosing the simplest and most regular, we shall therefore seek out that circle that moves uniformly about its own centre while effecting for us what is proposed. *Therefore, beginning from* A, *mark off equal lengths* AK, AL *upon the lines* AK, AL; *let the points* K *and* L *be joined by a straight line intersecting* MP *at* C; *and about* C *with radius* CK *let the eccentric circle* MN *be described, whose motion shall be regular about its centre. This hypothesis will represent the planet in the correct position, on the*

*four lines* AM, AN, AK, AL. But it is not this hypothesis alone, but many others as well, that could have had this effect. For they have this in common, and it is indeed perfectly true, that the point of uniform
120 motion is on the line that connects the positions of the planet falling upon the lines AK, AL, and at that point upon it where the line intersects MP. *Now it follows from the premises that this hypothesis has absorbed the entire maximum error of the former hypothesis* OP, *namely,* KAV, LAX, *at about the quarters of the period, nor does it commit a new error (since at* AM, AP *it is equivalent to the former). Therefore, if this hypothesis is still in error, that error will be much smaller than* KAV. *And since it is correct at* CM, CN, CK, CL, *the error (if any) will retreat to the four regions intermediate to those just mentioned, and will occur at the eighths of the period, since the time is measured about* C. *Therefore, let the angles* MCK, KCN *be bisected by two new lines through* C *intersecting the circumference at* Q, T, R, S. *The maximum error, if any, will be about these points. But the hypothesis also places the planet on the lines* AQ, AR, AS, AT, *at the eighths of the period. Now suppose (as is true for Mars) that after the eighths of the periodic time the planet is not destined to appear on the lines* AQ, AR, AS, AT, *but instead is above the former two at* AF, AE, *and below the latter two at* AG, AD. *Therefore, if the former error* KAV *was* $10\frac{1}{2}°$, *the present error* QAF *will hardly amount to a few minutes*. For Mars, the magnitude of QAF or RAE is observed to be about 9′, while SAG or TAD is about 28′.

Now as a third step, let this hypothesis too be corrected. As this can happen in a variety of ways (specifically, by a reciprocation of the point C along the line CA), we avoid disturbing the hypothesis anywhere by *keeping the point of uniform motion* C *fixed at distance* CA, *on account of the angle* KAV, *and also keeping the planet's path circular. These three, taken by choice and not forced by demonstration, lead us to move the centre of the eccentric downwards to* B *from the point of uniform motion* C, *substituting* HI *for* MN. *The body of the planet would depart from the points* Q, R, S, T, *nevertheless remaining on the lines* CQ, CR, CS, CT *(because the measure of time stays at* C*), and would arrive at the points marked* F, E, G, D. *And* QF, ER, SG, TD *would be such as to make* QAF, EAR *9′ and* SAG, TAD *28′*. With this done, that error at the eighths of the period will also be absorbed, and the hypothesis will exhibit the longitude perfectly accurately at eight places. Thus if any error remains, it will be at the sixteenths of the period, the points in between. Also, since

this third eccentric HI is equivalent to the first at positions AM, AP, as well as to the second at the additional positions AK, AL, it introduces no new error. And because the error of the second was greatest at the eighths of the period, and this is now absorbed, the part of the old error remaining at the sixteenths will be much smaller. Let us estimate it proportionally: just as the error of the first eccentric was 10½° while that of the second was 9′ or 28″, that is, one seventieth or one twenty-fifth of the former, let us now make the errors of the second that many times the errors of the third. Plainly, already at the
121 sixteenths of the period, we will have driven the business down to within the limits of observational accuracy.

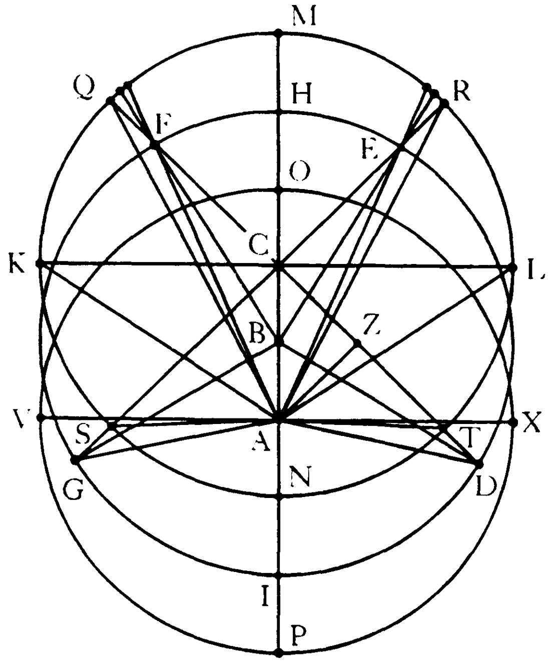
It is at least now clear to what extent and in what manner the truth may follow from false principles: whatever is false in these hypotheses is peculiar to them and can be absent, while whatever endows truth with necessity is in general aspect wholly true and nothing else.

Further, as these false principles are fitted only to certain positions throughout the whole circle, it follows that they will not be entirely correct outside those positions, except to the extent (as shown in this example) that the difference can no longer be appraised by the acuteness of the senses.

Also, this same dullness of the senses hides the following additional small error which remains at the eighths of the period. That there is such a remainder I prove thus:

*Once again, let a perfect eccentric be described about* B *so that* BD, BE, BF, BG *are equal, and let us have made* BC *such that the angle* QAF *is of the required magnitude. Now it is not likewise left to our discretion how great we want angles* SAG *to be, since it is completely determined. From* A *draw a perpendicular to* QT, *and let this be* AZ. *Now, as above, let* AC *be 18,564 where* CQ *is 100,000. And because* ACZ *is 45°, this makes* AZ *or* ZC *13,127 (both in the same units). Therefore,* ZQ *is 113,127, and* AQZ *6° 37′ 5″, and* QAZ *83° 22′ 55″, whose tangent is 864092*[1]. *Now let* FAZ *be taken as 9′ less: its tangent* FZ *will be 844900*[2]. *But where* AZ *is 13,127,* ZF *will be 110,910. Therefore,* QF *will be 2217. Now* QF *is larger than* TD, *which I prove thus:* QT *is the diameter of the circle, and is therefore equal to the two semidiameters* FB, BD *taken together. But* BF, BD *taken together are*

[1] This is actually the tangent of 83° 23′ 55″. The correct value is 861,896.
[2] This is the tangent of 83° 15′. The correct value is 842,621.




*greater than* FD. *Therefore,* QT *is also greater than* FD. *Let the common part* FT *be subtracted. The remainder* QF *is then greater than* TD. *And yet, over and above what is required, we shall allow them to be equal. Let* CZ, *13,127, be subtracted from* CT, *so as to leave* ZT, *86,873. Now from* AZ, ZT, ATZ *is known, and it is 8° 35′ 33″. So* ZAT *is 81° 24′ 27″. And because* ZT *is 86,873, I shall add to it a magnitude equal to* QF, *as if it were* TD, *namely, 2217. This makes* ZD *89,090. But where* AZ *is 100,000,* ZD *becomes the tangent of the angle* ZAD, *686,291. Thus this angle is 81° 42′ 35″. But* ZAT *was 81° 24′ 27″. Therefore* TAD, *or* SAG, *is less than the difference, 18′ 8″, since* TD *is less than 2217.*

This then is the required angle TAD, which ought to have been $27\frac{3}{5}'$. And so if you make QAF 12′ instead of 9′, TAD becomes 24′. And in both places the planet is made to be 3 minutes higher than it should be. The equation therefore will be seen to be too large, and thus the eccentricity [of the equant] is too large. It will straightaway be diminished, then, so that the planet is about $1\frac{1}{2}'$ lower at the lines

AK, AL, and the same amount (that is, $1\frac{1}{2}'$) higher at D, E, F, and G[3].

This mutual tempering of various influences causes one error to compensate for another, brings the calculation within the limits of observational precision, and makes it impossible to perceive the falsity of this particular hypothesis. And so this sly Jezebel cannot gloat over the dragging of truth (a most chaste maiden) into her bordello. Any honest woman following this false predecessor would stay closely in her tracks owing to the narrowness of the streets and the press of the crowd, and the stupid, bleary-eyed professors of the
122 subtleties of logic, who cannot tell a candid appearance from a shameless one, judge her to be the liar's maidservant.

This is without doubt the reason for the remaining discrepancies of one or two minutes in chapter 18, in Cancer, Leo, Scorpio, and several other places. But the error cannot easily be seen, since the observations used do not fall at the apsides and at the quarters and eighths of the period.

## Conclusion of Part II

Up to the present, the hypothesis accounting for the first inequality (in which Brahe and Copernicus are in agreement, both differing somewhat in form from Ptolemy) has been presented using the sun's mean motion, which all three authors had substituted for the sun's apparent motion. Thereafter, it was shown that whether we follow the sun's apparent motion and the hypothesis found in chapter 16, or the sun's mean motion and the hypothesis proposed in chapter 8 according to Brahe's rendition, in both instances there result false distances of the planet from the centre, whether of the sun (for Copernicus or Brahe) or of the world (for Ptolemy). Consequently, what we had previously constructed from the Brahean observations we have later in turn destroyed using other observations of his. This was the necessary consequence of our having observed (in imitation

[3] Reading 'D, E, F, G,' instead of 'DE, FG', which makes no sense although it stands thus in all editions.

An elucidation of Kepler's procedure here may be helpful. The angle QAF retains its adjusted magnitude of 12′, which places the planet 3′ too high. However, the eccentricity of the equant (C in the diagram) is decreased so as to diminish this error to $1\frac{1}{2}'$, introducing a $1\frac{1}{2}'$ error in the opposite direction at points K and L. Here, as often elsewhere, when Kepler says 'up' and 'down' he means 'toward aphelion' and 'toward perihelion', respectively.

of previous theorists) several things that were plausible but really false.

And this much of the work is dedicated to this imitation of previous theorists, with which I am concluding this second part of the *Commentaries*.

PART III

# Investigation of the second inequality, that is, of the motions of the sun or earth, or the key to a deeper astronomy, wherein there is much on the physical causes of the motions

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# 22

## The epicycle, or annual orb, is not equally situated about the point of equality of motion

That, then, is the method which our predecessors used to measure the first inequality. With this calculation established, which would represent the planet's eccentric position at any desired moment, they turned to exploring the second inequality (which depends upon the sun), comparing the observed or apparent position with that which the eccentric and the planet's first inequality alone would assign.

When I was on this same path and was confronted with this equivocal fork in the road (in chapters 19 and 20 above), and the observations (most faithful guides) were seen to be at war with observations, the thought occurred to me to alter completely the way the path was set out, using the method which follows.

In this third part I first approach the second inequality. Here I shall use unquestionable observations to demonstrate, with either a confirmation or a refutation, all that I have hitherto supposed as principles but had doubts about. Once this is found it will be like a key: the rest will be opened up. Afterwards, in Part IV, I shall proceed to the first inequality.

In chapter 22 of the *Mysterium cosmographicum*, when I was giving the physical cause of the Ptolemaic equant or of the Copernican-Tychonic second epicycle, I raised an objection against myself at the end of the chapter: if the cause I proposed were true, it ought to hold universally for all planets. But since the earth, one of the celestial bodies (for Copernicus), or the sun (for the rest) had not hitherto required this equant, I decided to leave that speculation open, until the matter were clearer to astronomers. I nevertheless entertained a

suspicion that this theory might perchance also have its equant. After I gained the recognition of Tycho, this suspicion was confirmed in me. For in a letter to me in Styria in 1598 Brahe said the following:

> The annual orb according to Copernicus, or the epicycle according to Ptolemy, does not appear always of the same size, in comparisons made to the eccentric, but introduces a perceptible alteration in all three superior planets, so much so that for Mars the angle of difference reaches one degree 45′[1].

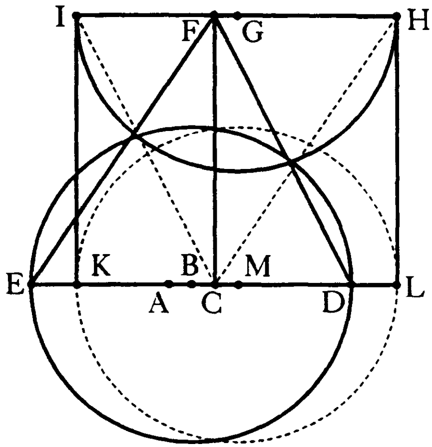
He also touched upon this point at the same time in the appendix to his *Mechanica*[2], an account of his studies. Also, his words in volume I p. 209 of his letters[3] are not much different, where he states the opinion that as an effect of the solar eccentricity a certain amount of nonuniformity is also mixed in with the eccentric equations and the acronychal positions. This is, in fact, refuted in part I: there is no particular disturbance at acronychal positions, or at least very little. But it appears that this should be corrected slightly so as to refer to those occasions when Mars is 90° from the sun.

**125** Now when I heard that the annual orb grows and shrinks, a sprite whispered to me that this illusion arises thus: Copernicus's annual orb, or Ptolemy's epicycle, is not everywhere distant from that centre about which by supposition it is sweeping out equal angles in equal times. For what physical cause could make the circuit of the centre of the planetary system (Tychonic) or of the circuit of the earth (for Copernicus) or of the epicycle bearing the star (for Ptolemy) grow and shrink? What, I ask, is this novelty unprecedented in astronomy, this unlikely absurdity? Wouldn't it seem more worthy of belief that the sun (for Copernicus) or the centre of the planetary system (Tychonic) or the body of the planet (for Ptolemy) would in certain places be farther from, and in others nearer to, the selected point of uniform motion (at rest for Copernicus and Tycho, and moving around on the circumference of the eccentric for Ptolemy) – and this especially on the line of apsides? And for this, that suspicion of mine arising from my *Mysterium cosmographicum* – that an equant might be introduced into the theory of the sun (or, as I call it, the theory of the Ptolemaic epicycle) – provides a convenient occasion.

Term. The centre of the planetary system is the common point of intersection of the lines drawn through the apsides of the individual planets. And that point is either near to the sun's body, as Brahe originally thought, or in the sun's very centre, as I would say, correcting him.

Let us suppose that the second inequality is measured starting from

[1] Brahe to Kepler, April 1, 1598 (old style), letter number 92 in KGW 13 pp. 197–202.
[2] TBOO 5 p. 115. (This and the following reference are from KGW 3 p. 465).
[3] TBOO 6 p. 239.




the line of the sun's mean motion, as the theorists have hitherto been pleased to hold (lest anyone hold suspect here my innovation of using the sun's apparent motion), *and in the present diagram let the planet's eccentricity for Copernicus originate not from the sun's centre* A, *but from the point* C *about which the earth's motion is supposed regular. But let that point* C *not be the centre of the earth's orb* DE *but only of uniformity of motion, and let its distance from the sun* A *be greater than that of* B, *the centre of the earth's orb* ED. I say that, these things being granted, the resulting observations will have been such that one might suspect the annual orb of growing and shrinking. *Let a line* CF *be drawn from* C *perpendicular to* DE, *and let the star Mars be at* F *twice: once when the earth is at* D *and again when it is at* E; *and let* F *be joined with the points* D *and* E. *Now because* C *is the point of uniformity of the earth's motion on* DE, FCD *and* FCE *will be the anomaly of commutation, and (as we suppose) equal on both sides. Now if* CD *and* CE *were equal (as has hitherto been thought) then the angles* DFC *and* EFC, *parallaxes of the orb, would be equal on both sides and for both anomalies of commutation. But because* CE *is greater than* CD, *the angle* CFE *will also appear greater than the angle* CFD. *Therefore, anyone not noticing that this growth occurs only at or about* E, *and that the contrary diminution occurs only at the contrary position* D, *will think that the entire annual orb sometimes gets larger, with radius* CE, *and sometimes smaller, with radius* CD. *This is because such a person presupposes, along with astronomy as it has*

*hitherto been practiced, that the point of equal motion* C *is at the same time also the centre of the circle* DE.

In the Ptolemaic form, *let the earth be at* C, *and the lines of the sun's mean motion be* CK, CL, *in place of* DC *and* EC *in the preceding Copernican arrangement. And let the centre about which the epicyclic* 126 *motion is regular be at* F, *and* IH *be equal and parallel to* ED *so that,* CI *being drawn, it is parallel to* DF *and* CH *to* EF. *For when the earth (or the observer) is transferred to the centre of the world* C, *as Ptolemy has it, Mars at* F *is likewise transferred to* H. *Similarly, owing to the translation of* D *to* C, F *is transferred to* I. *Now Ptolemy, thinking that the point* F, *about which the motion of the epicycle* IH *is uniform, is also the centre of the epicycle* IH, *supposed* FI *and* FH *to be exactly equal. Consequently, for both of the equated anomalies* HFC *as well as* IFC *(that is, at both 90° and 270°, in this diagram) he supposed one and the same equation of the epicycle, namely, the equal angles* HCF *and* ICF. *So if observation affirms that* HCF *is greater than* ICF, *then the centre of the epicycle will not be at the point of uniform motion* F, *but at* G, *in the direction of* H. *Further, on the supposition that* F *be nevertheless considered the centre of the epicycle, the epicycle will appear distinctly enlarged at anomaly 90° at* H, *and diminished at 270° at* I, *while in both instances Mars, in its eccentric position (that is, on the line* CF*), is in the same position with respect to the fixed stars.*

In the Tychonic form, *let* C *remain the earth,* DE *the sun's circle with centre* B, *but let the centre of uniform motion be* A. *And let the lines along which the planet is seen (namely,* CI *and* CH*) be the same as in Ptolemy. Accordingly, let* HL, IK *be drawn from* H *and* I *parallel to* FC. *In order that* K *and* L *be the centre of the planetary system, the centre of whose circuit would be* M, *which is in the direction of the sun's perigee, let the point* M, *the centre of the circuit* KL *(in which is found the point from which Mars's eccentricity originates) descend as far below* C *as the point* B, *the true centre of the sun's circuit (contrary to common opinion), descends below* A, *the putative centre of the same circuit of the sun. And let* AC *and* BM *be equal. The line of equated motion on the eccentric (that is,* KI, LH*) will be parallel to itself after an integral number of returns of the planet. Therefore Tycho, thinking that the earth* C *is in the middle of the circuit* KL *bearing the planets' eccentrics, will make angles* CIK, CHL *equal when the angles of commutation* CLH, CKI *are equal. But if these are perceived to be unequal,* CHL *being the greater,* CL *will be longer than* CK: and the orb KL, the deferent of the centre of the system, will

appear to grow at L and to shrink at K, because M, the centre of the orb which is the planetary system's deferent, is not believed to be elsewhere than at the earth C, about whose centre the motion of that orb is uniform.

It is an important consequence of this account*, which makes the distance CK of the centre of the system from the earth short where CE, the sun's distance from the earth, is long, and which, contrariwise, makes the former (CL) long where the latter (CD) is short, that the true cause of the discrepancy is hidden; that is, that the sun's eccentricity is freed from suspicion.

*Here is a notable incongruity. If the general Ptolemaic or Brahean hypothesis concerning the world system is true, and if at the same time we make use of the sun's mean motion, then the epicycle in the former or the deferent circle of the planetary system in the latter is made to be an eccentric whose apogee is exactly opposite the sun's apogee, while its eccentricity (as will appear below) is exactly equal to the sun's true eccentricity, that is, half of what it has hitherto been thought to be.

The reason why the apsides have thus been reversed is this. For Copernicus, the earth traverses the regions opposite the Tychonic sun and the Ptolemaic epicycle, and also DC, CE, the distances of the earth from the sun, of the sun from the earth, and of Mars H or I from the epicycle's centre of uniform motion F, subtend angles of the same magnitude in all three forms of hypothesis. Therefore, it also happens that the Copernican distances of the sun and the earth are transferred
127 to the opposite sides by Brahe and Ptolemy; that is, CE to CL or FH, and CD to CK or FI.

Next, in order either to confirm or undermine this speculation by observations, this is the road upon which I set out. Since the sun's apogee is at $5\frac{1}{2}^\circ$ Cancer, I enquired whether there might exist an observation in which Mars, reckoned by the first inequality only, would be twice at $5\frac{1}{2}^\circ$ Libra or Aries, while the sun would be at $5\frac{1}{2}^\circ$ Cancer at one time, and then at $5\frac{1}{2}^\circ$ Capricorn. As it turns out, this is not possible within such a short space of time (20 or 30 years). For the periodic motions of Mars and the sun are incommensurable, nor does it ever happen that they are 90° from one another, or at opposition, after a certain number of complete periods, or quarter or half periods, of either. I therefore had to choose the next best thing, which was to find many days throughout those 20 years on which the planet was observed, and in which the equated anomaly of commutation was 90° or 270° or nearly that much, with Mars at 6 Aries or Libra or thereabouts. Afterwards, it was necessary to look up all those dates in the catalog of observations of Mars, to see whether it had been observed at those moments. Had the indefatigable Tycho Brahe not observed Mars very frequently, the selection would have been so exclusive that I would not have been able to accomplish what I wished. Now since Tycho put the apogee of Mars at $23\frac{1}{2}^\circ$ Leo, while the required position of Mars, corrected by the eccentric equation,

was $5\frac{1}{2}°$ Libra, an equated anomaly of 42° was required. And from his table, to an equated anomaly of 42° there corresponds an equation of 8° $15\frac{3}{5}'$; therefore, a mean anomaly on the eccentric of 50° 16′ is required, and this occurred at twelve points in time in the twenty years between 1579 and 1600.

What had to be skilfully tracked down now is whether for any of these times there was an equated anomaly of commutation that was at one time 90° and again 270°, or if the former were greater or less, the latter would be correspondingly less or greater.

One revolution of Mars has 687 days, and two of the sun have $730\frac{1}{2}$. The difference is $43\frac{1}{2}$ days, to which corresponds 42° 54′ 23″ of the sun's mean motion. This, therefore, is how much the anomaly of commutation changes at the end of any revolution of Mars. Therefore, if in any two year period one seeks two anomalies of commutation that are equal, with Mars at the same eccentric position, each angle of commutation would be 21° 27′. Over four years 42° 54′ is required; over six years, 64° 22′; over eight years, 85° 49′. And we were supposing 90°, if it were possible. Therefore, we had to look for our two observations eight years apart. However, a team of two such observations is not to be found in the catalog of observations we had.

I next turned to the interval of six years, and found at length that from 1585 May 18 and 1591 January 22 suitable observations exist. For they corresponded to 1585 May 30 at $5^h$ and 1591 January 20 at $0^h$[4]. For both, the mean longitude of Mars was $6^s$ 22° 43′, and the Tychonic equation was 9° 14′ 52″, to be subtracted. Therefore, Mars's eccentric position was 13° 28′ 16″ Libra. The equated commutation
128 for 1585 was $8^s$ 4° 23′ 30″, by which it was argued, Ptolemaic style, that the planet was 64° 23′ 30″ beyond the perigee of the epicycle. Similarly, the equated commutation for 1591 was $3^s$ 25° 36′ 30″, by which it was argued that the planet was 64° 23′ 30″ before the perigee of the epicycle. Therefore, both the angles of commutation, FCD and FCE (or CFI, CFH) in the diagram, are equal. However, in 1585 the sun was in 18° Gemini, 18° before apogee, and in 1591 in 9° Aquarius, 33° beyond perigee, and this inequality could not be avoided.

Now to the observations: on 1585 May 18 at $10\frac{1}{2}^h$ at night Mars was observed at 0° 50′ 45″ Virgo with latitude 1° 19′ 30″ north. Magini

[4] The conditions that must be satisfied are: 1. that Mars have the same eccentric position at the two times, and 2. that the two angles between Mars and the earth about the centre of the earth's orbit be equal. These conditions are satisfied on 1585 May 30 and 1591 January 20 (old style). The observations which Kepler had were not made on those dates; however, they were near enough that adjustments could readily be made.

puts it at 1° 5′ Virgo, 14′ or 15′ too much. Therefore, when on the 30th at $5^h$ in the evening he puts it at 6° 48′ Virgo, we shall again subtract the discrepancy of eleven days previously. So he will be left with 6° 34′ Virgo. Here we incur an error of some very few minutes because the deduction over 12 days is too great, and the diurnal motion is not exactly the same as that obtained here from Magini. For consider that on April 18 preceding at $10^h$ Mars was found at 17° $37\frac{1}{2}'$ Leo, while Magini puts it at 18° 0′ Leo. The difference is $22\frac{1}{2}'$. Over the 33 days elapsed to May 18 this difference was diminished to the amount of $14\frac{1}{4}'$. So if we extrapolate, since over 33 days eight minutes vanished, in the same ratio, over the 12 following days 3 minutes will vanish. Therefore, on May 30 the difference will be $11\frac{1}{4}'$. So, more accurately, Mars is at 6° 37′ Virgo.

Similarly, on 1591 January 22 at $7^h$ in the morning, Mars was 34° 32′ 45″ from Spica with declination 17° 25′ south, at an altitude of 16°. Therefore, after correction for horizontal variations, the declination was 17° 30′. Hence, the right ascension was 230° 23′ 12″[5], longitude 22° 33′ Scorpio, latitude 1° 0′ 30″ north. Now this time differs from ours by 1 day 19 hours, and the diurnal motion, from Magini, is 33′. Therefore, 59′ are required for the intervening time. Therefore, the remainder is the position of Mars on January 20 at $0^h$ (which, as we said, corresponds to the other time): 21° 34′ Scorpio.

And since from Tycho's rendition,

CF is 13° 28′ Libra (fairly accurately)
while DF or CI in 1585 is 6 37 Virgo

Therefore DFC or FCI will be 36° 51′.

Likewise, because CF in 1591 is again 13° 28′ Libra*
while EF or CH is. . . . . . . . 21 34 Scorpio

Therefore EFC or FCH will be . . 38° $5\frac{1}{2}'$[6].

*The precession over the intervening time does not amount to five minutes: hence, it is neglected.

Witness the great difference in the equations of the annual orb, despite the promise that the two anomalies of commutation would be

[5] A serious error. Spica's right ascension in January 1591 was about 195° 55′, and its declination about 8° 58′, from Tycho's tables (TBOO 3 p. 376). Since the difference in declinations was 8° 32′, while the distance between the stars was 34° 32′ 45″, the difference in right ascensions was about 33° 36′, and Mars's right ascension was therefore about 229° 31′, or about a degree less than Kepler has. This places Mars at 21° $44\frac{1}{2}'$ Scorpio, with latitude 0° 48′ north.

[6] As was remarked in the above note, Kepler's error in calculating Mars's right ascension resulted in an error in Mars's longitude. This error affects the present subtraction: the angle should be 37° $17\frac{1}{2}'$. This brings the angles much nearer one another, and considerably weakens Kepler's argument.

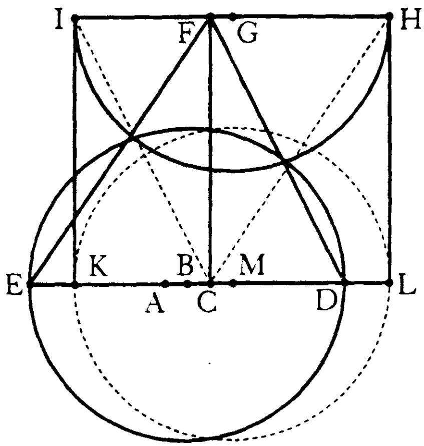
equal. The Copernican hypothesis shows us the cause. The earth at D and E was considered to be equally distant from the point of uniform motion C, but it was found to be at unequal distances, in such a way that the centre of its circuit would be at B towards the sun A. And by equivalence, in the Ptolemaic form the epicycle HI is not equally
129 situated about point F, whose eccentric path the acronychal observations have been describing to us, and about which the epicycle's motion is regular. And the centre of the epicycle G is towards E, on the same side as the solar perigee. Similarly, in the Tychonic form the deferent KL of the planetary system does not encircle the earth C at a constant distance, although its motion is regular about this point, but the centre M of its circuit is situated on the same side as the sun's perigee.

# 23

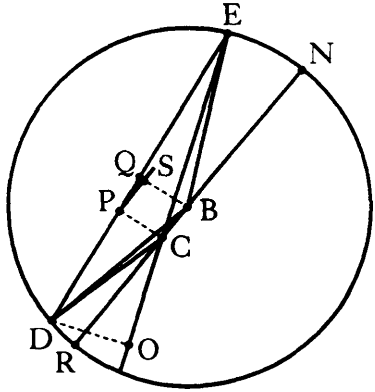
## From the knowledge of two distances of the sun from the earth and of the zodiacal positions and the sun's apogee, to find the eccentricity of the sun's path (or the earth's, for Copernicus)

From this it is not difficult for us to find, in addition, a tentative measure for the line BC. *Let* FC *be 100,000. And because* DFC *is 36° 51′ and* FCD *is 64° 23′ 30″, therefore the remaining angle* FDC *is 78° 45′ 27″. And as the sine of this angle is to* FC *(100,000), so is the sine of* DFC *to* DC, *61,148.*

*In the same way, because* EFC *is 38° 5½′ minus a little*[1], *and* FCE *is*




[1] As indicated in the previous chapter, this angle is actually 37° 17½′. This makes the angle FEC 78° 19′, and EC will be 61,869. Thus the difference between the two is much less pronounced than Kepler believed, and the points B and C will nearly coincide.




64° 23′ 30″, FEC will be 77° 31′ 0″ plus a little. Therefore, EC is 63,186 minus a little.

Let the earth's orb NED be set out, and on it let CBN be the line of apsides, and N perihelion, R aphelion, B the centre, C the point of equality of motion, E and D the positions of the two observations, which shall be joined to C and to B with straight lines. Now EC and CD are known in the same units, and angle ECD is known, viz. 128° 47′ 19″. Let EC be extended, and let DO be dropped perpendicular to it from D; and also to DE let two perpendiculars CP, BQ be dropped from C and B. DCO is therefore 51° 12′ 41″, and CDO is 38° 47′ 19″. Therefore, where DC is 61,148, DO will be 47,660 and CO 38,305. And this placed on the end of CE makes EO 101,491. But from the two given sides DO, OE about a right angle, the magnitude of DEO is obtained: 25° 9′ 20″. Wherefore DE is 112,125. The half of this, 56,062½, is the magnitude of DQ, since DB, BE are equal. And because DEC was 25° 9′ 20″, EDC or PDC will be 26° 3′ 21″. Therefore, where DC is 61,148, CP comes out to be 26,858 and PD is 54,932. Subtract this from QD, and the remainder, PQ, is 1130½. And now from the known inclination of the lines ED and NC the length CB is easily obtained. For since CR is the line of the aphelion, at 5° 30′ Capricorn, while CD is at 17° 52′ Sagittarius (because the sun is at 17° 52′ Gemini), DCR will be 17° 38′. But EDC was 26° 3′ 21″. Therefore, after subtraction, there remains the inclination of the lines in question: 8° 25′ 21″. From P let PS be drawn parallel to CB. This will be equal to
130 CB, and CP will be equal to BS. So in right triangle PQS, as the whole sine is to the tangent and secant of the angle QPS, 8° 25′ 21″, so is the known magnitude of PQ to QS, 167, and SP, 1143, which is CB. And

*because* PC *and* SB *are equal, with magnitude 26,858, add* QS, *and* QB *will come out to be 27,025, So in the right triangle* DQB, *given the sides about the right angle,* DB *will also be given, 62,237. Therefore, the ratio of* DB *to* BC *(the radius to the eccentricity, which is being sought) is the same as 62,237 to 1143. And as 62,237 is to 100,000 so is 1143 to 1837*[2]. *This, at last, is the eccentricity sought. It would have been less if we had accounted for the precession of the equinoxes, for then* CE *would have been less.*

And so from these two observations and the accepted true position of the sun's aphelion [sic] there is provided the distance of our equalizing point C or F (which we were considering as centre) from the orbit's true centre B or C or M, which is 1837 where the radius of its orb is 100,000. In contrast, Tycho Brahe found the sun's eccentricity, that is, the distance of the equalizing point C from the centre A of the solar body (in Copernicus) or the distance of the equalizing point A of the solar motion from the centre of the earth C (in the Tychonic-Ptolemaic supposition) to be 3584, whose half, 1792, is only slightly different from 1837. It is therefore fitting that the halving of the eccentricity hold in the theory of the sun, which halving previously held for the eccentric of Mars (in chapter 19 and 20). For owing to the large corrections and the use of a controverted value for the diurnal motion, the observations which I have presented are not exact enough to allow anything to be concluded from 45 parts in one hundred thousand, not to mention the ignoring of precession in Mars's and the sun's eccentric motions for the time interval involved.

What has been demonstrated here of the circuit of the earth can be demonstrated in exactly the same way concerning the Ptolemaic epicycle and the Tychonic deferent of the system, provided only that in the diagram the apsides be reversed.

I have supposed here that the sun's apogee established by Tycho was in the right place, and that the orbit of the sun (or earth) which it bodily traverses is arranged in a circle. Of this, too, by analogy with other planets, different testimony will be given in ch. 44 below; however, the small breadth of the deflection does not in the least vitiate our demonstration.

[2] Had Kepler not made the error noted above in chapter 22, he would have found this eccentricity to be 655, less than one fifth of the Tychonic solar eccentricity.

# 24

## A more evident proof that the epicycle or annual orb is eccentric with respect to the point of uniformity

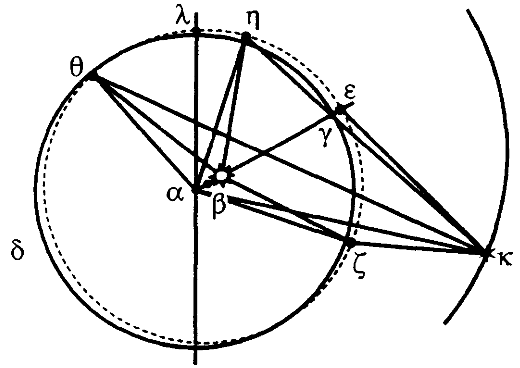
Such, then, was the beginning of this enquiry, timid and encumbered with such concern that the anomaly of commutation be equal on both sides.

131 Now that we have once made a hazard of this, we are buoyed by audacity to sally forth again more freely onto the battlefield. For I shall seek out three or more observed positions of Mars with the planet always at the same eccentric position, and from these find by trigonometry the distances of that number of points on the epicycle or annual orb from the point of uniform motion. And since a circle is defined by three points, I shall use three such observations to find the position of the circle, its apsides (previously taken as a presupposition), and its eccentricity with respect to the point of uniform motion. Should a fourth observation be at hand, it will serve as a test.

The first time shall be 1590 March 5 at $7^h\ 10^m$ in the evening, since then Mars had hardly any latitude, and thus no one looking at the demonstration could be troubled by any irrelevant suspicions about the intermingling of latitude. To this there correspond these moments, in which Mars returns to the same sidereal position: 1592 Jan. 21 at $6^h\ 41^m$; 1593 Dec. 8 at $6^h\ 12^m$; 1595 Oct. 26 at $5^h\ 44^m$. For the first of these times, Mars's [mean] longitude, according to Tycho's rendition, is $1^s\ 4°\ 38'\ 50''$, and for subsequent times $1'\ 36''$ greater for each. For this is the motion of precession corresponding to the periodic time of one return of Mars. And since Tycho places the apogee at $23\frac{1}{2}°$ Leo, its equation will be $11°\ 14'\ 55''$, and consequently the equated anomaly in 1590 will be $1^s\ 15°\ 53'\ 45''$.

Now over the same time, the commutation, or difference of the mean motions of the sun and Mars, is reckoned to be $10^s$ 18° 19′ 56″, so the equated [commutation], or the difference between the sun's mean motion and Mars's equated eccentric motion, is $10^s$ 7° 5′ 1″.

We shall present this first in the Copernican form, since it is simpler to perceive.




*Let α be the point of uniform motion of the earth's circuit, which shall be considered to be the circle δγ described about α, and let the sun*

132 *be on the side β, such that the line of the sun's apogee αβ lies in the direction of 5½° Cancer, despite our being about to investigate this freely, as if unknown, in ch. 25. And let the earth be at αθ in 1590, αη in 1592, αε in 1593, and αζ in 1595. And the angles θαη, ηαε, εαζ are equal, since α is the point of uniform motion and the periodic times of Mars are presupposed equal. And let the planet at these four times be at κ, and its line of apsides be αλ. Thus the angle θακ, according to the measure of the equated anomaly of commutation, is 127° 5′ 1″*[1].

*As for the observed position of Mars, at the same time on the preceding day, the fourth, it was at 24° 22′ Aries. Its diurnal motion for the day would be 44′. Therefore, at our time it was seen at 25° 6′ Aries, which is the position of the line θκ. But ακ is directed towards 15° 53′ 45″ Taurus. Therefore, θκα is 20° 47′ 45″. So the remainder αθκ to make up two right angles is 32° 7′ 14″.*

*Now as the sine of αθκ is to ακ, which we shall say is 100,000 units,*

[1] This is simply the equated commutation as given above, except that 180° is subtracted because of the change of viewpoint from the earth to the sun (θ to α, in the diagram).

*so is [the sine of]* $\theta\kappa\alpha$ *to* $\theta\alpha$*, which is what is sought. Therefore,* $\theta\alpha$ *is 66,774.*

Now if the remaining lines $\eta\alpha$, $\epsilon\alpha$, $\zeta\alpha$, turn out to be of the same length, my suspicion will be false, but if they are different, my triumph will be complete.

*Second, then, in 1592 at our moment the equated longitude was* $1^s\ 15^\circ\ 55'\ 23''$*; the equated commutation was* $8^s\ 24^\circ\ 10'\ 34''$ *– that is, the angle* $\eta\alpha\kappa$ *is* $84^\circ\ 10'\ 34''$*. It was observed on January 23 at* $7^h\ 15^m$ *at* $11^\circ\ 34\frac{1}{2}'$ *Aries, with the correction for parallax. And its motion over two days was* $1^\circ\ 25'$*. Therefore on the 21st at* $7^h\ 15^m$ *it was seen at* $10^\circ\ 9\frac{1}{2}'$ *Aries. The remaining parts of an hour would deduct the half minute. Therefore, the angle* $\eta\kappa\alpha$ *is* $35^\circ\ 46'\ 23''$*, and* $\alpha\eta\kappa$ *is* $60^\circ\ 3'\ 3''$ *and* $\alpha\eta$ *is 67,467, now longer than* $\alpha\theta$*. This is doubtless because the sun has*
133 *descended towards perigee, and the earth has been moved from* $\theta$ *to* $\eta$*; thus, in this region the sun is found beyond* $\beta$ *at a nearer point.*

*Third, in 1593 at our moment the equated longitude was* $1^s\ 15^\circ\ 56'\ 56''$*, the equated commutation was* $7^s\ 11^\circ\ 16'\ 16''$*, which makes* $\epsilon\alpha\kappa$ $41^\circ\ 16'\ 16''$*.*

*It was observed December 10 at* $7^h\ 20^m$ *at* $4^\circ\ 45'$ *Aries, parallax corrected. Its motion over two days was* $1^\circ\ 8'$*. Therefore, on December 8 at* $7^h\ 20^m$ *it was seen at* $3^\circ\ 37'$ *Aries, while at our time of* $6^h\ 12^m$ *it was at* $3^\circ\ 35\frac{1}{2}'$ *Aries. Hence,* $\epsilon\kappa\alpha$ *is* $42^\circ\ 21'\ 30''$ *and* $\kappa\epsilon\alpha$ *is* $96^\circ\ 22'\ 14''$*, and* $\alpha\epsilon$ *is 67,794, again longer, for it is yet closer to the sun's perigee.*

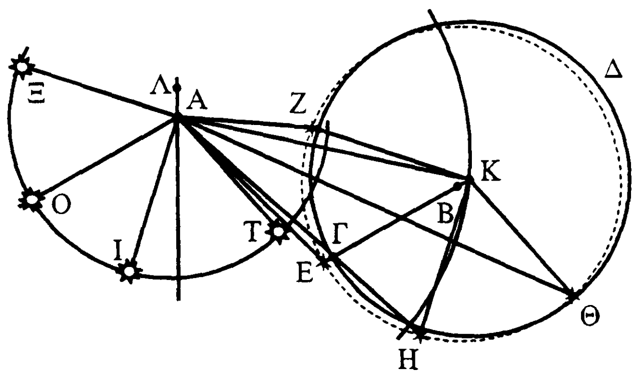
*Fourth, in 1595 at our moment the equated longitude was* $1^s\ 15^\circ\ 58'\ 30''$ *and the [equated] commutation was* $5^s\ 28^\circ\ 21'\ 55''$*, which makes the angle* $\kappa\alpha\zeta$ $1^\circ\ 38'\ 5''$*.*

*It was observed on October 27 at* $12^h\ 20^m$ *at* $18^\circ\ 52'\ 15''$ *Taurus, retrograde. Its diurnal motion was 23'. And so on the 26th at* $12^h\ 20^m$ *it was at* $19^\circ\ 15'\ 15''$ *Taurus, while at our time it was at* $19^\circ\ 21'\ 35''$ *Taurus. Therefore,* $\alpha\kappa\zeta$ *is* $3^\circ\ 23'\ 5''$ *and the supplement of* $\alpha\zeta\kappa$ *is* $5^\circ\ 1'\ 10''$*, and* $\alpha\zeta$ *is 67,478.* But this last operation is untrustworthy, owing to the small angles of the triangle. For if an error of one or two minutes is made either in observing or in computing Mars's eccentric position using Tycho's hypothesis, the ratio of the angles is easily changed noticeably. But for now I shall present all four lines for inspection.

| | | | | |
|---|---|---|---|---|
| Sun's mean position | 22° | 59′ | Pisces | $\alpha\theta$ 66,774 |
| | 10 | 6 | Aquarius | $\alpha\eta$ 67,467 |
| | 27 | 13 | Sagittarius | $\alpha\epsilon$ 67,794 |
| | 14 | 20 | Scorpio | $\alpha\zeta$ 67,478 |

So the longest is $\alpha\epsilon$, which is also the closest to the sun's perigee; the shortest is $\alpha\theta$, which is also the farthest from the sun's perigee; and $\alpha\zeta$ and $\alpha\eta$ are about equal, because they are also nearly equally removed from perigee.

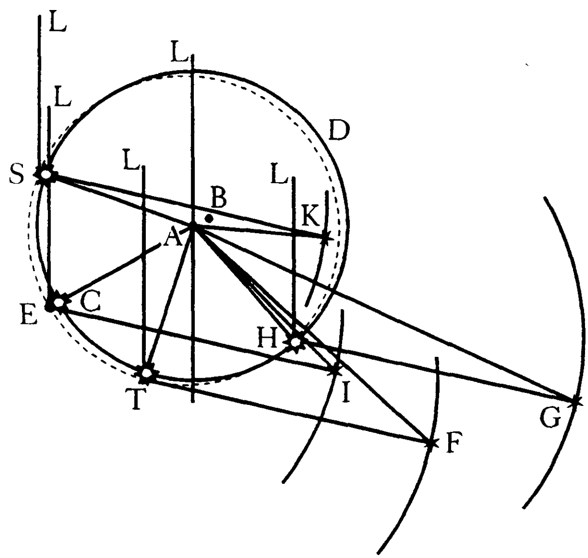
Moreover, even if $\alpha\zeta$ is a little longer than $\alpha\eta$ which is nearer the perigee, this should be attributed to the smallness of the angles at $\zeta$, through which such a small error as this could easily be introduced. Therefore, the circle $\delta\gamma$, which Copernicus described about the point $\alpha$ of uniformity of the earth's motion, is not the earth's path. There is instead some other circle $\theta\eta\epsilon\zeta$ on which the earth is found, whose centre lies in the same direction as the sun – that is, at $\beta$.




In the Ptolemaic form, *let the earth be at* A, *the sun's sphere be* ΞOIT, K *the putative centre of the epicycle; that is, the centre about which [is described] the epicycle* ΔΓ, *itself putative, which is equal to the [circle of the] theory of the sun. The total equivalence between the hypotheses of Copernicus and Brahe requires this, although in the present demonstration it doesn't matter what ratio the sun's orb and the planet's epicycle have, provided that they have equal periods. And let* AΛ *be Mars's line of apsides. Let* AK, AΛ *be parallel to* $\alpha\kappa$, $\alpha\lambda$ *in the preceding Copernican form. From the centre of the earth* A, *let the lines* AΘ, AH, AE, AZ *be drawn parallel to the former* $\kappa\theta$, $\kappa\eta$, $\kappa\epsilon$, $\kappa\zeta$, *and equal to them, so that Mars is at* Θ *in 1590, at* H *in 1592, at* E *in 1593, and at* Z *in 1595; and at the same time the sun's mean position at those times* AT, AI, AO, AΞ, *respectively, so that* KΘ *and* AT *are parallel (as is characteristic of the Ptolemaic hypothesis), and so on for the rest.*

134 *With Θ, H, E, and Z connected with K, it will be demonstrated as before, with exactly the same numbers, lines, and angles, that these lines are unequal, contrary to common opinion, and consequently that Mars does not traverse the circle ΓΔ, whose centre is at the centre of uniform motion K, but the circle ZHEΘ instead, whose centre lies in the direction of B from K, very near to the line KB, which is parallel to the line drawn from the earth A through the sun's perigee.*

Therefore, the epicycle's apogee lies in the direction of the sun's perigee. And because the epicycle, owing to the total equivalence just mentioned, is to be supposed equal to the sun's circuit, with ZK parallel to ΞA, EK to OA, HK to IA, and ΘK to TA, it is also likely that ΞA, OA, IA, and TA are unequal, and that the point of the sun's mean position (the centre of the sun's epicycle, in the Brahean conception) does not stand at the same distance from the point of uniform motion throughout its circuit. This I remark only in passing: it has no effect upon the present demonstration, but serves as an extension to it.



In the Tychonic form, *let* A *be the earth, and about it let the sun's concentric* CD *be described, which is considered to be the deferent of the system of the planets, since* A *is the point of uniform motion of the sun's concentric. Therefore the sun itself will be on another eccentric circle. Let its centre be in the direction of* B *from* A. *Now let* AL *be the reference for Mars's line of apsides, so that the line of apsides, in the circling and translating of its eccentric, remains ever parallel to* AL. *And let the lines of the sun's mean motion at our four moments be* AH, AT, AE, AS, *and from* A *let the lines along which Mars is observed be extended in the direction of one or another degree of the zodiac in accordance with the description above. And since at all four times*
135 *Mars is supposed to be at the same place on the eccentric, its distances from the points of the sun's mean position will all be equal and parallel. Let them be* GH, FT, IE, *and* KS, *all equal, and let the angles* LHG, LTF, LEI, LSK, *all be equal to the previous angle* ΛAK *or* λακ, *so that at our moments Mars might be at* G, F, I, K. I would note in passing that these four points G, F, I, K, in actual fact make an arc exactly equal in length and placement to the previous arc ΘHEZ in the Ptolemaic form. This is because there is no difference between the two other than that Ptolemy has an epicycle, equal to the [circle of the] theory of the sun, carried around on an eccentric, while Tycho has the eccentric carried around on the [circle of the] theory of the sun or on a circle equal to the Ptolemaic epicycle.

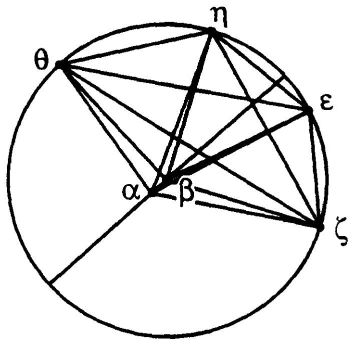
*Once again, then, it will be demonstrated, contrary to common opinion, that the lines* AH, AT, AE, AS *are unequal.* And so that point on the eccentric whence originates the eccentricity of Mars and all the planets (which is here considered to be on the line of the sun's mean motion, following earlier theorists) does not go around on that circle DC about whose centre A it makes equal angles in equal times, but on the circle HTES whose centre lies in the direction opposite the centre of the sun's eccentric B, as has been shown in an approximate way by the lines themselves.

# 25

## From three distances of the sun from the centre of the world, with known zodiacal positions, to find the apogee and eccentricity of the sun or earth

I shall now once again test the quantity of the eccentricity and the position of the apogee in a single circle adapted to all three forms. For it is easily seen that they are simply opposites: for example, in the Copernican form the longest line is towards Gemini, while in the other forms it is towards Sagittarius. This is because Copernicus's observer is looking towards the centre, and the others are looking away from it. Thus Copernicus too looks across the centre at the same parts of the zodiac as the others.

Let the circle θηεζ, with centre β, be set out, in which from the given point α there are the given lines αθ, αη, αε, and αζ, as before, with the angles about α also given, for each of them is 42° 52′ 47″. It is required to find both the magnitude αβ and the direction of that line



with respect to the fixed stars or to the other lines. Let $\theta$, $\eta$, and $\epsilon$ be selected and joined with one another, since three points suffice for investigating this.

136 First, in the triangle $\theta\alpha\eta$ the sides and the included angle are given. $\theta\eta$ is sought, and is shown by trigonometry to be 49,169[1] in the same units in which $\alpha\theta$ and $\alpha\eta$ were expressed.

Second, in triangle $\alpha\epsilon\eta$ the angle $\alpha\epsilon\eta$ is sought, and found to be 68° 12′ 26″.

Third, in the triangle $\theta\alpha\epsilon$ the angle $\alpha\epsilon\theta$ is sought, and is found to be 46° 39′ 10″, which, subtracted from $\alpha\epsilon\eta$ leaves 21° 33′ 16″, which is the angle at the circumference $\theta\epsilon\eta$. Therefore, twice this amount, 43° 6′ 32″, will be $\theta\beta\eta$, the angle at the centre, because $\beta$ is by supposition the centre of the circle. So in the isosceles triangle $\theta\beta\eta$ the angles are given, as well as the side $\theta\eta$ found previously. The size of $\theta\beta$, the radius of the circle, is sought, and found to be 66,923. And since $\beta\theta\eta$ is 68° 26′ 44″, while before, when $\theta\eta$ was being found, $\alpha\theta\eta$ was 69° 18′ 46″, $\beta\theta\alpha$ is therefore 0° 52′ 2″. Next, in the triangle $\beta\theta\alpha$, from the sides and the included angle, $\theta\alpha\beta$ and $\alpha\beta$ are sought. And the angle $\theta\alpha\beta$ is found to be 97° 50′ 30″, so that $\alpha\beta$ is at 15° 8′ 30″ Gemini, because $\alpha\theta$ is at 22° 59′ Virgo. Tycho, however, places the sun's apogee at 5½° Cancer. So you see that this very free enquiry has brought us within 20° of the correct Tychonic position. Also, $\alpha\beta$ is found to be 1023, and if $\theta\beta$ be taken as 100,000, $\alpha\beta$ becomes 1530. But the whole solar eccentricity is 3592, and its half is 1796 or 1800. So here somewhat less than half of the solar eccentricity is claimed for the eccentricity of our circle. But you should bear in mind that observations near minimum values may be somewhat in error, and that use was made of a questionable mean longitude and equation from Tycho. This will become quite clear if you carry out the same operation with the angle $\theta\eta\zeta$, and then with $\eta\epsilon\zeta$ and $\theta\epsilon\zeta$. For each time, $\alpha\beta$ has a somewhat different magnitude, and its position beneath the fixed stars will fall one side or the other of 5½° Capricorn and Cancer.

Consequently, we shall look into this more carefully below, where in many instances, through a fine demonstration, the eccentricity will be found to be half the solar eccentricity, and the apogee very near the Tychonic one.

It has thus been demonstrated in the Copernican form that the

[1] This figure is in error: the correct value would be 49,073. However, this does not affect the value of $\alpha\beta$ much, and brings the line of apsides some six degrees nearer the Tychonic position.

centre of the earth's circuit is halfway between the sun's body and the point of uniform motion of that circuit. That is, that the earth proceeds nonuniformly on its orbit, slowing when it recedes from the sun and speeding up when it approaches. This is in conformity with physical principles and with the analogy with other planets.

In the same way it has been demonstrated in the Ptolemaic form that the epicycle is eccentric with respect to the point about which its motion is uniform, and that its eccentricity is half of the sun's eccentricity as it is commonly determined, and in the opposite direction.

Finally, in the Tychonic form it has been demonstrated that the point whence originate the planets' eccentricities does not move on the sun's concentric, but throughout its course is unequally distant from the earth, about which it moves regularly and uniformly; and that it is farther away near the sun's perigee, and nearer at apogee, again by half of the sun's eccentricity. And so, since this Ptolemaic
137 epicycle and this Brahean deferent have this much of an analogy with the theory of the sun, it is likely that they also have a greater analogy. That is, it seems likely that the sun's true eccentricity, too, is only half of that computed from the maximum equation; or, what is the same thing, that the sun makes use of an equant, whose eccentricity is twice the eccentric's eccentricity.

I admit that this line of argument is a little weaker when applied to the Ptolemaic and Tychonic form, insofar as we are using the sun's mean motion, following the authorities. The argument consequently will become clearer when, moved by those reasons adduced in chapter 6 above, I measure out the planet's motion by the sun's apparent motion.

# 26

## Demonstration from the same observations that the epicycle is eccentric with respect to the point of attachment or axis, and that the annual orb (and so also the earth's path around the sun, or the sun's around the earth) is eccentric with respect to the body of the sun or earth, with an eccentricity just half that which Tycho Brahe found through equations of the sun's motion

We shall go through the observations[1] once more, carefully: On 1590 March 4 at $7^h\ 10^m$, Mars was found by careful observation and calculation to be at 24° 22′ 56″ Aries with latitude 0° 3′ 20″ S. At that time, 8° Aries was setting, so Mars was rather low. Therefore, it was raised up towards the east by refraction, so it seems right that without refraction it would have been seen at 24° 20′ Aries. Its parallax can only be very small, particularly at this distance, for Mars was near the sun and therefore had receded very far from the centre of the earth.

On 1592 January 23 at $7^h\ 20^m$, using only one stellar distance from Mars, without the confirmation of another, Mars was found at 11° 32′ 44″ Aries with latitude 0° 1′ 36″ S. And so we shall make no changes for horizontal variations, although we suspect an uncertainty of one or two minutes.

On 1593 December 7 at $8^h\ 0^m$ Mars was found at 3° 6′ 50″ Aries with no danger of horizontal variation, with latitude 7′ 9″ S. However, the right ascension found using three stars showed a discrepancy of 4′, and the value taken as true was the mean between the extremes.

On 1595 October 25 at $8^h\ 10^m$, the planet's distance from three fixed stars was observed, and by unanimous consensus the planet was found to be at 19° 39′ 25″ [Taurus] with latitude 0° 12′ 41″ S.

[1] That is, the observations presented in ch. 24.

138 Now we shall reduce the three subsequent times to the first. Accordingly, to its sidereal position

| | | | |
|---|---|---|---|
| on the eccentric in | 1590 | 4 March | $7^h$ $10^m$ |
| Mars will return in | 1592 | 20 January | 6 45 |
| | 1593 | 7 Decemb. | 6 15 |
| | 1595 | 25 October | 5 45 |

The motion over three days and 35 minutes of time in 1592 was 2° 9′ 4″, according to Magini. Therefore, at our time Mars was seen at 9° 23′ 40″ [Aries]. In 1593 the motion over 1 hour 45 min., from the diurnal motion of 33′, is 2′ 25″. And so at our time Mars's position comes out to be 3° 4′ 27″ Aries. Likewise, in 1595 the motion over 2 hours 25 min., from a diurnal motion of 22′ 11″, is 2′ 14″. Therefore at our time the position of Mars comes out to be 19° 41′ 39″ Taurus.

**From this there follows the table of positions**

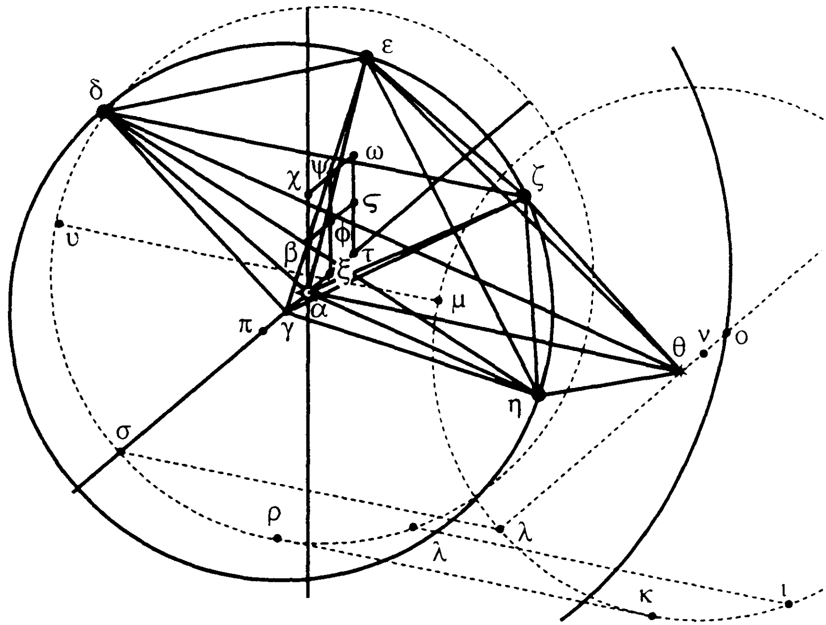
of Mars, from observation; of the sun, from Tycho's calculation

| | Mars | | Sun | |
|---|---|---|---|---|
| 1590 | 24° 20′ | Aries | 24° 0′ 25″ | Pisces |
| 1592 | 9 24 | Aries | 10 17 8 | Aquarius |
| 1593 | 3 4½ | Aries | 25 53 24 | Sagittarius |
| 1595 | 19 42 | Taurus | 11 41 34 | Scorpio |

Now because we have proposed to enquire how far the earth is from the centre of the sun, we need first of all to use the hypothesis constructed above in ch. 16, from oppositions to the sun's apparent position, to find the position of the line drawn from the centre of the sun through the body of Mars to the zodiac. And on 1595 October 25 at $5^h$ $45^m$ that line is found at 14° 19′ 52″ Taurus. Therefore, for the three other times it is set back each time by 1′ 36″. Thus, in 1593 it was at 14° 18′ 16″ Taurus; in 1592, at 14° 16′ 40″ Taurus; and in 1590, at 14° 15′ 4″ Taurus.

**Let the first diagram be in Copernicus's form**

139 *And let $\alpha$ be the centre of the sun; $\beta$ the centre of Mars's eccentric, drawn through* o; *$\chi$ the centre of uniform motion for Mars's eccentric; $\gamma$ the centre of the earth's eccentric; $\delta$, $\epsilon$, $\zeta$, $\eta$ four positions of the earth, opposite the sun's apparent positions; $\theta$ the position of Mars on its eccentric. Let all points be connected with one another.*




| | | | | | |
|---|---|---|---|---|---|
| *Now, in triangle δαθ* | | | *Likewise, in triangle εαθ* | | |
| *because δα is* | *24° 0′ 25″* | *Pisces* | *because εα is* | *10° 17′ 8″* | *Aquarius* |
| *and δθ* | *24 20 0* | *Aries* | *and εθ* | *9 24 0* | *Aries* |
| *Angle αδθ is* | *30 19 35* | | *Angle αεθ is* | *59 6 52* | |
| *And because δθ is* | *24 20 0* | *Aries* | *And because εθ is* | *9 24 0* | *Aries* |
| *and αθ* | *14 15 4* | *Taurus* | *and αθ* | *14 16 40* | *Taurus* |
| *Angle δθα is* | *19 55 4* | | *Angle εθα is* | *34 52 40* | |
| *Let αθ be taken as 100,000. By trigonometry, the required magnitude αδ comes out to be 67,467.* | | | *So εα comes out to be 66,632.* | | |
| *In triangle ζαθ* | | | *Finally, in triangle ηθα* | | |
| *because ζα is* | *25° 53′ 24″* | *Sagit.* | *because ηα is* | *11° 41′ 34″* | *Scorpio* |
| *and ζθ* | *3 4 30* | *Aries* | *and ηθ* | *19 42 0* | *Taurus* |
| *The suppl. of αζθ is* | *82 48 54* | | *The suppl. of αηθ is* | *8 0 26* | |
| *And because ζθ is* | *3 4 30* | *Aries* | *And because ηθ is* | *19 42 0* | *Taurus* |
| *and αθ is* | *14 18 16* | *Taurus* | *and αθ is* | *14 19 52* | *Taurus* |
| *ζθα is* | *41 13 46* | | *ηθα is* | *5 22 8* | |

*So $\zeta\alpha$ comes out to be 66,429* *So $\eta\alpha$ comes out to be 67,220*[2]

*Here are the distances of the centre of the sun from the earth, gathered together for you:* $\delta\alpha$ *67,467*
$\epsilon\alpha$ *66,632*
$\zeta\alpha$ *66,429*
$\eta\alpha$ *67,220*

We shall now test the magnitude of the eccentricity that may be deduced from these distances. If the sun's theory lacks a equant, the eccentricity of its circle will turn out to be about 3600. This is because we used the true or apparent positions of the sun, whose point of uniform motion has to be that far (namely, 3600) from the centre of the world, as Brahe has proved from solar observations. But if, on the other hand, the eccentricity turns out to be less, and is about half the Brahean value, we have won, and vindicated our contention, that the point of uniform motion found by Brahe is not the centre of the sun's eccentric.

*You can see at a glance (I would note in passing) that $\alpha\zeta$ is the shortest, it being near the sun's perigee; next, that $\alpha\epsilon$ is longer, it being in Aquarius, 34 degrees from perigee; then $\alpha\eta$, it being 54 degrees from perigee; and lastly, that $\alpha\delta$ is longest, because it is 80 degrees from perigee. And since $\alpha\zeta$ is almost at perigee, it will be hardly any longer than the shortest. Similarly, since $\alpha\delta$ is near the middle longitudes, it will be only a little less than the mean distance. Therefore, the eccentricity will come out to be a little greater than 1038, which is the difference between $\delta\alpha$ and $\zeta\alpha$. And if $\delta\alpha$ is assigned the measure of 100,000, 1038 will have the value 1539; and this is about what the eccentricity amounts to (it is actually a little greater). And this is much closer to 1800, half the Tychonic value, than to the full value of 3600.*

140 *The same is to be said of the sun's apogee. For because $\zeta\alpha$ is shortest, the perigee is about 25° 53′ Sagittarius. And because $\epsilon\alpha$ is shorter than $\eta\alpha$, the perigee is closer to 10° 17′ Aquarius than to 11° 42′ Scorpio. But the mean is 25° 57′ Sagittarius. Therefore, the perigee is beyond 25° 57′ Sagittarius and before 10° 17′ Aquarius; that is, in Capricorn.*

This I wanted to give as a foretaste to make up for the coming labours. For now I shall follow the geometric path to investigate the

[2] An erroneous figure: this should be 67,171. The translator completed the calculation using this corrected value, and found the eccentricity $\alpha\gamma$ to be 2382. This is nearly the same as the value 2401 given by Delambre (*Histoire de l'Astronomie Moderne*, vol. I p. 433, cited in KGW 3 p. 466).

position of the apogee and the eccentricity. And since three points determine a circle, I am at first using the points δ, ζ, and η.

*I proceed as in ch. 25 above. Since the points δ, ζ, η are positions on a single circumference with centre γ, the angle δηζ will be half of the angle δγζ and of its measure, the arc δζ. Therefore, the ratio of δζ to the radius δγ, and to the eccentricity γα, will be given, together with the angle δαγ, because αγ lies in the direction of the apsides. But for the knowledge of the angle δηζ and of the line δζ, we need to solve the three triangles.*

| | |
|---|---|
| *First, in δαζ, because αδ is in* | *24° 0′ 25″ Pisces* |
| *and αζ* | *25 53 24 Sagittarius* |
| *Therefore, δαζ is* | *88 7 1* |
| *Add 3′ 12″ for precession* | *88 10 13* |
| *The two remaining angles δ and ζ* | *91 49 47* |
| *Half* | *45 54 54* |
| *Tangent of this* | *103246.* |
| *From this and αδ,* | *67,467* |
| *and αζ,* | *66,429* |
| *is found angle αδζ* | *45° 27′ 22″* |
| *and its sine* | *71271;* |
| *from which, with side αζ,* | |
| *δζ is found:* | *93,159.* |

| | |
|---|---|
| *Second, in δαη, since αδ is* | *24° 0′ 25″ Pisces* |
| *and αη is* | *11 41 34 Scorpio* |
| *Therefore δαη is* | *132 18 51* |
| *Add for precession* | *4 48* |
| | *132 23 39* |
| *The two remaining angles δ and η* | *47 36 21* |
| *Half* | *23 48 11* |
| *Tangent* | *44110.* |
| *From this and αδ,* | *67,467* |
| *and αη,* | *67,220* |
| *is found angle αηδ* | *23° 51′ 0″* |

| | |
|---|---|
| 141 *Third, in ζαη, since αζ is* | *25° 53′ 24″ Sagittarius* |
| *and αη is* | *11 41 34 Scorpio* |
| *Therefore ζαη is* | *44 11 50* |
| *Add for precession* | *1 36* |
| | *44 13 26* |

*The two remaining angles* $\zeta$, $\eta$ 135 46 34
*Half* 67 53 17
*Tangent* 246120.

*From this and* $\alpha\zeta$, 66,429
*and* $\alpha\eta$, 67,220
*is found angle* $\alpha\eta\zeta$ 67° 3′ 12″.

| | | | |
|---|---|---|---|
| *So, since* $\alpha\eta\delta$ *is* | 23° 51′ 0″ | *And since* $\alpha\delta\zeta$ *is* | 45° 27′ 22″ |
| *and* $\alpha\eta\zeta$ *is* | 67 3 12 | *and* $\gamma\delta\zeta$ *is* | 46 47 48 |
| *Therefore* $\delta\eta\zeta$ *is* | 43 12 12 | *Therefore* $\gamma\delta\alpha$ *is* | 1 20 26 |
| *Hence,* $\delta\gamma\zeta$ *is* | 86 24 24 | *The other two* $\gamma$, $\alpha$, | 178 39 34 |
| *The other two* $\delta$, $\zeta$, | 93 35 36 | *Half* | 89 19 47 |
| *Half,* $\gamma\delta\zeta$, | 46 47 48 | *Tangent* | 8540000 |
| *Whose sine is* | 72893 | *Let* $\gamma\delta$ *be taken as* | 100,000 |
| *From this and* $\delta\zeta$, | | *Of these units,* | |
| $\delta\gamma$ *is found to be* | 68,141 | $\alpha\delta$ *will be* | 99,011 |
| | | *Hence* $\delta\gamma\alpha$ *is found:* | 68° 26′ 7″ |
| | | *So that* $\alpha\gamma$ *is at* | 15° 34′ 18″ *Capricorn*[3] |
| | | *But the sine of* $\delta\alpha\gamma$, | 93000 |
| | | *And the sine of* $\gamma\delta\alpha$, | 2340 |
| | | *Show that the eccentricity* $\alpha\gamma$ *is* | 2516 |

And yet it was said before that using $\delta$ and $\zeta$ the eccentricity comes out to be a little greater than 1539, supposing that $\zeta$ is nearest to perigee. But since here (letting $\eta$ be received into the company in place of $\zeta$) the eccentricity comes out much greater, this hints, although erroneously, that there is some line at perigee which is still shorter than $\alpha\zeta$. In order that this distance at perigee might be able to be shorter than $\alpha\zeta$, this line of argument has moved the perigee to 16 Capricorn, farther from $\alpha\zeta$.

But since we already know that the sun's perigee is not in 16 Capricorn but in 6 Capricorn, it is appropriate to assign the cause of the slight error to the point $\eta$ and to the excessive length of the line

[3] Here Kepler seems to have made an elementary blunder: he ostensibly subtracted the angle $\delta\gamma\alpha$ from the position of the line $\delta\alpha$ to obtain the position of the line $\alpha\gamma$. He needed to use the position of the line $\delta\gamma$ instead, whose longitude is 1° 20′ 26″ less. However, the value he gives for the angle $\delta\gamma\alpha$ is inconsistent with his values for the sides and included angle of the triangle $\delta\gamma\alpha$. According to the translator's calculations, $\delta\gamma\alpha$ should be 66° 18′ 53″, and $\delta\alpha\gamma$ should be 112° 20′ 41″, whose supplement is 67° 39′ 19″, 1° 20′ 26″ greater than $\delta\gamma\alpha$. Perhaps the number he gives is intended to be the supplement of $\delta\alpha\gamma$.

Using correct values throughout, the translator has found the resulting perigee to be at 11° 57′ 44″ Capricorn.

$\alpha\eta$[4]. For the result of this is that the circle $\delta\epsilon\eta$ would be too large, and its radius $\delta\gamma$ too long; and consequently $\gamma\alpha$ would be too long, and $\gamma$ would move perpendicularly away from the line $\delta\eta$, but obliquely from the point $\zeta$. Thus, the line $\gamma\alpha$ would now be placed too far forward in longitude. Therefore, letting $\delta$ and $\gamma$ remain, let $\alpha\eta$ be supposed shorter. Then the centre $\gamma$ moves perpendicularly toward the line $\delta\eta$, and $\delta\gamma$ thus becomes shorter. And because $\gamma$ approaches $\delta\eta$ perpendicularly, it recedes obliquely from the present $\gamma\alpha$. Hence, if a straight line be drawn from $\alpha$ through the new position of $\gamma$, it will be inclined back towards $\delta$.

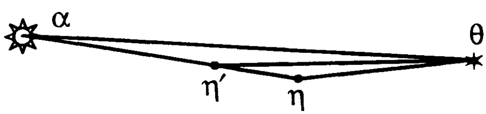
You thus see how shortening $\alpha\eta$ helps us in two ways. But to shorten $\alpha\eta$ is a very easy, small change, because the angles are small: it can be done by saying that the planet was seen in a slightly prior position along a line drawn from $\eta$ below $\theta$[5]. *For example, let the observed position of Mars be 19° 40′ Taurus, and the supplement of* $\alpha\eta\theta$ *be 7° 58′ 26″, and* $\eta\theta\alpha$ *5° 20′ 8″: then* $\alpha\eta$ *will be 67,030. The second and third triangles are then changed,* $\alpha\eta\delta$ *becoming 23° 53′ 0″, and* $\alpha\eta\zeta$ *67° 15′ 32″. Therefore,* $\delta\eta\zeta$ *is 43° 22′ 26″, and* $\delta\gamma\zeta$ *86° 44′ 52″. The*
142 *remaining angles are 93° 15′ 8″ whose half,* $\gamma\delta\zeta$, *is 46° 37′ 34, and* $\gamma\delta\alpha$ *is 1° 10′ 12″. Hence* $\delta\gamma$ *is 67,892. And where this is 100,000,* $\alpha\delta$ *will be 99,416, and* $\delta\gamma\alpha$ *73° 24′ 39″. Consequently the perigee is in 10° 36′ Capricorn, and the eccentricity is still about 2100.*

Thus, as the true perigee was approached the eccentricity decreased, and so when we get exactly to the correct perigee we shall also get exactly to the halving of the eccentricity.

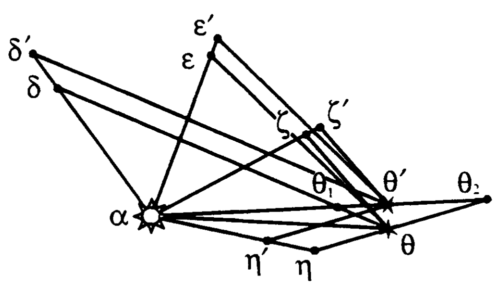
But it is nonetheless worthwhile to find out, in addition, how much

[4] Here Kepler displays his remarkable insight into the interrelationship of the variables in the orbit. Although he did not find the error in the computed length of $\alpha\eta$, he sensed it was too long, and adjusted the data to shorten it, in what is surely the first deliberate published example of what has come to be called the 'fudge factor'.

[5] The letters $\eta$ and $\theta$ have been exchanged to preserve the sense of the argument. It is clear that Kepler means that point $\theta$ will appear as if farther down in the diagram. However, it may be clearer to imagine the point $\eta$ moving directly toward $\alpha$ (to $\eta'$), $\theta$ meanwhile remaining fixed. The angle $\alpha\eta\theta$ thus increases, which is in fact what happens in the numerical example that follows.



we gain by changing the line $\alpha\theta$, adding one minute to the computed eccentric position of Mars while keeping fixed the point of observation (that is, the point $\eta$) for the year 1595. *With* $\alpha\theta$ *accordingly moved forward, if these lines of sight* $\eta\theta$, $\zeta\theta$, *and the rest were to stay the same, it would come about that* $\alpha\theta$ *would be intersected by* $\eta\theta$ *at a place higher than* $\theta$; *and on the other hand it would be intersected by* $\zeta\theta$ *and its counterparts at a place lower than* $\theta$[6]. *So* $\alpha\theta$ *would not keep the same length. But since we supposed that Mars is at the same eccentric position all four times, the length of* $\alpha\theta$ *will also be the same for all four times. Therefore, in order that the point of intersection* $\theta$ *be the same, and nonetheless the lines of sight go towards their original zodiacal positions, it will be necessary to draw a line parallel to* $\eta\theta$ *somewhat lower, thus making* $\alpha\eta$ *shorter; and also a line outside* $\zeta\theta$ *and parallel to it, by which* $\alpha\zeta$ *would be lengthened; and so on for the rest. Next, the whole labour is to be repeated from the beginning. For* $\delta\theta\alpha$ *will now be 19° 56′ 4″,* $\epsilon\theta\alpha$ *34° 53′ 40″,* $\zeta\theta\alpha$ *41° 14′ 46″, and* $\eta\theta\alpha$ *5° 21′ 8″. Therefore* $\delta\alpha$ *will be 67,522,* $\epsilon\alpha$ *66,660,* $\zeta\alpha$ *66,451,* $\eta\alpha$ *66,963. Hence* $\alpha\delta\zeta$ *will be 45° 26′ 37″,* $\alpha\eta\delta$ *23° 54′ 30″,* $\alpha\eta\zeta$ *67° 20′ 48″. And* $\delta\eta\zeta$ *will be 43° 26′ 18″, and* $\delta\gamma\zeta$ *86° 52′ 36″,* $\gamma\delta\zeta$ *46° 33′ 42″, and* $\gamma\delta\alpha$ *1° 7′ 5″ – a different angle from different beginnings. Now, when* $\alpha\zeta$ *is divided by the sine of* $\alpha\delta\zeta$, *and the quotient multiplied by the sine of* $\delta\alpha\zeta$, *the product is* $\delta\zeta$, *93,252. Again, when this is divided into the sine of* $\delta\gamma\zeta$, *and the quotient is multiplied by the sine of* $\delta\zeta\gamma$, *the product will be* $\delta\gamma$, *67,823. Hence the angle* $\delta\gamma\alpha$ *is 76° 37′ 30″, and the perigee is at 7° 23′*




6 'Higher' and 'lower' here (and in the third sentence following) mean 'farther from $\alpha$' and 'nearer $\alpha$', respectively.

Kepler is saying that if the only line to change position is $\alpha\theta$, then the old lines from $\delta$, $\epsilon$, $\zeta$, and $\eta$ will intersect it at different points (for instance, at $\theta_1$ and $\theta_2$ in the adjacent diagram). Kepler's solution is to take a point $\theta'$ on the new line, with $\alpha\theta' = \alpha\theta$, and to draw lines from $\theta'$ parallel to the old lines from $\delta$, $\epsilon$, $\zeta$, and $\eta$, intersecting the lines (or their extensions) from $\alpha$ to those points at $\delta'$, $\epsilon'$, $\zeta'$, and $\eta'$.

*Capricorn, with an eccentricity about 1880,* which would clearly be 1800 if the perigee were brought back to $5\frac{1}{2}$ Capricorn, as could happen through the combined effect of both causes.

143 For if you now subtract only half a minute from the apparent position for 1595, we will be within range. And there could easily be a one minute error in the equations of the eccentric found by the hypothesis of chapter 16.

So, since the result is easily vitiated by the data from 1595, let us now omit them and use the remaining three points $\delta$, $\epsilon$, $\zeta$, keeping the most recent correction of eccentric position, and forming the new triangles $\delta\alpha\epsilon$ and $\epsilon\alpha\zeta$.

| | |
|---|---|
| *Now because* $\alpha\delta$ *is* | *24° 0' 25" Pisces* |
| *and* $\alpha\epsilon$ | *10 17 8 Aquarius* |
| *Therefore angle* $\delta\alpha\epsilon$ *is* | *43 43 17* |
| *For precession of equinoxes add* | *1 36* |
| | *43 44 53* |
| *From this and* $\alpha\delta$, | *67,522* |
| *and* $\alpha\epsilon$, | *66,660* |
| $\alpha\delta\epsilon$ *is found to be* | *67° 12' 35"* |
| *But* $\alpha\delta\zeta$ *was, and remains,* | *45 26 37* |
| *Therefore* $\epsilon\delta\zeta$ *is* | *21 45 58* |
| *and* $\epsilon\gamma\zeta$ *is* | *43 31 56* |
| *Similarly because* $\alpha\epsilon$ *is* | *10° 17' 8" Aquarius* |
| *and* $\alpha\zeta$ | *25 53 23 Sagittarius* |
| *Therefore angle* $\epsilon\alpha\zeta$ *is* | *44 23 44* |
| *Precession of equinoxes* | *1 36* |
| | *44 25 20* |
| *From this and* $\alpha\epsilon$, | *66,660* |
| *and* $\alpha\zeta$, | *66,451* |
| $\alpha\zeta\epsilon$ *is found to be* | *68° 0' 34".* |
| *Add to* $\alpha\delta\zeta$ | *45 26 37* |
| *Angle* $\delta\alpha\zeta$ | *88 10 13* |
| | *133 36 50* |

| | |
|---|---|
| *αζδ will be* | *46° 23′ 10″* |
| *Therefore εζδ will be* | *21 37 24* |
| *and εγδ* | *43 14 48* |
| *Hence δγζ* | *86 46 44*[7] |
| *and γδζ* | *46 36 38* |
| *While αδζ remains* | *45 26 37* |
| *Therefore γδα will be* | *1 10 1* |

*And because δζ remains 93,252, as before, dividing the sine of γδζ by the sine of δγζ and multiplying the quotient by δζ gives γδ as 67,873. But αδ is 67,522. From this and γδ,*

*δγα is found to be 75° 8′ 40″*

*and the perigee in 8 51 45 Capricorn,*

*about what it was before. The eccentricity, somewhat more than 2000, is to be decreased to 1800, if the perigee were to be moved back to $5\frac{1}{2}$° Capricorn. This is accomplished by lengthening αε. And αε is lengthened if we say that the planet appeared one or two minutes before 9° 24′ Aries. For then, from the point θ set up by the other lines of observation, some line would be drawn outside θε towards θζ.*

One might, however, hold suspect such license in making small changes in the data, thinking that by taking the same liberty in changing whatever we don't like in the observations, the full Tychonic eccentricity might also at last be obtained. Anyone who thinks this way should make a try at it, and, comparing his changes with ours, he should judge whether the changes remain within the limits of observational precision. He also needs to beware lest, elated by the results of one such iteration, he give himself much more hideous problems in what follows, because of the very divergent apogees found for the sun.

I, to be sure, have laid all my prejudices and preferences out in the open here, so that I am more afraid of appearing to the reader to be importunate than I am of seeming untrustworthy.

And by the way, a remark for future use: if γδ is made 100,000, αθ will come out to be 147,443, or even greater when the figures that are still missing are obtained correctly.

144 Finally, to avoid prolixity, if αθ be 147,700, and the eccentric position of Mars in 1595 be 14° 21′ 7″ Taurus, and the eccentricity of

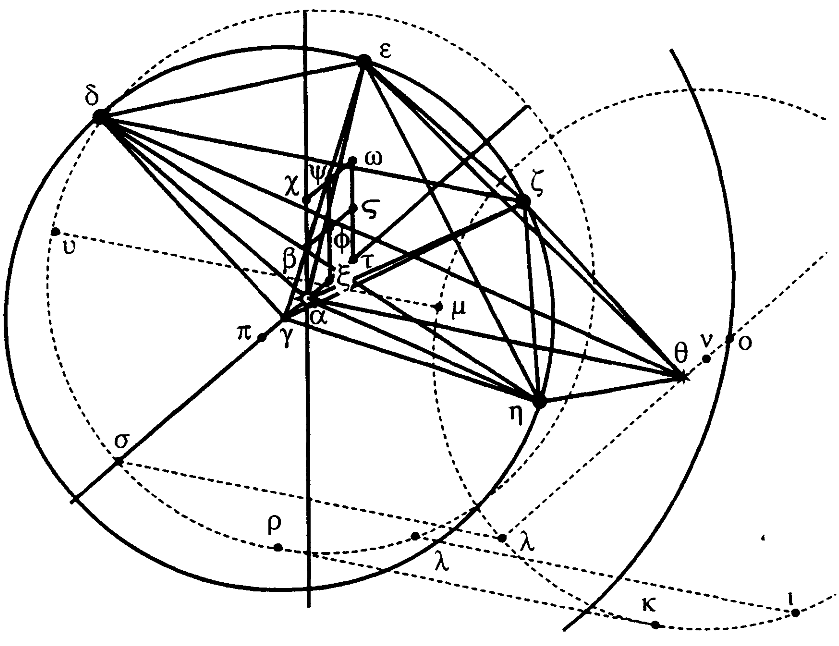
[7] This is obtained by adding εγζ and εγδ. Then γδζ is half the supplement of this sum.

the earth be 1800, and the earth's path oval, as will be said in chapters 30 and 44, the appearances come out to be

| | | | | | | |
|---|---|---|---|---|---|---|
| 24° | 21′ | 13″ | Aries; | Should be | 24° | 20′ |
| 9 | 23 | 20 | Aries | | 9 | 24 |
| 3 | 2 | 30 | Aries | | 3 | $4\frac{1}{2}$ |
| 19 | 42 | 40 | Taurus | | 19 | 42 |

From this figure, I conclude that αθ is about 147,750.

And thus it has been demonstrated that αγ is about 1800, although it ought to have been 3600 if Tycho's discoveries were accommodated to the Copernican form and the sun's apparent motion. Consequently the point π of the earth's uniformity of motion must be sought on the line απ, so that γπ and γα are equal. For if the earth is moved uniformly about π, so that δπε, επζ, ζπη are equal, Tycho's observations of the sun will remain unaltered, and πα will be 3600. And at the same time, since the earth will be at the same distance from point γ when at the points δ, ε, ζ, η, the observations of Mars will also remain unchanged.



In the Ptolemaic form, the delineation can be done in two ways. In the first, let the earth replace the solar body at α, *and then lines of vision be drawn out from α parallel to δθ, εθ, ζθ, ηθ, so that the Copernican positions of the earth δ, ε, ζ, η coalesce into one Ptolemaic position of the earth. Meanwhile, let the star Mars, which for Copernicus had stayed at one point θ, now be placed about θ at four locations: ι, κ, λ, μ. The description of this circle is as follows. Upward through θ let θν be drawn equal and parallel to γα. And about centre ν, with radius γε, let the circle ικλμ be described. Thus the point θ, which we can call the point of attachment, moves around on the eccentric, which before, in Copernicus, the planet bodily traversed. While the epicycle is thus being borne around, the centre ν is propelled around θ, so that at one time it is inside θα and at another it is outside; however, θν is always parallel to itself and to the line αγ. And the epicycle will be moved uniformly, not about θ to which it is attached, nor about its centre ν, but about a higher point, ο, such that θο is twice θν. For the earth too will thus be moved*[8] *uniformly about π, not about the centre of its orb γ, nor about the sun at α.*

Delineation of the theory of the epicycle.

Point of attachment. See Part I.

145 That these characteristics belong to the Ptolemaic epicycle, is properly demonstrated. But that they are carried over from the epicycle to the theory of the sun is shown by a probable argument only, pieced together from Ptolemaic opinions. *For, keeping everything as it was, let ατ be set up equal to απ and in the same line but in the opposite direction, so that τ may be the centre of the sun's uniform motion, which the theorists had thought to be the centre of the sun's orbit. Therefore, the line θνο will always be parallel to the line of the sun's apogee ατ. Now, if you have decided that the diurnal parallax of Mars should be kept in the same ratio to the sun's parallax as that given out by Tycho, ικλμ will also be equal to the [circle of the] sun's theory, and consequently θο will also be equal to the eccentricity of the point τ, about which the sun moves uniformly. But ικλμ also moves in the same direction in which the sun moves on its circle, according to Ptolemy, and at the same times both are found in the same, or at least corresponding, places, the sun on its eccentric, and the planet on its epicycle, so that lines from τ through the sun and from ο through the planet are ever parallel, again as taught by Ptolemy. So, since all other things are in agreement, why not this, too: that, just as ικλμ is moved uniformly, not about the centre ν but about the point ο above it, as is*

[8] Reading *movebitur* instead of *movebatur*.

*demonstrated here by the transposition of the eccentric of the earth into the epicycle, where we took* θ *in place of* α, ν *for* γ, *and* ο *for* π: *so likewise, these points are distinct in the sun, so that the eccentricity* ατ, *which is found from solar observations, is to be bisected at* ξ, *with* ξ *the centre of the sun's eccentric* λρσυ? *For Ptolemy made use of such a procedure to make it appear that if the sun's apparent positions were used, exactly the same eccentricity would be used on the planet's epicycle as was found in the sun. So since the observations give evidence of the double eccentricity of the Ptolemaic epicycle (because, as was said, the parallel relationships of the lines leave the triangles the same as in the Copernican form), the spirit of Ptolemy urges us to bisect the sun's eccentricity as well, so that the lines* λι, ρκ, σλ, υμ *remain parallel.*

So by this reasoning even Ptolemy will be persuaded that ατ, the eccentricity of the sun's motion found by Tycho, should be bisected at ξ, so that the centre of the sun's orbit is at ξ, and the centre of uniformity of motion at τ.

Ptolemy refuted in passing.

Now this argument in the Ptolemaic form (as I just now began to say) is no firmer than the Ptolemaic world system itself. For anyone who believes Ptolemy, thinking that for the three superior planets there are three theories of epicycles, exactly equal to the theory of the sun, in quantity and quality, in lines as well as motions, in absolutely all respects – this same person will not admit this one inconsistency, but will gladly derive the bisection, too, from the epicycle, transferring it to the solar theory as if from an image in a mirror to the face itself.

And finally, when a comparison of hypotheses has been made, and it has appeared that four theories of the sun (or rather, six, as will be said elsewhere) can be generated from a single theory of the earth, like many images from one substantial face, the sun itself, the clearest of truth, will melt all this Ptolemaic apparatus like butter, and will disperse the followers of Ptolemy, some to Copernicus's camp, and some to Brahe's.

**146** Here, one might raise a question. The Ptolemaic epicycle has three notable points: the centre ν, the point θ which we have called the point of attachment, and the point ο about which its motion is uniform. Now since it is said that the line θο remains parallel to ατ throughout the entire circuit, what are the properties of the circuits described by the other two points ν and ο? *In order to show this, let lines be drawn from* ξ *and* τ *parallel to* αβ, *and from* β *and* χ *parallel to*

*ατ, and let them be extended so as to intersect one another; and let the intersection of the lines from ξ and β be φ, from ξ and χ be ψ, from τ and β be ς, and from τ and χ be ω. Now, just as the point θ is moved regularly around χ, traversing an eccentric described about β, so also ν is moved regularly around ψ, traversing an eccentric described about φ. Also, ο is moved regularly about ω, traversing a third eccentric likewise equal to the others, described about ς. For all three of these eccentrics the zodiacal position of the apogee is the same, owing to the lines' αχ, ξψ, τω being parallel. But the word 'apogee' cannot be applied properly to any of these, apart from the first, that belongs to the point θ, since its line of apsides αβχ is drawn through the earth itself, which was placed at α, but not at ξ or τ.*

*It is indeed true that straight lines can be drawn from the earth α through the centres of the remaining two eccentrics φ and ς, which may properly be called 'lines of the apogee'. These will fall antecedently to the apogee αχ; that is, αφ will be at 24° Leo, and ας at 19° Leo, approximately. But then these lines will not pass through the point of uniform motion of whichever eccentric they belong to. Thus, if any of Ptolemy's followers does not wish to attach the epicycle to the eccentric at the point θ, but prefers to relate it to the centre ν, he will be driven to use two lines of apsides: one, αφ, for the eccentric, and the other, αψ, for the equant; and also two eccentricities, αφ and αψ. How intricate and inconvenient this is (for of its absurdity enough has been said in chapter 6), let anyone so inclined judge.*

*The same will happen if anyone wishes to attach the epicycle to the eccentric at the point ο, about which the epicycle revolves uniformly. For then the eccentric bearing the point ο will have two apogees and*
147 *eccentricities, one for the centre on the line ας, and the other for the point of uniformity of motion on the line αω. The remaining possibilities are either to attach the epicycle at θ, or to take improper apogees for the eccentrics that bear the points ν and ο, and to compute the eccentricities from the points ξ and τ, not from the reference point, the earth α.*

What has been given so far is the first delineation in the Ptolemaic form. The other can be constructed as follows. Let the Copernican positions of the earth δ, ε, ζ, η, merge, not in α, but in γ, so that in this diagram, γ, not α, would denote the earth, the centre of the world. Here, the epicycle too, as well as its three eccentrics belonging to the points θ, ν, ο, will be shifted from their positions by the amount αγ, and a perfect equivalence will result. I shall forego further expla-

nation, lest the reader become too confused, for this has really only been mentioned for smatterers or curiosity seekers.

In the Tychonic form, there is no need of any new delineation. A very brief sketch will suffice. *Let the eccentric's point of attachment be in the four different positions* λ, ρ, σ, υ, *so the planet would be at* ι, κ, λ, μ, *and* ιλ, κρ, λσ, μυ, *and* θα *parallel. Now Tycho would say that the centre of Mars circle, which he makes carry a double epicycle, goes around* α *uniformly on a circle concentric with the sun, and for this idea he is indebted to Ptolemy. And in this matter, he, along with Ptolemy and Copernicus, was strongly urged by me (in ch. 6 of Part I) to seek that point of attachment, whether it is the centre of the concentric or of an eccentric, in the centre of the solar body, this being supported by physical arguments and by the demonstration of its geometrical possibility. Additional support is provided by the valid argument of ch. 22 and 23, that unless this were done, even if the observations were referred to the sun's mean position, the Ptolemaic epicycle and the Brahean deferent would be made eccentric, in directions exactly opposite to the sun's eccentricity. I have also promised stronger arguments, deduced from Brahe's own observations, for abandoning the sun's concentric, and in ch. 52 and 57 below I will produce them. But it has already been proved here in chapter 26 that this centre of Mars's concentric (or the point from which Mars's eccentricity originates) is found, not on an equal eccentric described about the centre of the sun's point of uniform motion* τ, *as Brahe along with the other authorities had believed, but on an eccentric described about* ξ, *which is in the middle position between* α *and* τ.

*Therefore, if the centre of Mars's concentric goes around with the sun, it nevertheless goes around on an eccentric described about* ξ, *and consequently the sun itself will go around on an eccentric described about* ξ. *But its motion is uniform about* τ. *Therefore, the sun's eccentricity* ατ *must be bisected at* ξ. For it is not likely that, although the centres of the concentrics of Mars and the sun go around in the same way, reach apogee at the same time, transpose their apogees in the same way, go slowly or quickly in the same way, and describe the same circumferences, their circles would nevertheless make different digressions from the earth in the same direction.

Let it be enough for now, to present this form of demonstration in the three hypotheses. In what follows, whenever there is need of the

148 same demonstration, I shall use Copernicus's form alone, it being the simplest, so as not to be too long-winded. Here, in contrast, the industrious reader has seen how any of these diagrams can be transformed into either the Ptolemaic or the Copernican form using parallel lines.

# 27

## From four other observations of the star Mars outside the acronychal situation but still in the same eccentric position, to demonstrate the eccentricity of the earth's orb, with its aphelion and the ratio of the orbs at that place, together with the eccentric position of Mars on the zodiac

Hitherto we have almost exclusively used the aphelion of Mars, along with the correction of the mean motion and the hypothesis of the equations found above. If these should err by a single minute in defining the planet's zodiacal longitude, as can easily happen, this creates considerable difficulty for us in the present undertaking.

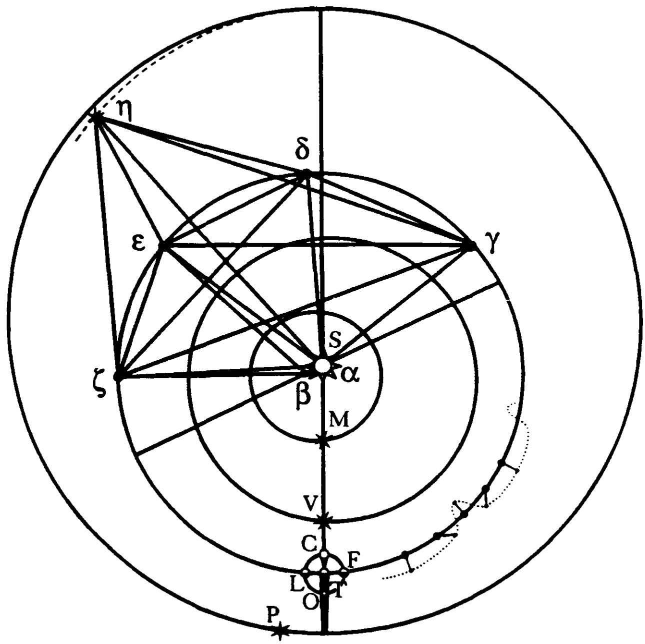
So now, at this point, we shall take for granted nothing whatever except Mars's periodic time, concerning which there can be no doubt, and the sun's zodiacal positions from Tycho's calculations. We shall also, I grant, assume an eccentric position, as is the common procedure in demonstrations leading to an impossibility, but we shall test that very position using repeated suppositions.

*These are the observations*

| | | H | M | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 1585 | May 7 | 11 | 26 | | at | 25° | 55′ | Leo | Lat. | 1° | 33′ | N. |
| | May 12 | 10 | 8 | | at | 28 | $3\frac{1}{2}$ | Leo | Lat. | 1 | $24\frac{1}{2}$ | N. |
| 1587 | March 27 | 9 | 40 | | at | 18 | $21\frac{3}{4}$ | Virgo | Lat. | 2 | $55\frac{2}{3}$ | N. |
| | April 1 | 9 | 30 | | at | 17 | 11 | Virgo | Lat. | 2 | $43\frac{1}{2}$ | N. |
| 1589 | Febr. 12 | 5 | 13 | am | at | 8 | 48 | Scorpio | Lat. | 2 | 9 | N. |
| 1590 | Dec. 28 | 7 | 8 | am | at | 8 | 6 | Scorpio | Lat. | 1 | 14 | N. |
| 1591 | Jan. 5 | 6 | 50 | am | at | 12 | $44\frac{2}{5}$ | Scorpio | Lat. | 1 | $23\frac{1}{4}$ | N. |

Since in 1589 there is but a single day that can be related to the others, and nothing else was observed for a long time before and after, let us refer the other times to this one. The catalog of them, along with the apparent positions of the sun and Mars, and with the eccentric position of Mars, is this:

| Time | (am) | Sun | Mars | For the first trial, let its eccentric position be |
|---|---|---|---|---|
| 1585 May 10 | 6 11 | 28° $55\frac{3}{4}$′ Taurus | 26° $54\frac{1}{2}$′ Leo | 5° 22′ 2″ Libra |
| 1587 March 28 | 5 42 | 16 $50\frac{4}{5}$ Aries | 18 12 Virgo | 5 23 38 |
| 1589 Feb. 12 | 5 13 | 3 $41\frac{3}{5}$ Pisces | 8 $48\frac{1}{4}$ Scorpio | 5 25 14 |
| 1590 Dec. 31 | 4 44 | 19 $6\frac{4}{5}$ Capricorn[1] | 9 $47\frac{1}{6}$ Scorpio | 5 26 50 |




*Let the diagram be made as before, with α the sun, β the centre of the earth's eccentric, ζ, δ, ε, γ the four positions of the earth, η the position of Mars on its eccentric, and let each point be connected with all the others. Now, from the data,*

149 the known angles will be — Hence are given

| | | | |
|---|---|---|---|
| αζη 87° 58′ 45″ | αηζ 38° 27′ 32″ | αζ $62{,}227\frac{1}{2}$ | by the |
| αεη 151 21 36 | αηε 17 11 38 | αε 61,675 | method of |
| αδη 114 53 25 | αηδ 33 23 1 | αδ 60,658 | chapter 26 |
| αγη 69 19 38 | αηγ 34 20 20 | αγ 60,291 | preceding. |

[1] This should be 19° 34′ Capricorn, a large error which escaped notice because it has little effect upon the size of angle εγζ, used as a tgest below. Although αγ is shortened to 60,113, εγζ becomes 21° 21′ 37″.

*Now because there stand two angles upon the arc ζε at the circumference of the circle, namely ζδε, ζγε, by Euclid III. 21 these must be equal. And in order that they come out equal, αη must be repeatedly moved forward and backward above α beneath the zodiac. And since in this first trial the zodiacal position for αη is given, let it therefore be tried whether ζδε, ζγε might be equal, for then it will be established that the position of αη is correct.*

*In the four triangles* . . . . . . . . . . ζαδ, δαε, εαγ, ζαγ
*the same number of angles is sought, namely* . ζδα, εδα, εγα, ζγα
*in order to get* . . . . . . . . . . . . . . . εδζ, εγζ

*Now in any of these triangles the angles at α are given by the position of the sun from Tycho, and the correction for the precession of the equinoxes. But the sides comprehending those angles have just been found. Therefore, the angles, too, will be given.*

*And*

| | | |
|---|---|---|
| *ζαδ is 85° 17′ 17″* | *ζδα 48° 8′ 59″* | |
| | *Hence εδζ is 21° 28′ 1″* | |
| *εαδ is 43 10 20* | *εδα 69 37 0* | *Differ* |
| *εαγ is 87 46 48* | *εγα 46 47 36* | *by 9′.* |
| | *Hence εγζ is 21° 19′ 6″* | |
| *ζαγ is 129 53 45* | *ζγα 25 28 30* | |

150 *Since these angles didn't quite come out equal, I made a second trial with αη's sidereal position moved forward 2′. And I found εδζ to be 21° 40′ 9″ εγζ 21° 22′ 14″, differing by 18′, which is twice the previous discrepancy. Whence it is understood that αη should have been moved backwards to a lesser longitude, rather than forwards.*

*For a third trial, then, supposing Mars's eccentric position in 1585 was 5° 20′ 2″ Libra, εδζ comes out to be 21° 15′ 54″, and εγζ 21° 13′ 54″. There remains a difference of 2′, which we may safely ignore.* Nevertheless, by extrapolating we realize that at this place Mars's eccentric position has to be moved back through $2\frac{1}{2}'$, just as previously in chapter 22 it was moved forward 1′ on the opposite semicircle. Both of these are brought about by an increase of the eccentricity and a slight retraction of the aphelion.

Limit of accuracy of the hypothesis of chapter 16 for longitudinal positions.

Let us now proceed to the investigation of the rest. *And because each of the angles in question has decreased, they will decrease further when αη is moved back. Therefore, let each be 21° 13′, and ζβε, the double angle at the centre, be 42° 26′. Therefore, ζεβ is 68° 47′.*

*In triangle* $\zeta\alpha\epsilon$ *angle* $\zeta\alpha\epsilon$ *is 42° 6′ 57″, and the sides are given by a new correction, so that* $\alpha\zeta$ *is 62,177 and* $\alpha\epsilon$ *61,525, approximately. Hence,* $\zeta\epsilon\alpha$ *is given as 69° 43′ 31″, and* $\zeta\epsilon$ *44,518. But this same* $\zeta\epsilon$*, from angle* $\zeta\beta\epsilon$ *(which* $\zeta\epsilon$ *subtends), is 72,379 where* $\epsilon\beta$ *is 100,000. Therefore, where* $\epsilon\beta$ *is 100,000,* $\alpha\eta$ *is 162,818, and thus* $\alpha\epsilon$ *is 100,174. But when* $\zeta\epsilon\beta$ *is subtracted from* $\zeta\epsilon\alpha$*, the remainder* $\beta\epsilon\alpha$ *is 0° 56′ 31″, and* $\beta\alpha\epsilon$ *is 83° 30′. Therefore, the aphelion is at 10° 19′ Capricorn, while the eccentricity* $\alpha\beta$ *is 1653.*

151 Again we have come rather close to half of 3600, and would doubtless obtain exactly that if we had the apogee perfectly.

Still, it should be noted that if we suppose that the earth's path is not a perfect circle, but is narrower at the sides, $\alpha\eta$ comes out here to be a little less than 163,100. And then, with $1\frac{1}{2}'$ subtracted from the eccentric position, and taking the value 1800 for the earth's eccentricity and $5\frac{1}{2}°$ Capricorn as the aphelion, the following apparent positions are produced:

26° 55′ Leo  18° $11\frac{2}{3}'$ Virgo  8° 49′ Scorpio  9° $44\frac{1}{3}'$ Scorpio

Should have been:

26° $54\frac{1}{2}'$  18° 12′  8° $48\frac{1}{4}'$  9° $47\frac{1}{6}'$

This supposition also agrees with my observations of 1604 February 29/March 10, for on the night following that day with my instruments I found Mars culminating at 26° $18\frac{4}{5}'$ Libra. And calculation based upon these assumptions puts it at 26° $17\frac{1}{2}'$ Libra. Moreover, at $8\frac{2}{3}^{h}$, a few hours before the observation, it was again in the same eccentric position.

Besides, since Mars has some latitude here, the value for $\alpha\eta$ just found is the distance in the plane of the ecliptic of the point $\eta$ from the centre of the sun, to which point a perpendicular is drawn from the body of Mars, as was noted in chapter 20 above. So the true distance of the planet's body itself from the centre of the sun is made longer by 37 units.

# 28

## Assuming not only the zodiacal positions of the sun, but also the sun's distances from the earth found using an eccentricity of 1800; through a number of observations of Mars at the same eccentric position, to see whether by unanimous consent the same distance of Mars from the sun, and the same eccentric position, are elicited. By which argument it will be confirmed that the solar eccentricity of 1800 is correct, and was properly assumed

The reader should not be surprised that in this third turn I am now declining to presuppose the eccentric position of Mars as given by the hypothesis of acronychal observations found above. For I have said that hypothesis was only vicarious, not natural, and thus possesses only as much trustworthiness as is permitted by the observations; and it could deviate somewhat in the intermediate positions between
152 observations. Besides, it helps us to have at hand various methods of demonstration by which to explore carefully the distances of Mars at all places throughout the entire circle. Accordingly, the new form of demonstration follows here.

*The observations are these*

| | | D | H | | | | | Lat. |
|---|---|---|---|---|---|---|---|---|
| 1583 | April | 22 | $9\frac{2}{3}$ | it was at | 1° 17′ | Leo, | 1° $50\frac{2}{3}$′ | N. |
| 1585 | March | 9 | $9\frac{1}{6}$ | it was at | 11 $49\frac{1}{10}$ | Leo, | 3 $29\frac{1}{10}$ | N. |
| | March | 11 | 5 | it was at | 11 $45\frac{1}{2}$ | Leo, | 3 $24\frac{1}{6}$ | N. |
| | March | 12 | 5 | it was at | 11 $45\frac{3}{4}$ | Leo, | 3 $21\frac{2}{3}$ | N. |
| 1587 | January | 26 | 5 am | it was at | 4 $41\frac{3}{4}$ | Libra, | 3 26 | N. |
| | January | 28 | 5 am | it was at | 4 41 | Libra, | 3 27 | N. |
| 1588 | December | 5 | $6\frac{1}{2}$ am | it was at | 9 23 | Libra, | 1 $44\frac{3}{4}$ | N. |
| | December | 15 | $6\frac{1}{6}$ am | it was at | 14 $35\frac{2}{3}$ | Libra, | 1 54 | N. |
| 1590 | October | 31 | $6\frac{1}{4}$ am | it was at | 2 $57\frac{1}{3}$ | Libra, | 1 $15\frac{1}{2}$ | N. |

When the times of the other observations are adjusted so as to return Mars to the same eccentric position which it had in the last, we are given the following times. Along with these are added the requisite positions of the sun, and the distances of the sun and the earth computed from the hypothesis so far established. And indeed, it is these very things for the testing of which we have taken up this labour. A little later, in ch. 30, a technique for computing these distances follows.[1]

| | D.H.(am) | Mars at | Sun at | Distance Sun to earth |
|---|---|---|---|---|
| 1583 April | 23 $8\frac{1}{10}$ | 1° $29\frac{1}{2}$′ Leo | 12° 10′ 3″ Taurus | 101,049 |
| 1585 March | 10 $7\frac{2}{3}$ | 11 $48\frac{1}{3}$ Leo | 29 41 4 Pisces | 99,770 |
| 1587 January | 26 $7\frac{1}{6}$ | 4 $41\frac{3}{4}$ Libra | 16 5 55 Aquarius | 98,613 |
| 1588 December | 13 $6\frac{3}{4}$ | 13 $35\frac{2}{3}$ Libra | 1 44 53 Capricorn | 98,203 |
| 1590 October | 31 $6\frac{1}{4}$ | 2 $57\frac{1}{3}$ Libra | 17 28 33 Scorpio | 98,770 |

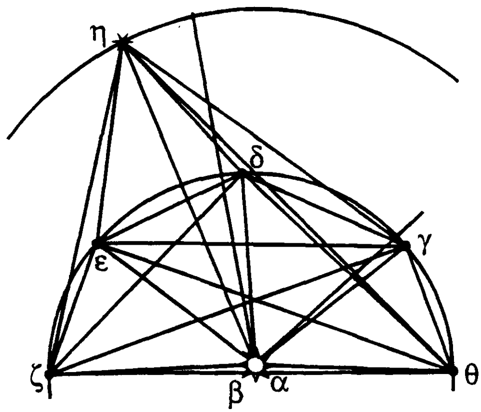
A few remarks would be in order on the deduction of the observation to the times we have chosen. For the first time, the diurnal motion was taken from Magini, since over the space of a few hours there is no danger of error. The other times are supported by

[1] Kepler is somewhat less than candid here about how he obtained his figures for the distances. And understandably so, since the numbers he gives are derived from an elliptical orbit, not from the circle he is ostensibly considering here. So instead of raising that question here, he refers the reader ahead to ch. 30, where he presents his table of distances and admits his use of the ellipse.

What this shows is that, in this chapter at least, Kepler is not reproducing the arguments that led him to introduce an equant into the theory of the sun or earth. Rather, he is presenting an additional argument, worked out after his discovery of the elliptical orbit, to confirm the sun/earth equant.

observations before and after. However, for the penultimate time I also looked up the sequence of diurnal motions in Magini. Around December 15 the diurnal motion was 30′, and around December 5 it was 32′. For the last time, although Mars, being at an altitude of 23°, was affected by refraction, so that 2′ might easily be wanting in the latitude, this refraction nevertheless hardly affects the longitude of Mars. (Tycho claimed that the refraction of the fixed stars, also used for the planets, ceases at this altitude, although the solar refraction reaches higher, and at this altitude is about 4′. This distinction was discussed and demolished in my *Optical astronomy* p. 137,[2] and would be rendered even more dubious if any changes are to be made in the parallax of the sun.)

*Let* α *be the sun's body,* αβ *the eccentricity of the earth's orb (1800), and the line of apsides be at* $5\frac{1}{2}$° *Cancer, positions of the earth* ζ, ε, δ, γ, θ, *and the body of the planet at the same eccentric position* η *all five times, since the intervals span complete periods of Mars. And let all points be connected with one another.* It is desired to find αη, and its
153 zodiacal position, that is, angle ηαθ, ηαγ, or some other angle at α. We shall do this from two earth positions in the following manner. *For a start, let these be* ε, δ. *And in triangle* εαδ, *given the sides* εα *99,770,* αδ *98,613, and angle* εαδ, *the rest are sought, namely, the angles* δ *and* ε *and the side* δε.




[2] *Astronomiae pars optica* (Frankfurt 1604), in KGW 2 pp. 126–7.

| | | |
|---|---|---|
| αϵ 99770 | | |
| αδ 98613 | | |
| 1157 | | [αϵ − αδ[3]] |
| 198383 | | [αϵ + αδ] |
| 991915 | 5 | |
| 165085 | | |
| 158706 | 8 | |
| 6379 | 3 | [583 = (αϵ − αδ)/(αϵ + αδ)] |

| | | | | | |
|---|---|---|---|---|---|
| | 29° | 41′ | 4″ | *Pisces* | [sun's position in 1585] |
| | 16 | 5 | 55 | *Aquarius* | [sun's position in 1587] |
| | 43 | 35 | 9 | | |
| *Precession* | | 1 | 36 | | |
| ϵαδ | 43 | 36 | 45 | | |
| | 136 | 23 | 15 | | [supplement of ϵαδ] |
| | 68 | 11 | 38 | | [half the supplement] |
| *Tang.* 249813 | | | | | |
| 583 | | | | | [found above] |
| 12491 | | | | | |
| 1998 | | | | | |
| 75 | | | | | |
| 1456 | | | | | [product] |
| | 68° | 11′ | 38″ | | [half the supplement, from above] |
| | | 50 | 3 | | [arc tan .01456, from above] |
| αδϵ | 69 | 1 | 41 | | |
| αϵδ | 67 | 21 | 35 | | [remaining angle] |

| | | |
|---|---|---|
| 68977 | | [sin ϵαδ] |
| 93376 | | [sin αδϵ] |
| 653632 | 7 | |
| 36138 | | |
| 28013 | 3 | |
| 8125 | | |
| 7470 | 8 | |
| 655 | 7 | |
| 654 | 0 | [73870 = sin ϵαδ/sin αδϵ] |
| 1 | | |

[3] Bracketed material provided by the translator.

| | |
|---|---|
| 99770 | [αϵ] |
| 73870 | [sin ϵαδ/sin αδϵ, from above] |
| 664830 | |
| 66483 | |
| 5171 | |
| 517 | |
| 73700 δϵ. | |

*With these matters investigated, one goes on up to triangle ϵηδ.*

| | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| *For since ϵα is* | *29°* | *41′* | *4″* | *Pisces* | | *δα* | *16°* | *5′* | *55″* | *Aquarius* |
| *and ϵη* | *11* | *48* | *20* | *Leo* | | *δη* | *4* | *41* | *45* | *Libra* |
| *αϵη will be* | *132* | *7* | *16* | | | *αδη* | *131* | *24* | *10* | |
| *But αϵδ was just* | *67* | *21* | *35* | | | *αδϵ* | *69* | *1* | *41* | |
| *Remainder ηϵδ* | *64* | *45* | *41* | | | *ηδϵ* | *62* | *22* | *29* | |

*The remaining angle ϵηδ to make up two right angles with these is 52° 51′ 49″.*

154 So, angles ϵ, η, δ and one side ϵδ being given, side ϵη will also be given.

| | | |
|---|---|---|
| *Sine ϵηδ* | 79718 | |
| *sine ηδϵ* | 88600 | 8 |
| | 8838 | |
| | 7974 | 9 |
| | 864 | |
| | 797 | 9 |
| | 67 | |
| | 62 | 7 |
| | 5 | 1 |

| | |
|---|---|
| ϵδ 73700 | |
| 89972 | ϵη |
| 719776 | 8 |
| 17224 | |
| 8997 | 1 |
| 8226 | |
| 8097 | 9 |
| 129 | |
| 90 | 1 |
| 39 | 5 |

*Finally, let the triangle* ηεα be solved, in which now are given:

| | | |
|---|---|---|
| εη | 81915 | |
| εα | 99770 | |
| *Dif.* | 17855 | |
| *Sum* | 181685 | |
| | 1635165 | 9* |
| | 150335 | |
| | 145348 | 8 |
| | 4987 | |
| | 3834 | 2 |
| | 1153 | |
| | 1150 | 3 |

| | | | |
|---|---|---|---|
| *And* αεη, *as before, is* | 132° | 7′ | 16″ |
| *Supplement* | 47 | 52 | 44 |
| *Half* | 23 | 56 | 22 |
| *Tangent* | 44396 | | |
| | *9823 | | |
| | 39956 | | |
| | 3552 | | |
| | 89 | | |
| | 13 | | |
| | 4361 | | |

| | | | | |
|---|---|---|---|---|
| | 23° | 56′ | 22″ | [half the supplement, above] |
| | 2 | 29 | 50 | [arc tan .04361, from above] |
| εαη | 21 | 26 | 32 | |
| *But in 1585,* αε *was in* | *29°* | *41′* | *4″* | *Virgo* |
| *Therefore* αη *in 1585 was in* | *8* | *44* | *32* | *Virgo* |

| | | |
|---|---|---|
| *Sine* εαη | 36556 | |
| *Sine* αεη | 74173 | 2 |
| | 73112 | 0 |
| | 10610 | |
| | 7311 | 2 |
| | 3299 | 9 |
| | 3290 | 0 |
| | 9 | 3 |

| | |
|---|---|
| εη | 81915 |
| | 202903 |
| | 1638300 |
| | 16383 |
| | 7372 |
| | 25 |

*The magnitude sought,* αη, *comes out to be 166208*

If the other three observations at ζ, γ, θ, allow this same position and length for αη, we shall have excellent confirmation of them.

So, in the same manner in which we have hitherto worked with ε and δ, we shall now work with ζ, γ, seeking the same αη.

155 For the angles, and line, γ, ζ

| | | |
|---|---|---|
| αζ | 101049 | |
| αγ | 98203 | |
| | 2846 | |
| | 19925 | 1 |
| | 8535 | |
| | 7970 | 4 |
| | 565 | |
| | 399 | 3 |
| | 166 | |
| | 159 | 8 |

| | | | |
|---|---|---|---|
| 12° | 10′ | 3″ | *Taurus* |
| 1 | 44 | 53 | *Capricorn* |
| 130 | 25 | 10 | |

*Precession* 4 48

| | | |
|---|---|---|
| 130 | 29 | 58 |
| 49 | 30 | 2 |
| 24 | 45 | 1 |

| |
|---|
| 46101 |
| 1438 |
| 4610 |
| 1844 |
| 138 |
| 37 |
| 663 |

| | | | |
|---|---|---|---|
| | 24° | 45′ | 1″ |
| | | 22 | 48 |
| αγζ | 25 | 7 | 49 |
| αζγ | 24 | 22 | 13 |

| | |
|---|---|
| 76041 | |
| 42468 | 1 |
| 335730 | |
| 297276 | 7 |
| 38454 | |
| 38221 | 9 |
| 233 | 0 |
| 212 | 5 |
| 21 | 5 |

| | |
|---|---|
| 179055 | |
| 101049 | |
| 1790550 | |
| 17905 | |
| 716 | |
| 162 | |
| 180933 | ζγ |

And now in ζγη

| | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| Because | *ζη is* | *1° 29½′* | | *Leo* | *γη* | *13°* | *35′* | *40″* | *Libra* |
| *and* | *ζα is* | *12* | *10* | *3* *Taurus* | *γα* | *1* | *44* | *53* | *Capricorn* |
| *Therefore* | *ηζα is* | *79* | *19* | *27* | *ηγα* | *78* | *9* | *13* | |
| *But* | *γζα is* | *24* | *22* | *13* | *ζγα* | *25* | *7* | *49* | |
| *Therefore* | *ηζγ is* | *54* | *57* | *14* | *ηγζ* | *53* | *1* | *24, and the* | |

*remaining angle γηζ to make up two right angles with these is*
*72° 1′ 22″.*

| | | | | | |
|---|---|---|---|---|---|
| *The same is also proved from this:* | *ζη is at* | *1° 29½′* | | | *Leo* |
| | *and γη is at* | *13* | *35* | *40* | *Libra,* |
| *or, with the precession over the interval subtracted,* | *at* | *13* | *30* | *52* | *Libra* |
| *Therefore, γηζ is* | | *72* | *1* | *22* | |

[See figure on page 347.]
Next, given the angles of the triangle ζηγ, and side γζ, the side ζη is sought

| | |
|---|---|
| 79887 | |
| 95118 | |
| 760944 | 8 |
| 37926 | |
| 28535 | 3 |
| 9391 | 9 |
| 8560 | 8 |
| 831 | 7½ |
| 761 | |

180933
83987½

1447464
54280
16284
1447

151960: *this is ζη.*

156 Finally, in triangle ηζα the sides and the comprehended angle are given

| | |
|---|---|
| ζη 151960 | |
| ζα 101049 | |
| 50911 | 2 |
| 253009 | 0 |
| 506018 | |
| 3092 | |
| 2530 | 1 |
| 562 | 2 |
| 506 | |
| 56 | 2 |

| | | |
|---|---|---|
| 79° | 19′ | 27″ |
| 100 | 40 | 33 |
| 50 | 20 | 16 |

| |
|---|
| 120612 |
| 20122 |
| 241224 |
| 1206 |
| 241 |
| 24 |
| 24270 |

| | | | | |
|---|---|---|---|---|
| | 50° | 20′ | 16″ | |
| | 13 | 38 | 39 | |
| *ζαη comes out to be* | *63* | *58* | | |
| *But αζ is at* | *12* | *10* | *3* | *Scorpio in 1583* |
| *Therefore αη is at* | *8* | *11* | *31* | *Virgo in 1583* |
| *Precession* | | *1* | *36* | |
| *So it would be at* | *8* | *13* | *8* | *Virgo in 1585* |
| *Previous position* | *8* | *14* | *32* | *Virgo in 1585* |
| *Difference* | | *1* | *24* | |

| | |
|---|---|
| 98269 | |
| 89861 | 10 |
| 84080 | |
| 80875 | 9 |
| 3205 | |
| 2696 | 3 |
| 509 | 5 |
| 449 | |
| 60 | 7 |

| | |
|---|---|
| | 151960 |
| | 109357 |
| | 1519600 |
| | 136764 |
| | 4559 |
| | 760 |
| | 106 |
| *$\alpha\eta$ comes out to be* | *166179* |
| *Previous value* | *166208* |
| *Difference* | *29* |

And so it appears that with two other observations, at $\zeta$ and $\gamma$, we have come upon the same thing, within the limits of observational accuracy. For an error of a minute and a half in observing, or in deducing the observed position to a day when it was not observed, can easily be committed.

But let us see the evidence of the fifth position, $\theta$, that is, of the observation at $\theta$.

| | | |
|---|---|---|
| *We know that $\theta\alpha$ is at* | *17° 28′ 33″ Scorpio,* | *and suppose that* |
| *and $\theta\eta$ was observed at* | *2 57 20 Libra* | *$\theta\alpha$ is 98,770* |

*Therefore the angle $\alpha\theta\eta$ is 44° 31′ 13″. If I lengthen $\alpha\eta$ subtending this angle, I will thereby move $\alpha\eta$ farther forward in longitude, and vice versa.*

157 *So let $\alpha\eta$ be 166,208, as initially determined.*

*Now as* $\alpha\eta$ *is to [sin]* $\alpha\theta\eta$ *so is* $\alpha\theta$ *to [sin]* $\alpha\eta\theta$.

| | |
|---|---|
| *98770* | |
| *166208* | |
| *831040* | *5* |
| *156660* | |
| *149587* | *9* |
| *7073* | |
| *6648* | *4* |
| *425* | |
| *332* | *2* |
| *93* | *6* |

| |
|---|
| *70116* |
| *59426* |
| *415982* |
| *594* |
| *59* |
| *41665* |

| | | | | |
|---|---|---|---|---|
| $\alpha\eta\theta$ *comes out to be* | *24°* | *37′* | *28″* | |
| *But in 1590* $\theta\eta$ *is directed toward* | *2* | *57* | *20* | *Libra* |
| *Therefore, in 1590* $\alpha\eta$ *is at* | *8* | *19* | *52* | *Virgo* |
| *Precession* | | *4* | *48* | |
| *Therefore in 1585 it is at* | *8* | *15* | *4* | |
| *Original position* | *8* | *14* | *32* | |
| *Difference* | | *0′* | *32″* | |

*Therefore by a very small shortening of* $\alpha\eta$, *it will fall in exactly with the first two observations.*

And so it appears from this that the distances $\alpha\zeta$, $\alpha\epsilon$, $\alpha\delta$, $\alpha\gamma$, $\alpha\theta$, assumed by us, and the eccentricity $\alpha\beta$ as well, were assumed and posited correctly. For it is impossible to take other distances than these, which nonetheless also fit approximately on a circle and have the appropriate zodiacal positions, and still obtain the same magnitude for $\alpha\eta$, and the same zodiacal position, from all five observations.

But concerning the length of $\alpha\eta$, we shall put our trust mostly in observations $\zeta$, $\gamma$, $\theta$. For also in the common method of measuring distances of things on earth, where the standing points are farther from one another, the distance of the mark is obtained with greater accuracy.

For the zodiacal position, however, we shall put our trust in the observations at $\epsilon$, $\delta$, instead. For if there is some slight error in the length of $\alpha\eta$, it is presented to the observer at $\epsilon$, $\delta$ quite obliquely, and the angle does not change perceptibly.

Nor is this to be forgotten: that there is no perceptible lengthening of $\alpha\eta$ over the seven years from 1583 to 1590, because the progression of the aphelion is very slow.

Summary: On 1590 October 31 at $6\frac{1}{4}^h$ in the morning, Mars's eccentric position was 8° 19′ 20″ Virgo, while the hypothesis constructed using acronychal observations places it at 8° 19′ 29″ Virgo. Its distance was 166,180, which must be lengthened because of the latitude, making the distance from the body of Mars itself to the centre of the sun about 166,228.

# 29

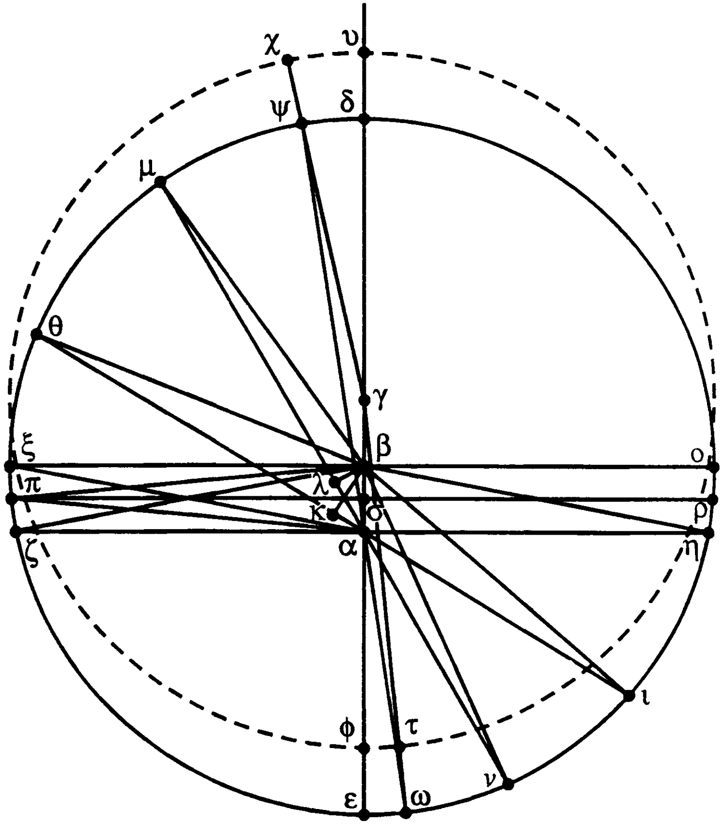
## A method of deducing the distances of the sun and earth from the known eccentricity

I think it is well enough confirmed that the distances of the sun and the earth are to be deduced by halving the eccentricity obtained by Tycho. This is also abundantly confirmed by observation of the sun's
158 summer and winter diameter, as I have shown in the *Astronomiae pars optica* chapter 11[1]. But it is also wonderfully confirmed in the *Mysterium cosmographicum*, in the table in ch. 15 p. 53[2], where the equations of the centre for Mars, Venus, and Mercury were deficient when the lunar orb was interposed, but excessive when it was omitted. Now, the orb of the moon being retained while the eccentricity of the sun is bisected, they come out about right.

Furthermore, the same thing will again be confirmed more frequently and much more clearly when we use the distances forthcoming from the bisection (as we have just begun to do in the last chapter) and see the phenomena follow from them. Therefore, in order that these distances be ready at hand for our future use, I shall show how they may easily be computed, using a geometrical demonstration.

*On the line* $\alpha\delta$ *let* $\alpha$ *be the body of the sun (or the earth for Tycho, or the centre of attachment of the epicycle for Ptolemy);* $\beta$ *the centre of the eccentric* $\zeta\delta\eta$ *of the earth (or of the sun and of the annual orb for Tycho, or of the epicycle for Ptolemy); and* $\alpha\beta$ *being extended, let it intersect the eccentric at* $\delta$, $\epsilon$, *so that* $\delta$ *is the aphelion or apogee and* $\epsilon$ *the perihelion or perigee; and let* $\beta\gamma$ *be equal to* $\alpha\beta$. *Also, let* $\gamma$ *be the*

[1] Chapter 11, Problema II, pp. 341–2, in KGW 2 p. 292.
[2] *Mysterium Cosmographicum: The Secret of the Universe*, A. M. Duncan, trans., New York 1981, pp. 162–3 (second table).




*centre of motion or of uniformity, about which the earth (for Ptolemy, the centre of the epicycle, for Tycho the sun and the point of attachment of all the eccentrics) sets out equal angles in equal times. And let* αγ *be 3600, from the observations of Tycho and the Landgrave*[3], *but* αβ, *according to my recently introduced adjustment, be 1800. Now let* ζη *be drawn through* α *perpendicular to* δε, *intersecting the circle at* ζ, η, *and also through* α *let the straight line* θι *be drawn, at any inclination whatever, intersecting the circumference at* θ, ι; *and let the four points* θ, ι, ζ, η *be connected with the centre* β. *And let this also be posited at*

[3] The observations of William, Landgrave of Hesse, which were known to Kepler through Tycho, were published several years later by W. Snel under the title *Coeli et siderum in eo errantium observationes* . . ., Leiden 1618 (cited in KGW 3 p. 466).

*the start: that although the earth (sun, or planet) is moved uniformly around $\gamma$ and thus nonuniformly around $\beta$, it nevertheless remains on the circumference of the circle described about $\beta$. Now, by the equivalence shown in the second chapter (which, for the sake of avoiding confusion, I shall not apply to the general Ptolemaic hypothesis), this is exactly as if one were to say that the earth (or sun) is moved nonuniformly on a concentric with an epicycle about centre $\alpha$, the semidiameter of the epicycle being equal to $\alpha\beta$; and the arcs described on the concentric by the centre of the epicycle being similar to the arcs of the epicycle described by the earth (or sun), so that both the earth (or sun) and the centre of the epicycle are moved unequally in equal times, so as to become slow, or again speed up, simultaneously.* I am going to postpone for a little while the physical explanation of this hypothesis.

Longest and shortest distance.

Now, with these things supposed. I shall proceed to the work of finding the distances. *And because $\beta\delta$ is 100,000 and $\beta\alpha$ 1800, and $\alpha\beta\delta$ is a straight line, by addition of the two the aphelial distance $\alpha\delta$ is obtained; and because $\beta\epsilon$ too is 100,000, when $\alpha\beta$ is subtracted the perihelial distance $\alpha\epsilon$ remains.*

Distances of intermediate positions.

*And because $\beta\alpha\zeta$ is right, and $\zeta\beta$ is 100,000 (that is, the whole sine), therefore $\alpha\beta$ is the sine of the angle $\alpha\zeta\beta$. Consequently $\alpha\zeta\beta$ is 1° 1′ 53″,*
159 *which is the optical part of the equation of the sun or earth.* Now the maximum equation at the middle longitudes, which is composed of the optical and physical parts, has the whole eccentricity 3600 (or 3592) as its sine. Therefore, when the sun or earth goes from $\delta$ to $\zeta$, it takes two days longer than one fourth of the periodic time, but it nevertheless does only one day's journey beyond one fourth of its total circuit, So in this distance, or quarter of the periodic time[4], it takes one day longer than it should, owing to the physical weakening.

But to proceed to the distance $\alpha\zeta$. *In the right triangle $\zeta\alpha\beta$, since one of the acute angles is given, the other, $\zeta\beta\alpha$, will be the difference between the first and one right angle, that is, 88° 58′ 7″. And therefore $\alpha\zeta$ will be the sine of this angle, 99984. And the opposite line $\alpha\eta$ is the same size.*

Remaining distances.

For finding the intermediate distances of two opposite degrees of equated anomaly, let $\theta\iota$ be inspected, passing through the body $\alpha$ whence the eccentricity is computed. *Now $\delta\alpha\theta$ and $\delta\alpha\iota$ are equated anomalies, and are opposite, in that $\alpha$ is between them and is collinear with them. Now let a line $\beta\kappa$ fall from $\beta$ perpendicular to $\theta\iota$, so as to*

4 Kepler surely means 'quarter of the circumference' here.

*make $\theta\kappa$, $\kappa\iota$ equal. In the right triangle $\beta\kappa\alpha$ the base $\beta\alpha$ is given, as well as the angles $\kappa\alpha\beta$ (from the integral number of degrees of equated anomaly chosen) and its complement $\kappa\beta\alpha$. Therefore, the sides $\kappa\alpha$, $\kappa\beta$ will not be unknown. And $\kappa\beta$ is the sine of the angle $\kappa\theta\beta$ or $\kappa\iota\beta$. This being given, its complement, $\theta\beta\kappa$ or $\iota\beta\kappa$, will also be known, and its sine, which is the line $\theta\kappa$ or $\kappa\iota$. And when $\kappa\alpha$ is added to $\kappa\theta$, $\alpha\theta$ is obtained; and when the same is subtracted from $\kappa\iota$, $\alpha\iota$ is obtained. The former distance corresponds to the equated anomaly $\delta\alpha\theta$, and the latter to the equated anomaly $\delta\alpha\iota$, which has a line equal to itself in the preceding semicircle, which stands as far from aphelion in semicircle $\delta\theta$ as $\alpha\iota$ itself does in semicircle $\delta\eta$.*

Short cut

*Now through $\alpha$ let the straight line $\mu\nu$ be drawn intersecting the circle in $\mu$, $\nu$, and making the angle $\mu\alpha\delta$ equal to the angle $\kappa\beta\alpha$. And from $\beta$ let $\beta\lambda$ fall perpendicular to $\mu\nu$ bisecting $\mu\nu$ at $\lambda$. And let $\mu$, $\nu$ be connected to $\beta$. Now, since $\kappa\alpha\beta$ is an angle of an integral number of degrees, the remainder $\kappa\beta\alpha$, and $\mu\alpha\delta$ equal to it, are also an integral number of degrees, and in the triangles $\beta\kappa\alpha$, $\beta\lambda\alpha$ side $\kappa\alpha$ is equal to the similar side $\lambda\beta$, and $\kappa\beta$ to $\lambda\alpha$. But $\lambda\beta$ is the sine of angle $\lambda\mu\beta$, $\lambda\nu\beta$, and the complement of $\lambda\mu\beta$ is $\lambda\beta\mu$, $\lambda\beta\nu$. And the sine of this is the lines $\lambda\mu$, $\lambda\nu$, and the difference between these and $\alpha\mu$, $\alpha\nu$ is $\lambda\alpha$. But the magnitudes $\lambda\alpha$, $\lambda\beta$ have just now been found in triangle $\alpha\beta\kappa$.* Therefore, by the use of one triangle, four distances can be found making equal angles about $\alpha$ with the line of apsides and its perpendicular $\zeta\eta$ drawn through $\alpha$. For $\mu\alpha\zeta$ is equal to $\theta\alpha\delta$, and $\nu\alpha\eta$ to $\iota\alpha\epsilon$[5].

Where the mean between the longest and shortest distances is.

So the greatest distance is at $\delta$, the shortest at $\epsilon$, but the mean, which is equal to $\beta\zeta$, is not at $\zeta\eta$. Neither is it on a line through $\beta$ parallel to $\zeta\alpha$ which shall be called $\xi o$. *For $\alpha\zeta$ is less than $\beta\zeta$, since $\zeta\beta\alpha$ is subtended by a smaller line than $\zeta\alpha\beta$, which is right, and, $\alpha\xi$ being drawn, it is longer than $\beta\xi$, since it subtends a greater angle $\xi\beta\alpha$ (which is right) while $\xi\beta$ subtends a lesser angle $\xi\alpha\beta$.*

But in order that the distance at the mean position be defined geometrically, *let $\alpha\beta$ be bisected at $\sigma$, and through this let $\pi\rho$ be drawn perpendicular to $\alpha\beta$, intersecting the circle at $\pi$, $\rho$. I say that these are the points that are equally distant from $\alpha$ and $\beta$.*

*For let one of the points $\pi$ be connected with $\alpha$ and $\beta$. The lines $\pi\alpha$ and $\pi\beta$ will subtend equal angles $\pi\sigma\alpha$ and $\pi\sigma\beta$ (since they are right), and $\alpha\sigma$, $\sigma\beta$ are equal, and $\pi\sigma$ is common. Therefore $\pi\alpha$, $\pi\beta$ are equal.*

[5] Stated algebraically, the problem here is to find the distance from the sun to the earth ($\alpha\theta$, say) as a function of the equated anomaly $\delta\alpha\theta$. Where $\beta\delta = 1$.

$$\alpha\theta = \cos\,[\sin^{-1}(\alpha\beta \sin \delta\alpha\theta)] + \alpha\beta \cos \delta\alpha\theta.$$

And thus the demonstration taken from Reinhold[6] concerning the whole $\alpha\gamma$ and its midpoint $\beta$ remains true for the point $\sigma$ and its half $\alpha\beta$.

**160** One might think that since at $\pi$ the distance $\alpha\pi$ becomes equal to the semidiameter $\beta\pi$, the angle $\beta\pi\alpha$ is also greater than $\beta\zeta\alpha$, and thus the greatest equation occurs at $\pi$, *on the argument that the straight line* $\beta\alpha$ *is presented more directly from* $\pi$ *than from* $\zeta$ . *However, this proposed line of reasoning is not true. For to the same extent that* $\beta\alpha$ *is more oblique with respect to* $\zeta$, $\pi$ *in turn is more distant than* $\zeta$, *since* $\pi\sigma$ *is longer than* $\zeta\alpha$. *For* $\pi\beta\sigma$ *is greater than* $\zeta\beta\alpha$, *which* $\zeta\alpha$ *subtends.*

Where is the equation greatest?

So Ptolemy, and after him Reinhold in the *Theoricae*, demonstrated correctly that the greatest equation (eccentric equation alone, or optical part) occurs at $\zeta$. I shall, however, set up this demonstration in another, simpler form. *Let any point be taken above* $\zeta$, *such as* $\theta$, *and any below* $\eta$ *or* $\zeta$, *such as* $\iota$. *Let them be connected with* $\alpha$, *and from* $\beta$ *let perpendiculars* $\beta\kappa$ *fall to* $\theta\alpha$ *or* $\iota\alpha$ *extended. Now since* $\delta\alpha\zeta$ *and* $\beta\kappa\alpha$ *are equal, they being right angles, and* $\kappa\beta\alpha$, $\kappa\alpha\beta$ *together are equal to one right angle, when the same angle* $\delta\alpha\theta$ *or* $\beta\alpha\kappa$ *is subtracted from the equals, the equals* $\theta\alpha\zeta$, $\kappa\beta\alpha$ *will remain. And first, let a line be drawn through* $\alpha$ *above* $\zeta$, *as* $\theta\alpha$ *was just drawn, whether* $\theta$ *is next to* $\zeta$ *or remote. At the same time let its perpendicular* $\beta\kappa$ *be inclined to* $\beta\alpha$. *Now* $\beta\alpha$ *is greater than any of the perpendiculars* $\beta\kappa$, *since* $\beta\alpha$ *subtends the right angle* $\beta\kappa\alpha$, *while* $\beta\kappa$ *subtends the smaller, acute angle* $\beta\alpha\kappa$. *Now since* $\beta\zeta$, $\beta\theta$, $\beta\iota$ *are equal, and* $\beta\alpha\zeta$, $\beta\kappa\theta$, $\beta\kappa\iota$ *are right, they fit onto the same semicircle, whose diameter is equal to* $\beta\zeta$, $\beta\theta$, $\beta\iota$. *And* $\beta\alpha$, *being longer, subtends a greater part of the circumference of any such semicircle than does* $\beta\kappa$ *or any of the perpendiculars; and for that reason, its angle* $\beta\zeta\alpha$ *will be greater than* $\beta\theta\kappa$, *or the angle of the equation for any other point above* $\zeta$, *such as* $\pi$ *or* $\xi$. *Which was to be demonstrated.*

Everything said in this chapter about the computing of distances of the sun and earth will also be valid for Mars, as long as one of the suppositions is that the orbits of planets are perfect circles. When this is seen as false, another method of computing them will be given.

[6] Erasmus Reinhold published the *Theoricae planetarum* of Georg Peurbach with detailed scholia in 1542 in Wittenberg, and the work was later reprinted. The passage cited is in the scholium to the theory of the sun, part II prop. iv. (citation from KGW 3 p. 467).

# 30

## Table of the distance of the sun from the earth and its use

We have gathered together here into a table, the columns of which are three, the distances of the sun accumulated in this way [that is, in the way described in ch. 29], as if they were done for integral degrees of equated anomaly for the whole semicircle (for those in the other semicircle at equal distances from apogee are also equal to these). In the first column, which we have called 'mean anomaly', are the angles $\delta\beta\mu$, $\delta\beta\theta$, $\delta\beta\xi$, $\delta\beta\iota$, $\delta\beta\nu$, composed of the integral angles $\delta\alpha\mu$, $\delta\alpha\theta$, $\delta\alpha\xi$, $\delta\alpha\iota$, $\delta\alpha\nu$, and their optical or eccentric equations, namely, $\beta\mu\alpha$, $\beta\theta\alpha$, $\beta\xi\alpha$, $\beta\iota\alpha$, $\beta\nu\alpha$. In the second, the distances themselves, $\alpha\mu$, $\alpha\theta$, $\alpha\xi$, $\alpha\iota$, $\alpha\nu$, are placed together opposite [the mean anomalies]. In the
161 third, under the heading 'equated anomaly', are tabulated angles not depicted here, but the procedure for whose generation is revealed in part here and in part in chapters 31 and 40. For they are constituted by subtraction of the optical equations $\alpha\mu\beta$ and so forth from $\delta\alpha\mu$ and so forth. Thus we have given no column to the integral angles like $\delta\alpha\mu$, because they are the arithmetic mean between the angles of the columns at the sides, and thus are easily found in themselves, and are not of any use, as we shall hear.

Instructions for use of the table.

Therefore, beginning with either the mean or the equated anomaly, either one going into its proper respective column as usual; or, where it exceeds a semicircle, beginning with the difference between either of these and the whole circle; you find the requisite distance of the sun from the earth, in units of which the radius of the orb is 100,000 and the eccentricity is 1800.

It is true that in this way (that is, in associating the distance $\alpha\zeta$ of the

In this table I have supposed the path of the sun or earth to be oval.

angle $\delta\alpha\zeta$ with an angle which is as much smaller than $\delta\alpha\zeta$ as $\delta\alpha\zeta$ is smaller than $\delta\beta\zeta$) a path is attached to the circuit of the earth (or sun) about $\alpha$ which is oval rather than exactly circular. For the distance $\alpha\zeta$ (for example) was determined by the angle $\delta\alpha\zeta$, an integral 90°, and it was assumed in the operation that this angle $\delta\alpha\zeta$ was the equated anomaly. Now, however, you are told to get the distances using the angles of anomaly which are called 'equated' in our table, which have been diminished by the equation $\beta\zeta\alpha$. It thus happens that at 90° you do not get 99,984, although you would previously have determined it to be 99,984. For here, opposite 99,984, you find an equated anomaly of 88° 58′ 7″, which is not your value. For it was 90° that was proposed, which, standing farther down, shows [a distance of] 99,953, while, according to the law of the circle, $\alpha\zeta$ or $\alpha\eta$ should be 99,984. So all the distances are diminished at the sides, most greatly about $\zeta$, $\eta$, none at $\delta$, $\epsilon$. Clearly, an oval is thus substituted for the circular path. You will obtain the same result if you begin with a mean anomaly obtained from whatever source. For when the diagram was set out above, the mean anomaly denoted angles about $\gamma$. But now you would begin with angles about $\beta$, smaller than the optical equations about $\gamma$. And 91° 1′ 53″ of mean anomaly shows you a distance of 99,984. But that was the magnitude of $\delta\beta\zeta$ above. Nor was the mean anomaly there, for it was $\delta\gamma\zeta$, which is still greater. So the former mean anomaly of 91° 1′ 53″ had generated a longer distance there than a mean anomaly of the same magnitude, 91° 1′ 53″, shows here. All this, I say, is true. But there is no reason why that should hinder you. For since we are considering differences of one degree, you can observe that the distances within one degree vary no more than 31 parts in one hundred thousand. Therefore, there is no perceptible error, even if what has been done is preposterous. But the reason why this arrangement is deduced, by analogy with the rest of the planets, in the theory of the sun also, you will find below in ch. 44 and the following. Therefore, this is not preposterous, but perfectly correct, with regard to the property of the figure which the planet describes, which has been substituted.

As regards the quantity of the figure, the cure is excessive. The equated anomaly of 88° 58′ 7″, to which corresponds a mean anomaly of 91° 1′ 53″, ought not to show a value of 99,984, but 100,000, which is the mean between the distances of the figure and of the table. The reason for this assertion must be postponed until ch. 55 and the following.

It was just said, however, that we shall not err perceptibly if we err by 31 units. It will therefore hurt us much less perceptibly if we err by only half that, or 16 units. So for the time being we safely admit this small error, in order to accommodate ourselves to an understanding of what has been advanced in the reading so far, and to avoid appearing to presuppose what was to be demonstrated.

162

| Mean Anomaly | | | Distance | Equated Anomaly | | |
|---|---|---|---|---|---|---|
| D. | M. | S. | | D. | M. | S. |
| 0 | 0 | 0 | 101800 | 0 | 0 | 0 |
| 1 | 1 | 5 | 101800 | 0 | 58 | 55 |
| 2 | 2 | 10 | 101799 | 1 | 57 | 50 |
| 3 | 3 | 14 | 101797 | 2 | 56 | 46 |
| 4 | 4 | 18 | 101795 | 3 | 55 | 42 |
| 5 | 5 | 23 | 101793 | 4 | 54 | 37 |
| 6 | 6 | 27 | 101790 | 5 | 53 | 33 |
| 7 | 7 | 31 | 101786 | 6 | 52 | 29 |
| 8 | 8 | 36 | 101782 | 7 | 51 | 24 |
| 9 | 9 | 40 | 101777 | 8 | 50 | 20 |
| 10 | 10 | 44 | 101772 | 9 | 49 | 16 |
| 11 | 11 | 48 | 101766 | 10 | 48 | 12 |
| 12 | 12 | 52 | 101760 | 11 | 47 | 8 |
| 13 | 13 | 55 | 101753 | 12 | 46 | 5 |
| 14 | 14 | 58 | 101746 | 13 | 45 | 2 |
| 15 | 16 | 1 | 101738 | 14 | 43 | 59 |
| 16 | 17 | 3 | 101729 | 15 | 42 | 57 |
| 17 | 18 | 6 | 101720 | 16 | 41 | 54 |
| 18 | 19 | 8 | 101710 | 17 | 40 | 52 |
| 19 | 20 | 9 | 101700 | 18 | 39 | 51 |
| 20 | 21 | 10 | 101689 | 19 | 38 | 50 |
| 21 | 22 | 11 | 101678 | 20 | 37 | 49 |
| 22 | 23 | 11 | 101667 | 21 | 36 | 49 |
| 23 | 24 | 11 | 101654 | 22 | 35 | 49 |
| 24 | 25 | 10 | 101642 | 23 | 34 | 50 |
| 25 | 26 | 9 | 101628 | 24 | 33 | 51 |
| 26 | 27 | 8 | 101615 | 25 | 32 | 52 |
| 27 | 28 | 6 | 101600 | 26 | 31 | 54 |
| 28 | 29 | 3 | 101586 | 27 | 30 | 57 |
| 29 | 30 | 0 | 101570 | 28 | 30 | 0 |
| 30 | 30 | 56 | 101555 | 29 | 29 | 4 |
| 31 | 31 | 52 | 101539 | 30 | 28 | 8 |
| 32 | 32 | 47 | 101522 | 31 | 27 | 13 |
| 33 | 33 | 42 | 101505 | 32 | 26 | 18 |
| 34 | 34 | 36 | 101487 | 33 | 25 | 24 |
| 35 | 35 | 29 | 101469 | 34 | 24 | 31 |
| 36 | 36 | 22 | 101451 | 35 | 23 | 43 |
| 37 | 37 | 14 | 101432 | 36 | 22 | 46 |

| Mean Anomaly | | | Distance | Equated Anomaly | | |
|---|---|---|---|---|---|---|
| D. | M. | S. | | D. | M. | S. |
| 38 | 38 | 6 | 101412 | 37 | 21 | 54 |
| 39 | 38 | 57 | 101392 | 38 | 21 | 3 |
| 40 | 39 | 47 | 101372 | 39 | 20 | 13 |
| 41 | 40 | 36 | 101351 | 40 | 19 | 24 |
| 42 | 41 | 24 | 101330 | 41 | 18 | 36 |
| 43 | 42 | 12 | 101309 | 42 | 17 | 48 |
| 44 | 42 | 59 | 101287 | 43 | 17 | 1 |
| 45 | 43 | 45 | 101265 | 44 | 16 | 15 |
| 45 | 43 | 45 | 101265 | 44 | 16 | 15 |
| 46 | 44 | 30 | 101242 | 45 | 15 | 30 |
| 47 | 45 | 15 | 101219 | 46 | 14 | 45 |
| 48 | 45 | 59 | 101195 | 47 | 14 | 1 |
| 49 | 46 | 42 | 101172 | 48 | 13 | 18 |
| 50 | 47 | 24 | 101147 | 49 | 12 | 36 |
| 51 | 48 | 5 | 101123 | 50 | 11 | 55 |
| 52 | 48 | 46 | 101098 | 51 | 11 | 14 |
| 53 | 49 | 25 | 101073 | 52 | 10 | 35 |
| 54 | 50 | 4 | 101047 | 53 | 9 | 56 |
| 55 | 50 | 41 | 101022 | 54 | 9 | 19 |
| 56 | 51 | 18 | 100995 | 55 | 8 | 42 |
| 57 | 51 | 54 | 100969 | 56 | 8 | 6 |
| 58 | 52 | 29 | 100942 | 57 | 7 | 31 |
| 59 | 53 | 3 | 100915 | 58 | 6 | 57 |
| 60 | 53 | 35 | 100888 | 59 | 6 | 25 |
| 61 | 54 | 7 | 100860 | 60 | 5 | 53 |
| 62 | 54 | 38 | 100832 | 61 | 5 | 22 |
| 63 | 55 | 8 | 100804 | 62 | 4 | 52 |
| 64 | 55 | 37 | 100775 | 63 | 4 | 23 |
| 65 | 56 | 5 | 100747 | 64 | 3 | 55 |
| 66 | 56 | 32 | 100719 | 65 | 3 | 28 |
| 67 | 56 | 58 | 100690 | 66 | 3 | 2 |
| 68 | 57 | 22 | 100660 | 67 | 2 | 38 |
| 69 | 57 | 46 | 100631 | 68 | 2 | 14 |
| 70 | 58 | 9 | 100601 | 69 | 1 | 51 |
| 71 | 58 | 30 | 100571 | 70 | 1 | 30 |
| 72 | 58 | 51 | 100542 | 71 | 1 | 9 |
| 73 | 59 | 11 | 100511 | 72 | 0 | 49 |
| 74 | 59 | 29 | 100481 | 73 | 0 | 31 |

| | Mean Anomaly | | | Distance | Equated Anomaly | | |
|---|---|---|---|---|---|---|---|
| | D. | M. | S. | | D. | M. | S. |
| | 75 | 59 | 46 | 100451 | 74 | 0 | 14 |
| | 77 | 0 | 2 | 100420 | 74 | 59 | 58 |
| | 78 | 0 | 18 | 100389 | 75 | 59 | 42 |
| | 79 | 0 | 32 | 100359 | 76 | 59 | 28 |
| | 80 | 0 | 45 | 100328 | 77 | 59 | 15 |
| | 81 | 0 | 57 | 100297 | 78 | 59 | 3 |
| | 82 | 1 | 7 | 100266 | 79 | 58 | 53 |
| | 83 | 1 | 16 | 100235 | 80 | 58 | 44 |
| | 84 | 1 | 25 | 100203 | 81 | 58 | 36 |
| | 85 | 1 | 32 | 100172 | 82 | 58 | 28 |
| | 86 | 1 | 38 | 100141 | 83 | 58 | 22 |
| | 87 | 1 | 43 | 100109 | 84 | 58 | 17 |
| | 88 | 1 | 46 | 100078 | 85 | 58 | 14 |
| | 89 | 1 | 49 | 100047 | 86 | 58 | 11 |
| | 90 | 1 | 51 | 100015 | 87 | 58 | 9 |
| 163 | 91 | 1 | 53 | 99984 | 88 | 58 | 7 |
| | 91 | 1 | 53 | 99984 | 88 | 58 | 7 |
| | 92 | 1 | 51 | 99952 | 89 | 58 | 9 |
| | 93 | 1 | 49 | 99921 | 90 | 58 | 11 |
| | 94 | 1 | 46 | 99890 | 91 | 58 | 14 |
| | 95 | 1 | 43 | 99858 | 92 | 58 | 17 |
| | 96 | 1 | 38 | 99827 | 93 | 58 | 22 |
| | 97 | 1 | 32 | 99796 | 94 | 58 | 28 |
| | 98 | 1 | 25 | 99765 | 95 | 58 | 35 |
| | 99 | 1 | 16 | 99734 | 96 | 58 | 44 |
| | 100 | 1 | 7 | 99703 | 97 | 58 | 53 |
| | 101 | 0 | 57 | 99672 | 98 | 59 | 3 |
| | 102 | 0 | 45 | 99641 | 99 | 59 | 15 |
| | 103 | 0 | 31 | 99610 | 100 | 59 | 29 |
| | 104 | 0 | 18 | 99580 | 101 | 59 | 42 |
| | 105 | 0 | 2 | 99549 | 102 | 59 | 58 |
| | 105 | 59 | 46 | 99519 | 104 | 0 | 14 |
| | 106 | 59 | 29 | 99489 | 105 | 0 | 31 |
| | 107 | 59 | 11 | 99459 | 106 | 0 | 49 |
| | 108 | 58 | 51 | 99429 | 107 | 1 | 9 |
| | 109 | 58 | 31 | 99399 | 108 | 1 | 29 |
| | 110 | 58 | 9 | 99370 | 109 | 1 | 51 |
| | 111 | 57 | 46 | 99341 | 110 | 2 | 14 |

| Mean Anomaly | | | Distance | Equated Anomaly | | |
|---|---|---|---|---|---|---|
| D. | M. | S. | | D. | M. | S. |
| 112 | 57 | 23 | 99312 | 111 | 2 | 37 |
| 113 | 56 | 58 | 99283 | 112 | 3 | 2 |
| 114 | 56 | 32 | 99254 | 113 | 3 | 28 |
| 115 | 56 | 5 | 99226 | 114 | 3 | 55 |
| 116 | 55 | 37 | 99198 | 115 | 4 | 23 |
| 117 | 55 | 8 | 99170 | 116 | 4 | 52 |
| 118 | 54 | 38 | 99142 | 117 | 5 | 22 |
| 119 | 54 | 7 | 99115 | 118 | 5 | 53 |
| 120 | 53 | 35 | 99088 | 119 | 6 | 25 |
| 121 | 53 | 3 | 99061 | 120 | 6 | 57 |
| 122 | 52 | 29 | 99034 | 121 | 7 | 31 |
| 123 | 51 | 54 | 99008 | 122 | 8 | 6 |
| 124 | 51 | 18 | 98982 | 123 | 8 | 42 |
| 125 | 50 | 41 | 98957 | 124 | 9 | 19 |
| 126 | 50 | 4 | 98931 | 125 | 9 | 56 |
| 127 | 49 | 25 | 98906 | 126 | 10 | 35 |
| 128 | 48 | 46 | 98882 | 127 | 11 | 14 |
| 129 | 48 | 5 | 98857 | 128 | 11 | 55 |
| 130 | 47 | 25 | 98833 | 129 | 12 | 35 |
| 131 | 46 | 42 | 98810 | 130 | 13 | 18 |
| 132 | 45 | 59 | 98787 | 131 | 14 | 1 |
| 133 | 45 | 15 | 98764 | 132 | 14 | 45 |
| 134 | 44 | 31 | 98741 | 133 | 15 | 29 |
| 135 | 43 | 45 | 98719 | 134 | 16 | 15 |
| 135 | 43 | 45 | 98719 | 134 | 16 | 15 |
| 136 | 42 | 59 | 98698 | 135 | 17 | 1 |
| 137 | 42 | 12 | 98676 | 136 | 17 | 48 |
| 138 | 41 | 24 | 98655 | 137 | 18 | 36 |
| 139 | 40 | 36 | 98634 | 138 | 19 | 24 |
| 140 | 39 | 47 | 98614 | 139 | 20 | 13 |
| 141 | 38 | 57 | 98595 | 140 | 21 | 3 |
| 142 | 38 | 6 | 98575 | 141 | 21 | 54 |
| 143 | 37 | 14 | 98557 | 142 | 22 | 46 |
| 144 | 36 | 22 | 98538 | 143 | 23 | 38 |
| 145 | 35 | 30 | 98520 | 144 | 24 | 30 |
| 146 | 34 | 36 | 98503 | 145 | 25 | 24 |
| 147 | 33 | 42 | 98486 | 146 | 26 | 18 |
| 148 | 32 | 47 | 98469 | 147 | 27 | 13 |

| Mean Anomaly | | | Distance | Equated Anomaly | | |
|---|---|---|---|---|---|---|
| D. | M. | S. | | D. | M. | S. |
| 149 | 31 | 52 | 98453 | 148 | 28 | 8 |
| 150 | 30 | 56 | 98437 | 149 | 29 | 4 |
| 151 | 30 | 0 | 98422 | 150 | 30 | 0 |
| 152 | 29 | 3 | 98407 | 151 | 30 | 57 |
| 153 | 28 | 6 | 98393 | 152 | 31 | 54 |
| 154 | 27 | 8 | 98379 | 153 | 32 | 52 |
| 155 | 26 | 9 | 98366 | 154 | 33 | 51 |
| 156 | 25 | 10 | 98353 | 155 | 34 | 50 |
| 157 | 24 | 11 | 98341 | 156 | 35 | 49 |
| 158 | 23 | 11 | 98329 | 157 | 36 | 49 |
| 159 | 22 | 11 | 98317 | 158 | 37 | 49 |
| 160 | 21 | 10 | 98307 | 159 | 38 | 50 |
| 161 | 20 | 9 | 98296 | 160 | 39 | 51 |
| 162 | 19 | 8 | 98286 | 161 | 40 | 52 |
| 163 | 18 | 6 | 98277 | 162 | 41 | 54 |
| 164 | 17 | 3 | 98268 | 163 | 42 | 57 |

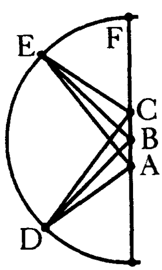
| Mean Anomaly | | | Distance | Equated Anomaly | | |
|---|---|---|---|---|---|---|
| D. | M. | S. | | D. | M. | S. |
| 165 | 16 | 1 | 98260 | 164 | 43 | 59 |
| 166 | 14 | 58 | 98253 | 165 | 45 | 2 |
| 167 | 13 | 55 | 98245 | 166 | 46 | 5 |
| 168 | 12 | 52 | 98239 | 167 | 47 | 8 |
| 169 | 11 | 48 | 98232 | 168 | 48 | 12 |
| 170 | 10 | 44 | 98227 | 169 | 49 | 16 |
| 171 | 9 | 40 | 98222 | 170 | 50 | 20 |
| 172 | 8 | 36 | 98217 | 171 | 51 | 24 |
| 173 | 7 | 31 | 98213 | 172 | 52 | 29 |
| 174 | 6 | 27 | 98210 | 173 | 53 | 33 |
| 175 | 5 | 23 | 98207 | 174 | 54 | 37 |
| 176 | 4 | 18 | 98204 | 175 | 55 | 42 |
| 177 | 3 | 14 | 98202 | 176 | 56 | 46 |
| 178 | 2 | 10 | 98201 | 177 | 57 | 50 |
| 179 | 1 | 5 | 98200 | 178 | 58 | 55 |
| 180 | 0 | 0 | 98200 | 180 | 0 | 0 |

# 31

## That the bisection of the sun's eccentricity does not perceptibly alter the equations of the sun set out by Tycho: and concerning four ways of computing them

But lest there remain any suspicions preventing our moving onwards, we shall investigate, in the usual Ptolemaic form of the first inequality*, whether there be any difference in the solar equations consequent upon the now bisected eccentricity.

*In the following chapters, the uncautious reader may become confused. The motion of the sun (for Brahe) or of the earth (for Copernicus) or of the epicycle (for Ptolemy), which is the cause of the second inequality for the other planets, also itself participates in the first inequality.




*First let there be an unbisected eccentricity of 3600 on the line of apsides* AF, *with* CE *and* CD *accordingly radii of the orb; and let the anomaly* FAE *be 45°, and* FAD *135°. Now it is obvious that, however great the discrepancy may be, it will reach its maximum around these positions of anomaly. For in the middle longitudes the equations come out exactly the same, since 3600, when investigated in both the sines and tangents*[1], *results in the same arc. Therefore, as the radius* CE *is*

[1] That is, with a simple eccentric the angle of the equation (AEC or ADC) can be found using the law of sines alone, while the equant is added, the angle is the sum of the optical part (AEB or ADB), found by the law of sines, and physical part (BEC or BDC), found by the law of tangents. Kepler is claiming that the two calculations produce identical results at the middle longitudes, and series expansions of the angular relations show this to be true when the angle at C is 90°.

*to the sine of the angle* CAE *or* CAD, *so is the eccentricity* CA *to the equation* CEA *or* CDA, *which are both 1° 27′ 31″*. And in this first way, Ptolemy computed the equations of the sun, and, following Ptolemy, Copernicus; and, following both, Brahe: each of them using only the eccentricity AC, whose magnitude they found through their observations.

There now follows a second way of computing the same equations, of which Ptolemy made use in the other planets, and of which I shall make use. For I have demonstrated in this third part that the centre of the eccentric is not at the point C, the centre of uniform motion, but at B, the midpoint between the centre of the world A and the point of uniformity C.

*Therefore let* CA *be bisected at* B, *and let* EB, BD, *be the radius of the orb. By the same method, the part of the equation* BDA, BEA, *will be 0° 43′ 46″, which, added to* EAB, DAB, *results in an angle* EBC *of 45° 43′ 46″, and* DBC, *of 135° 43′ 46″. As a result, from the sides and the included angle,* BEC *comes out to be 43′ 38″, and* BDC *43′ 42″. Thus the whole angle* CEA *is 1° 27′ 24″, and* CDA *is 1° 27′ 28″, within a hair's breadth of the previous value.* And so, in the appendix to Tycho Brahe's *Progymnasmata* p. 821, where the difference of the two calculations is given as $1\frac{1}{6}'$, you should read $\frac{1}{6}'$[2]. And this is in accord with the conclusions of ch. 4 applied to the form of the vicarious hypothesis.

And since you may observe that in this particular form of Ptolemaic hypothesis the parts of the equation are nearly equal (for the optical part was 43′ 46″ and the physical part was 43′ 38″ at E and 43′ 42″ at D), you see why, in the construction of the table in the preceding chapter, all I did was double the equation to establish the total equation. And this is the third way of computing the sun's equations. For at apogee and perigee both parts of the equation vanish, and in the middle longitudes the parts are again equal, as was just now said. Therefore, since these three ways of computing the equations co-

[2] Kepler made an error in computing BEC and BDC, which should have been 44′ 52″ and 42′ 39″, respectively. Thus CEA should have been 1° 28′ 38″, and CDA, 1° 26′ 25″. The differences are 1′ 7″ and 1′ 6″, respectively, not far from the $1\frac{1}{6}'$ in the *Progymnasmata* (TBOO 3 p. 322). But there is an interesting twist to the story: when Magini noticed the errors and pointed them out to Kepler (letters of 15 January and 23 February 1610, numbered 548 and 555 in KGW 16 pp. 270–274 and 285–7), Kepler replied (letter of 1 February 1610, number 551 in KGW 16 pp. 279–280) that he himself had written that appendix.

Caspar's analysis of the expanded equations for the orbits (mentioned in KGW 3 p. 467) shows that the maximum difference (which is also the difference between the physical and optical parts of the equation) amounts to 1′ 17″.

incide in eight places distributed about the entire circle, they will perceptibly coincide everywhere. This is a result of the smallness of the eccentricity: if it were greater, this coincidence would not take place everywhere.

165 Now I shall prepare myself for finding a fourth way to the equation, to be computed not through an arbitrary hypothesis but from the very nature of things. This will require eight chapters, so that this fourth way can finally follow in chapter 40.

# 32

## The power that moves the planet in a circle diminishes with removal from its source

I have said above that Ptolemy, well informed by the observations, bisected the eccentricities of the three superior planets, that Copernicus imitated this, and that Tycho's observations of Mars urge the same conclusion, as has been seen in chapters 19 and 20, and will appear with much greater certainty below in chapter 42. In addition, Tycho closely imitated this in his lunar theory. And now the same thing has been demonstrated in the theory of the sun (for Tycho) or of the earth (for Copernicus). Further, there is nothing to prevent our believing the same of Venus and Mercury. Indeed, I now have a proof that this is the origin of the belief that the centres of these planets' eccentrics move around on a small annual circle. Therefore all planets have this [double eccentricity]. Now in my *Mysterium cosmographicum*, published eight years ago*, I postponed arguing this case of the cause of the Ptolemaic equant for the sole reason that it could not be said on the basis of ordinary astronomy whether the sun or earth uses an equalizing point and has its eccentricity bisected. However, now that we have the confirmation of a sounder astronomy, it should be transparently clear that there is indeed an equant in the theory of the sun or earth. And, I say, now that this is demonstrated, it is proper to accept as true and legitimate the cause to which I assigned the Ptolemaic equant in the *Mysterium cosmographicum*, since it is universal and common to all the planets. So in this part of the work I shall make a further declaration of that cause.

*Now more than that.

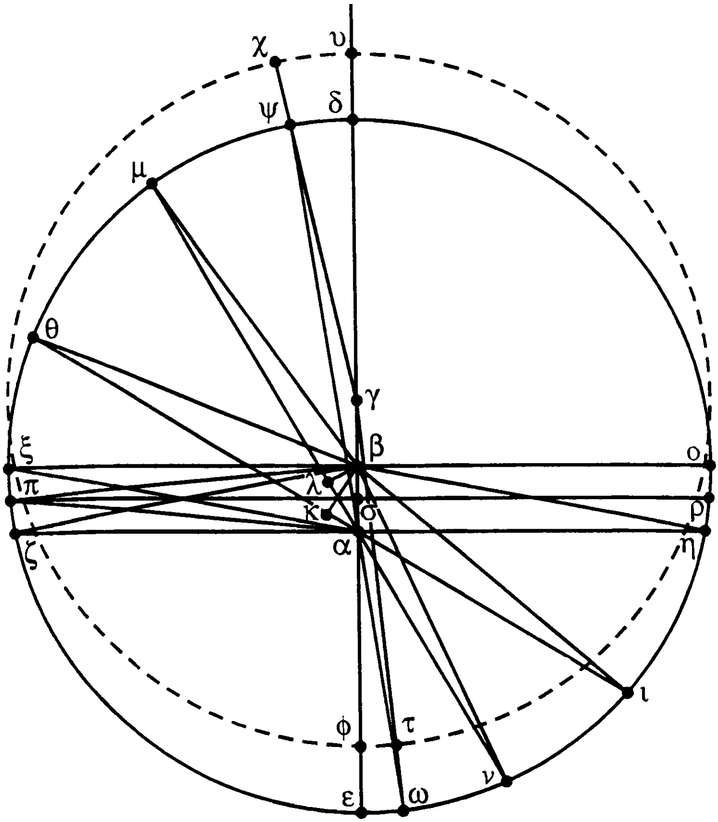
And since the declaration will be general, I shall use the word, 'planet'. However, in this and the next few chapters, the reader may

always understand this as denoting in particular the earth for Copernicus or the sun for Tycho.

First, the reader should know that in all hypotheses constructed according to this Ptolemaic form, however great the eccentricity, the speed at perihelion and slowness at aphelion are very closely proportional to the lines drawn from the centre of the world to the planet.

*In the diagram of chapter 29, in which* α *was the centre of the world,* β *was the centre of the eccentric* δε, *and* γ *was the point of the equant, let the equant circle* υφ *be described about centre* γ, *with radius equal to* βδ. *And through the centre of the world* α, *from which the eccentricity is reckoned (and in the business at hand, it is the sun for Copernicus and the earth for the others), let the straight line* ψω *be drawn, intersecting the eccentric at* ψ *and* ω, *so that the planet is at* ψ *and* ω,
166 *having described the arcs of the eccentric* δψ *and* εω, *from apogee or aphelion and from perigee or perihelion, respectively. It is supposed that these arcs appear equal from* α, *since the straight line* ψω *makes the vertical angles* ψαδ *and* ωαε, *which are equal. But since* δψ *and* εω *are taken as minimal arcs, as if at the apsides* δ *and* ε, *they do not differ appreciably from straight lines. And so, just as if* δαψ *and* εαω *were rectilinear triangles, with right angles at* δ *and* ε, *and a common vertex* α, δα *will be to* εα *as arc* δψ *is to arc* εω. *But* αδ *is longer than* αε. *Therefore, arc* δψ *is longer than* εω. *These arcs, which are in fact unequal, appear equal from* α. *The question now is, how much time will the planet take to traverse each arc, according to Ptolemy's theory and hypothesis, when it has an equant? So let straight lines be drawn from the centre* γ *through the points* ψ *and* ω, *intersecting the equant at* χ, τ. *Now Ptolemy will say, 'since the whole circle of the equant* υφ *denotes the periodic time of the planet, then* υχ *is the measure of the time which the planet takes to traverse the arc of the eccentric* ψδ, *and* φτ *is the measure of the time which the planet takes to traverse the arc of the eccentric* εω.'

*And now I myself say that* υχ, *thus designated as the arc of the time (as Ptolemy wished) is to the arc* δψ *which the planet traverses, approximately as* αδ, *the distance of the arc* δψ *from the centre of the world, is to* δβ *the mean distance of the points* π *and* ρ *from* α. *And likewise, the arc of the time* φτ *is to the arc of the planet's motion* εω, *approximately as* αε, *the distance of the arc* εω *from the centre of the world* α, *is to* εβ *and* απ, *the mean distance from the centre of the world, which may be found at the points* π *and* ρ. *And now, as before,*




*as* $\gamma\upsilon$ *is to* $\gamma\delta$ *so is* $\upsilon\chi$ *to* $\delta\psi$, *and as* $\gamma\phi$ *is to* $\gamma\epsilon$ *so is* $\phi\tau$ *to* $\epsilon\omega$. *But* $\gamma\upsilon$ *is to* $\gamma\delta$ *very nearly as* $\beta\delta$ *(or* $\gamma\upsilon$*) is to* $\alpha\delta$, *and this is shown by the fact that* $\beta\delta$ *is the arithmetic mean between* $\gamma\delta$ *and* $\alpha\delta$, *since Ptolemy makes* $\alpha\beta$, $\beta\gamma$ *equal. And further, the arithmetic mean between two terms whose ratio is near equality is only imperceptibly greater than the geometric mean. For example, the arithmetic mean between 10 and 12 is 11, and the geometric mean is about* $10\frac{19}{20}$, *so that there is less than the twentieth part of a unit between the two means. Nevertheless, these numbers are of the order of the eccentricity of Mars, which according to Ptolemy has the greatest eccentricity of all the planets.*

*And therefore, since the ratio* $\gamma\upsilon$ *to* $\gamma\delta$ *is imperceptibly greater than the ratio* $\alpha\delta$ *to* $\delta\beta$, $\chi\upsilon$ *will also have to* $\psi\delta$ *a ratio imperceptibly greater than* $\alpha\delta$ *to* $\delta\beta$. *Likewise, as* $\gamma\epsilon$ *is to* $\gamma\phi$, *so is* $\epsilon\omega$ *to* $\phi\tau$, *but* $\gamma\epsilon$ *is to* $\gamma\phi$

*approximately as* $\epsilon\beta$ *is to* $\alpha\epsilon$*; that is, the former ratio is only imperceptibly less than the latter. Therefore, too, the ratio* $\epsilon\omega$ *to* $\phi\tau$ *is only imperceptibly smaller than the ratio* $\epsilon\beta$ *to* $\alpha\epsilon$.

*Now let us permute the ratios. For the ratio* $\alpha\delta$ *to* $\delta\beta$ *is imperceptibly less than the ratio* $\delta\beta$ *or* $\beta\epsilon$ *to* $\epsilon\alpha$*, seeing that* $\beta\delta$ *or* $\beta\epsilon$ *is the arithmetic mean between* $\alpha\delta$ *and* $\alpha\epsilon$*, as before. But it was proved that the ratio* $\upsilon\chi$ *to* $\delta\psi$ *is greater than the ratio* $\alpha\delta$ *to* $\delta\beta$*, from the smaller pair, and the*
167 *ratio* $\epsilon\omega$ *to* $\phi\tau$ *is less than the ratio* $\epsilon\beta$ *to* $\alpha\epsilon$*, from the greater pair, so that in the two ratios* $\alpha\delta$ *to* $\delta\beta$ *and* $\epsilon\beta$ *to* $\alpha\epsilon$*, the former terms will be greater and the latter smaller to the extent that this is true concerning the two ratios* $\upsilon\chi$ *to* $\delta\psi$ *and* $\epsilon\omega$ *to* $\phi\tau$*. Consequently, there is some compensation even for that imperceptible difference, so that it is much more nearly true that the ratio of* $\upsilon\chi$ *to* $\delta\psi$ *is equal within a hair's breadth to the ratio of* $\epsilon\omega$ *to* $\phi\tau$.

*Therefore, taking as equal the arcs* $\delta\psi$ *and* $\epsilon\omega$ *(which hitherto were unequal), either* $\delta\psi$ *or* $\epsilon\omega$ *will be a mean proportional between* $\upsilon\chi$*, the elapsed time at aphelion, and* $\phi\tau$*, the time at perihelion. Consequently, the ratio* $\upsilon\chi$ *to* $\phi\tau$ *(*$\delta\psi$ *and* $\epsilon\omega$ *being equal) will be in the duplicate ratio of* $\alpha\delta$ *to* $\delta\beta$ *or* $\beta\epsilon$ *to* $\epsilon\alpha$*, the former smaller and the latter greater by an imperceptible difference. But since the ratio* $\alpha\delta$ *to* $\alpha\epsilon$ *is also the duplicate of either of these (for it is compounded of the two, which are nearly equal, taking the arithmetic mean* $\delta\beta$ *or* $\beta\epsilon$*), therefore, the arcs on the eccentric* $\delta\psi$ *and* $\epsilon\omega$ *being equal, the ratio of the times* $\upsilon\chi$ *to* $\phi\tau$ *will be equal to the ratio* $\alpha\delta$ *to* $\alpha\epsilon$*. Or, more clearly,* the planet takes a proportionally longer time to traverse an arc of the eccentric at $\delta$ than to traverse an equal arc at $\epsilon$, according as $\alpha\delta$ is greater than $\alpha\epsilon$. And this follows from the way the Ptolemaic form* is ordered, and from its equalizing point, by means of a certain and valid proof pertaining to points near apogee and perigee. At other points there appears a very small discrepancy, whose effect is in fact smaller where in the demonstration it is more evident. This is because (for example) the ratio $\alpha\mu$ to $\alpha\nu$ is smaller, and the ratio $\alpha\theta$ to $\alpha\iota$ is much smaller, than $\alpha\delta$ to $\alpha\epsilon$, the greatest ratio, where the effect is also greatest.

*This refers to the particular form that accounts for the first inequality.

# 33

## The power that moves the planets resides in the body of the sun

It was demonstrated in the previous chapter that the elapsed times of a planet on equal parts of the eccentric circle (or on equal distances in the aethereal air) are in the same ratio as the distances of those spaces from the point whence the eccentricity is reckoned; or, more simply, to the extent that a planet is farther from the point which is taken as the centre of the world, it is less strongly urged to move about that point. It is therefore necessary that the cause of this weakening is either in the very body of the planet, in a motive force placed therein, or right at the supposed centre of the world.

Now it is an axiom in natural philosophy of the most common and general application that of those things which can occur at the same time and in the same manner, and which are always subject to like measurements, either one is the cause of the other or both are effects of the same cause. Just so, in this instance, the intension and remission of motion is always in the same ratio as the approach and recession from the centre of the world. Thus, either that weakening will be the cause of the star's motion away from the centre of the world, or the motion away will be the cause of the weakening, or both
168 will have some cause in common. But it would be impossible for anyone to think up some third concurrent thing which would be the cause of these two, and in the following chapters it will become clear that we have no need of feigning any such cause, since the two are sufficient in themselves.

Further, it is not in accord with nature that strength or weakness in longitudinal motion should be the cause of distance from the centre.

For distance from the centre is prior both in thought and in nature to motion over an interval. Indeed, motion over an interval is never independent of distance from the centre, since it requires a space in which to be performed, while distance from the centre can be conceived without motion. Therefore, distance will be the cause of intensity of motion, and a greater or lesser distance will result in a greater or lesser amount of time.

And since distance is a kind of relation whose essence resides in end points, while of relation itself, without respect to end points, there can be no efficient cause, it therefore follows (as has been said) that the cause of the variation of intensity of motion inheres in one or the other of the end points.

Now the body of a planet is never by itself made heavier in receding, nor lighter in approaching.

Moreover, that an animal force, which the motion of the heavens suggests is seated in the mobile body of the planet, undergoes intension and remission so many times without ever becoming tired or growing old, – this will surely be absurd to say. Also, it is impossible to understand how this animal force could carry its body through the spaces of the world, since there are no solid orbs, as Tycho Brahe has proved. And on the other hand, a round body lacks such aids as wings or feet, by the moving of which the soul might carry its body through the aethereal air as birds do in the atmosphere, by some kind of pressure upon, and counter-pressure from, that air[1].

Therefore, the only remaining possibility is that the cause of this intensification and weakening resides in the other endpoint, namely, in that point which is taken to be the centre of the world, from which the distances are measured.

The motive power is in the centre of the system.

So now, if the distance of the centre of the world from the body of a planet governs its slowness, and approach governs its speeding up, it is a necessary consequence that the source of motive power is at that supposed centre of the world. And with this laid down, the manner in which the cause operates is also clear. For it gives us to understand

[1] This clear adumbration of Newton's Third Law, couched as it is in terms of animal locomotion, lends support to the view (for which the translator is indebted to Robert Sacks of St. John's College, Santa Fe, New Mexico) that the intellectual ancestry of that law is to be found in Aristotle's *On the Motion of Animals*, chapters 1 and 2. Aristotle's most pertinent statement is this: 'For just as in the animal there must be something which is immovable if it is to have any motion, so *a fortiori* there must be something which is immovable outside the animal, supported upon which that which is moved moves. . . . [T]here will be . . . no flying or swimming unless the air or sea were to offer resistance.' (698 b 13–18, translated by A. L. Peck, Loeb Classical Library, 1955.)

that the planets are moved rather in the manner of the steelyard or lever. For if the planet is moved with greater difficulty (and hence more slowly) by the power at the centre when it is farther from the centre, it is just as if I had said that where the weight is farther from the fulcrum, it is thereby rendered heavier, not of itself, but by the power of the arm supporting it at that distance. And this is true, both of the steelyard or lever, and of the motion of the planets: that the weakening of power is in the ratio of the distances.

The sun is in the centre of the planetary system.

But which body is it that is at the centre? Is there none, as for Copernicus when he is computing, and for Tycho in part? Is it the earth, as for Ptolemy and for Tycho in part? Or finally, is it the sun itself, as I, and Copernicus when he is speculating, would have it? This question I began to discuss in physical terms in Part I. I there supposed as one of the principles what has now been expressly and
169 geometrically proved in chapter 32: that a planet is moved less vigorously when it recedes from the point whence the eccentricity is computed.

From this principle I presented a probable argument that the sun is at that point and at the centre of the world (or the earth for Ptolemy) rather than its being some other point occupied by no body. Allow me, then, to recall that same probable argument, its principles now demonstrated, in the present chapter. Then, as you may remember, I demonstrated in the second part, that the phenomena at either end of the night follow beautifully if the oppositions of Mars are reckoned according to the sun's apparent position. If this is done, then we likewise set up the eccentricity and the distances from the very centre of the sun's body, so that the sun itself again comes to be at the centre of the world (for Copernicus), or at least at the centre of the planetary system (for Tycho). But of these two arguments, one depends upon physical probability, and the other proceeds from possibility to actuality. And so in the third place I have demonstrated from the observations (in a proof which, because of its conceptual difficulty, I have postponed until chapter 52) that we cannot avoid referring Mars to the apparent position of the sun, and drawing the line of apsides, which bisects the eccentric, directly through the sun's body, unless perhaps we wish to allow an eccentric such as will by no means be in accord with the parallax of the annual orb. Anyone who cannot tolerate the delay may read about this in chapter 52, and then may carry on here afterwards. For there nothing is assumed but the bare

observations. You will find a similar proof in Part V, from considerations of latitudes.

The motive power is in the sun.

Therefore, with the sun belonging in the centre of the system, the source of motive power, from what has now been demonstrated, belongs in the sun, since it too has now been located in the centre of the world.

But indeed, if this very thing which I have just demonstrated *a posteriori* (from the observations) by a rather long deduction, if, I say, I had taken this as something to be demonstrated *a priori* (from the worthiness and eminence of the sun), so that the source of the world's life (which is visible in the motion of the heavens) is the same as the source of the light which forms the adornment of the entire machine, and which is also the source of the heat by which everything grows, I think I would deserve an equal hearing.

The sun is in the centre of the world, and does not move from place to place.

Tycho Brahe himself, or anyone who prefers to follow his general hypothesis of the second inequality, should consider by how close a likeness to the truth this physically elegant combination has for the most part been accepted (since for him, too, this substitution of the apparent position of the sun brings the sun back to the centre of the planetary system) yet to some extent recoils from his hypothesis.

For it is obvious from what has been said that only one of the following can be true: either the power residing in the sun, which moves all the planets, by the same action moves the earth as well; or the sun, together with the planets linked to it through its motive force, is borne about the earth by some power which is seated in the earth.

Now Tycho himself destroyed the notion of real orbs, and I in turn have in this third part irrefutably demonstrated that there is an equant in the theory of the sun or earth. From this it follows that the motion of the sun itself (if it is moved) is intensified and remitted according as it is nearer or farther from the earth, and hence that the sun is moved by the earth. But if, on the other hand, the earth is in motion, it too will be moved by the sun with greater or less velocity
170 according as it is nearer or farther from it, while the power in the body of the sun remains perpetually constant. Between these two possibilities, therefore, there is no intermediate.

The moon is driven around by the earth, but the sun and the other

I myself agree with Copernicus, and allow that the earth is one of the planets.

Now it is true that the same objection may be raised against

planets are not: the earth, on the other hand, is driven around by the sun.

Copernicus concerning the moon, that I raised against Tycho concerning the five planets; namely, that it appears absurd for the moon to be moved by the earth, and to be associated with it and bound to it as well, so that it too, as a secondary planet, is swept around the sun by the sun. Nevertheless, I prefer to allow one moon, akin to the earth in its corporeal disposition[2] (as I have shown in the *Optics*[3]) to be moved by a power seated in the earth but extended towards the sun, as will be described a little later in ch. 37, than to ascribe to that same earth as well the motion of the sun and of all the planets bound to it.

The kinship of the solar motive power with light.

But let us carry on to a consideration of this motive power residing in the sun, and let us now again observe its very close kinship with light.

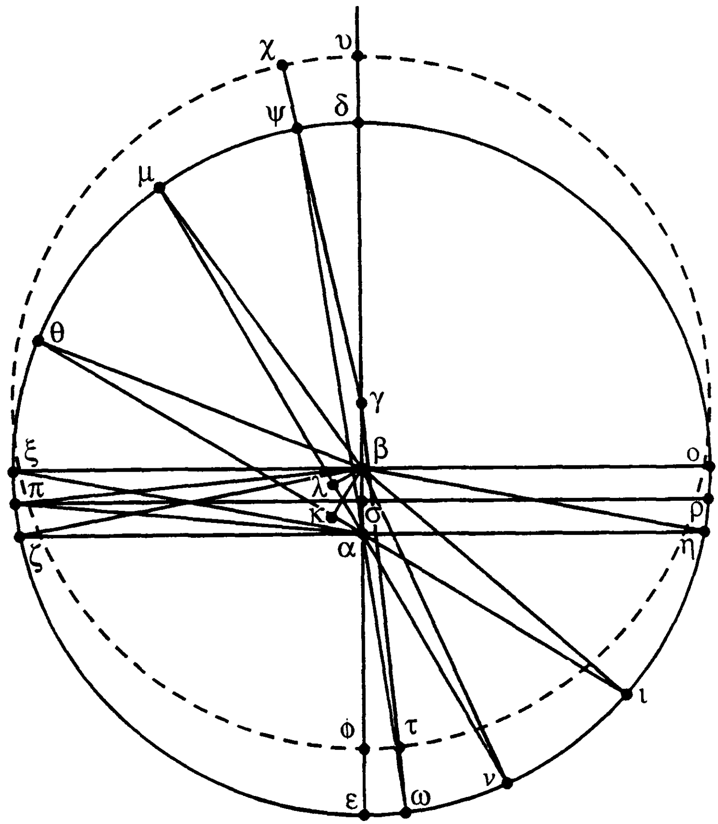
*Since the perimeters of similar regular figures, even of circles, are to one another as their semidiameters, therefore as* $\alpha\delta$ *is to* $\alpha\epsilon$, *so is the circumference of the circle described about* $\alpha$ *through* $\delta$ *to the circumference of the circle described about the same point* $\alpha$ *through* $\epsilon$. *But as* $\alpha\delta$ *is to* $\alpha\epsilon$, *so (inversely) is the strength of the power at* $\epsilon$ *to the strength of the power at* $\delta$, *by what was proved in chapter 32. Therefore, as the circle at* $\delta$ *is to the smaller circle at* $\epsilon$, *so, inversely, is the power at* $\epsilon$ *to the power at* $\delta$; *that is, the power is weaker to the extent that it is more spread out, and stronger to the extent that it is more concentrated. Hence we may understand that there is the same power in the whole circumference of the circle through* $\delta$ *as there is in the circumference of the smaller circle through* $\epsilon$. *This is shown to be true of light in exactly the same way in the* Astronomiae pars optica, *chapter 1*[4]. *Therefore, in all respects and in all its attributes, the motive power from the sun coincides with light.*

Whether light is the vehicle of the motive power.

And although this light of the sun cannot be the moving power itself, I leave it to others to see whether light may perhaps be so constituted as to be, as it were, a kind of instrument or vehicle, of which the moving power makes use.

This seems gainsaid by the following: first, light is hindered by the opaque, and therefore if the moving power had light as a vehicle, darkness would result in the movable bodies being at rest; again, light

[2] Changing the punctuation slightly so as to read *dispositione corporis* as modifying *cognatam* rather than *movendam*.
[3] Ch. 6, pp. 228–9, KGW 2 p. 203.
[4] Ch. 1 proposition 9, in KGW 2 p. 22.




spreads spherically in straight lines, while the moving power, though spreading in straight lines, does so circularly; that is, it is exerted in but one region of the world, from east to west, and not the opposite,
171 not at the poles, and so on. But we shall be able to reply plausibly to these objections in the chapters immediately following.

*The moving power is an immaterial species of the solar body.*

Finally, since there is just as much power in a larger and more distant circle as there is in a smaller and closer one, nothing of this power is lost in travelling from its source, nothing is scattered between the source and the movable body. The emission, then, in the same manner as light, is immaterial, unlike odours, which are accompanied by a diminution of substance, and unlike heat from a hot furnace, or anything similar which fills the intervening space. The remaining possibility, then, is that, just as light, which lights the

This *species* belongs to the second species of continuous quantity, and is a variety of surface.

whole earth, is an immaterial *species*[5] of that fire which is in the body of the sun, so this power which enfolds and bears the bodies of the planets, is an immaterial *species* residing in the sun itself, which is of inestimable strength, seeing that it is the primary agent of every motion in the universe.

Since, therefore, this *species* of the power, exactly as the *species* of light (for which see the *Astronomiae pars optica* ch. 1[6]), cannot be considered as dispersed throughout the intermediate space between the source and the mobile body, but is seen as collected in the body in proportion to the amount of the circumference it occupies, this power (or *species*) will therefore not be any geometrical body, but is like a variety of surface, just as light is. To generalize this, the *species* which proceed immaterially from things are not by that procession extended through the dimensions of a body, although they arise from a body (as this one does from the body of the sun). Instead, they proceed according to that very law of emission: they do not possess their own boundaries, but just as the surfaces of illuminated things cause light to be considered as surfaces in certain respects, because they receive and terminate its emission, so the bodies of things that are moved suggest that this moving power be considered as if a sort of geometrical body, because their whole masses terminate or receive this emission of the motive *species*, so that the *species* can exist or subsist nowhere in the world but in the bodies of the mobile things themselves. And, exactly like light, between the source and the movable thing it is in a state of becoming, rather than of being.

In what manner the immaterial *species* of the body of the sun may be quantified.

Moreover, at the same time, a reply can be made here to a possible objection. For it was said above that this motive power is extended throughout the space of the world, in some places more spread out and in others more concentrated, and that the intensification and remission of the motions of the planets are consequent upon this variation. Now, however, it has been said that this power is an immaterial *species* of its source, and never inheres in anything except a mobile subject, such as the body of a planet. But to lack matter and yet to be subject to geometrical dimensions appear to be contradictory. This implies that it is poured out throughout the whole world, and yet does not exist anywhere but where there is something movable.

[5] For the meaning of this technical term, for which there is no acceptable English equivalent, see the Glossary.

[6] Page 7, in KGW 2 p. 19.

The reply is this: although the motive power is not anything material, nevertheless, since it is destined to carry matter (namely, the body of a planet), it is not free from geometrical laws, at least on account of this material action of carrying things about. Nor is there need for more, for we see that those motions are carried out in space and time, and that this power arises and is poured out from the source through the space of the world, all of which are geometrical entities. So this power should indeed be subject to other geometrical necessities.

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Comparison of light and motive power on the basis of quantification:

But lest I appear to philosophize with excessive insolence, I shall propose to the reader the clearly authentic example of light, since it also makes its nest in the sun, thence to break forth into the whole world as a companion to this motive power. Who, I ask, will say that light is something material? Nevertheless, it carries out its operations with respect to place, suffers alteration[7], is reflected and refracted, and assumes quantities so as to be dense or rare, and to be capable of being taken as a surface wherever it falls upon something illuminable. Now just as it is said in optics, that light does not exist in the intermediate space between the source and the illuminable, this is equally true of the motive power.

And on the basis of time.

Moreover, although light itself does indeed flow forth in no time, while this power creates motion in time, nonetheless the way in which both do so is the same, if you consider them correctly. Light manifests those things which are proper to it instantaneously, but requires time to effect those which are associated with matter. It illuminates a surface in a moment, because here matter need not undergo any alteration, for all illumination takes place according to surfaces, or at least as if a property of surfaces and not as a property of corporeality as such. On the other hand, light bleaches colours in time, since here it acts upon matter *qua* matter, making it hot and expelling the contrary cold which is embedded in the body's matter and is not on its surface. In precisely the same manner, this moving power perpetually and without any interval of time is present from the sun wherever there is a suitable movable body, for it receives nothing from the movable body to cause it to be there. On the other hand, it causes motion in time, since the movable body is material.

Why the planets do not equal their mover, the immaterial *species* of the sun, in speed.

Or if it seems better, frame the comparison in this manner: light is constituted for illumination, and it is just as certain that power is

[7] Reading *mutationem* instead of *mutuum*.

constituted for motion. Light does everything that can be done to achieve the greatest illumination; nonetheless, it does not happen that colour is most greatly illuminated. For colour intermingles its own peculiar *species* with the illumination of light, thus forming some third entity. In like manner, there is no retardation in the moving power to prevent the planet's having as much speed as it has itself, but it does not follow that the planet's speed is that great, since something intervening prevents that, namely, some sort of matter possessed by the surrounding aether, or the disposition of the movable body itself to rest (others might say, 'weight', but I do not entirely approve of that, except, indeed, where the earth is concerned). It is the tempering effect of these, together with the weakening of the motive power, that determines a planet's periodic time.

# 34

## The sun is a magnetic body, and rotates in its space

Concerning that power that is closely attached to, and draws, the bodies of the planets, we have already said how it is formed, how it is akin to light, and what it is in its metaphysical being. Next, we shall contemplate the deeper nature of its source, shown by the outflowing
173 *species* (or archetype). For it may appear that there lies hidden in the body of the sun a sort of divinity, which may be compared to our soul, from which flows that *species* driving the planets around, just as from the soul of someone throwing pebbles a *species* of motion comes to inhere in the pebbles thrown by him, even when he who threw them removes his hand from them. And to those who proceed soberly, other reflections will soon be provided.

The power that moves the planets is whirled around.

The power that is extended from the sun to the planets moves them in a circular course around the immovable body of the sun. This cannot happen, or be conceived in thought, in any other way than this, that the power traverses the same path along which it carries the other planets. This has been observed to some extent in catapults and other violent motions. Thus, Fracastoro[1] and others, relying on a story told by the most ancient Egyptians, spoke with little probability when they said that some of the planets perchance would have their orbits deflected gradually beyond the poles of the world, and thus afterwards would move in a path opposite to the rest and to their modern course. For it is much more likely that the bodies of the

[1] Hieronymus Fracastorius, *Homocentrica*, Venice 1538, Sect. 3 Cap. 8. In this chapter, which bears the title, '*Cur solis declinatio minuatur*' (why the sun's declination changes), Fracastoro refers, in support of his remarkable opinion, to information received from the Egyptians by Herodotus and Pomponius Mela. (Citation from KGW 3 p. 468).

planets are always borne in that direction in which the power emanating from the sun tends.

But this *species* is immaterial, proceeding from its body out to this distance without the passing of any time, and is in all other respects like light. Therefore, it is not only required by the nature of the *species*, but likely in itself owing to this kinship with light, that along with the particles of its body or source it too is divided up, and when any particle of the solar body moves towards some part of the world, the particle of the immaterial *species* that from the beginning of creation corresponded to that particle of the body also always moves towards the same part. If this were not so, it would not be a *species*, and would come down from the body in curved rather than straight lines.

Since the *species* is moved in a circular course, in order thereby to confer motion upon the planets, the body of the sun, or source, must move with it, not, of course, from space to space in the world – for I have said, with Copernicus, that the body of the sun remains in the centre of the world – but upon its centre or axis, both immobile, its parts moving from place to place, while the whole body remains in the same place.

Example from light.

In order that the force of the analogical argument may be that much more evident, I would like you to recall, reader, the demonstration in optics that vision occurs through the emanation of small sparks of light[2] toward the eye from the surfaces of the seen object. Now imagine that some orator in a great crowd of men, encircling him in an orb[3], turns his face, or his whole body along with it, once around. Those of the audience to whom he turns his eyes directly will also see his eyes, but those who stand behind him then lack the view of his eyes. But when he turns himself around, he turns his eyes around to everyone in the orb. Therefore, in a very short interval of time, all get a glimpse of his eyes. This they get by the arrival of a

[2] *Luculae.* Kepler uses this odd word in his *Astronomiae pars optica*, ch. 1 propositions 15 and 32 (KGW 2 pp. 23 and 35). He says that in total darkness coloured objects may still emit *luculae*, and that there probably is a *lucula* in the heart accompanying the heart's fire. Evidently, a *lucula* is a very small spark of light. It is not a 'ray', for a ray 'is nothing but the motion of light' (*ibid.*, prop. 8, KGW 2 p. 21.), nor is it a particle in the usual sense of the word, since it 'lacks corporeal matter, but consists of its own sort of matter' (*ibid.*, p. 19). The chief distinguishing feature of this matter is that it is 'a kind of surface' (*ibid.*, prop. 8 p. 21). So it might be most nearly correct to imagine a *lucula* as a two dimensional 'particle' or very small part of the lucid *species* of a body, whose motion (at right angles to its surface) constitutes a 'ray'.

[3] In Latin, as in English, the word *orbis* (orb) is ambiguous: it can denote either a circle or a sphere. Care has therefore been taken to preserve the ambiguity in the translation.

small spark of light or *species* of colour descending from the eyes of the orator to the eyes of the spectators. Thus by turning his eyes around in the small space in which his head is located, he carries around along with it the rays of the small spark of light in the very
174 large orb in which the eyes of the spectators all around are situated. For unless the small spark of light went around, his spectators would not be recipients of his eyes' glance. Here you see clearly that the immaterial *species* of light either is moved around or stands still depending upon whether that of which it is the *species* either moves or stands still[4].

The sun rotates.

Therefore, since the *species* of the source, or the power moving the planets, rotates about the centre of the world, I conclude with good reason, following this example, that that of which it is the *species*, the sun, also rotates.

However, the same thing is also shown by the following argument. Motion which is local and subjected to time cannot inhere in a bare immaterial *species*, since such a *species* is incapable of receiving an applied motion unless the received motion is non-temporal, just as the power is immaterial. Also, although it has been proved that this moving power rotates, it cannot be allowed to have infinite speed (for then it would seem that infinite speed would also have to be imposed upon the bodies), and therefore it completes its rotation in some period of time. Therefore, it cannot carry out this motion by itself, and it is as a consequence necessary that it is moved only because the body upon which it depends is moved.

By the same argument, it appears to be a correct conclusion that there does not exist within the boundaries of the solar body anything immaterial by whose rotation the *species* descending from that immaterial something also rotates. For again, local motion which takes time cannot correctly be attributed to anything immaterial. It therefore remains that the body of the sun itself rotates in the manner

Natural cause of the zodiac.

described above, indicating the poles of the zodiac by the poles of its rotation (by extension to the fixed stars of the line from the centre of the body through the poles), and indicating the ecliptic by the greatest circle of its body, thus furnishing a natural cause for these astronomical entities.

4 In the *Dialogue on the Two Chief World Systems*, Galileo has Salviati say the following: 'So the turning motion made by the fowling piece in following the flight of the bird with the sights, though slow, must be communicated to the ball also: . . .' (p. 172, Stillman Drake translation p. 178). Evidently, the notion of a sort of circular impetus or inertia had its attractions. However, Galileo's interlocutors reject the idea shortly after.

In the bodies of the planets is a material inclination to rest in every place where they are put by themselves.

Further, we see that the individual planets are not carried along with equal swiftness at every distance from the sun, nor is the speed of all of them at their various distances equal. For Saturn takes 30 years, Jupiter 12, Mars 23 months, earth 12, Venus eight and one half, and Mercury three. Nevertheless, it follows from what has been said that every orb of power emanating from the sun (in the space embraced by the lowest, Mercury, as well as that embraced by the highest, Saturn) is twisted around with a whirl equal to that which spins the solar body, with an equal period. (There is nothing absurd in this statement, for the emanating power is immaterial, and by its own nature would be capable of infinite speed if it were possible to impress a motion upon it from elsewhere, for then it could be impeded neither by weight, which it lacks, nor by the obstruction of the corporeal medium.) It is consequently clear that the planets are not so constituted as to emulate the swiftness of the motive power. For Saturn is less receptive than Jupiter, since its returns are slower, while the orb of power at the path of Saturn returns with the same swiftness as the orb of power at the path of Jupiter, and so on in order, all the way to Mercury, which, by example of the superior planets, doubtless moves more slowly than the power that pulls it. It is therefore necessary that the nature of the planetary globes is material, from an inherent property, arising from the origin of things, to be inclined to rest or to the privation of motion. When the tension between these things leads to a fight, the power is more overcome by that planet which is placed in a weaker power, and is moved more slowly by it, and is less overcome by a planet which is closer to the sun.

The motion of the planets is extrinsic.

175 This analogy shows that there is in all planets, even in the lowest, Mercury, a material force of disengaging itself somewhat from the orb of the sun's power.

The amount of time in which the rotation of the solar body traverses its space.

From this it is concluded that the rotation of the solar body anticipates considerably the periodic times of all the planets; therefore, it must rotate in its space at least once in a third of a year.

However, in my *Mysterium cosmographicum* I pointed out that there is about the same ratio between the semidiameters of the sun's body and the orb of Mercury as there is between the semidiameters of the body of the earth and the orb of the moon. Hence, you may plausibly conclude that the period of the orb of Mercury would have the same ratio to the period of the body of the sun as the period of the orb of the moon has to the period of the body of the earth. And the semidiameter of the orb of the moon is sixty times the semidiameter

of the body of the earth, while the period of the orb of the moon (or the month) is a little less than thirty times the period of the body of the earth (or day), and thus the ratio of the distances is double the ratio of the periodic times. Therefore, if the doubled ratio also holds for the sun and Mercury, since the diameter of the sun's body is about one sixtieth of the diameter of Mercury's orb, the time of rotation of the solar globe will be one thirtieth of 88 days, which is the period of Mercury's orb. Hence it is likely that the sun rotates in about three days.

Whether the earth's diurnal rotation comes from the rotation of the solar globe.

You may, on the other hand, prefer to prescribe the sun's diurnal period in such a way that the diurnal rotation of the earth is dispensed by the diurnal rotation of the sun, by some sort of magnetic force. I would certainly not object. Such a rapid rotation appears not to be alien to that body in which lies the first impulse for all motion.

The monthly motion of the moon arises from the diurnal rotation of the earth.

This opinion (on the rotation of the solar body as the cause of the motion for the other planets) is beautifully confirmed by the example of the earth and the moon. For the chief, monthly motion of the moon, by the force of the demonstrations used in chapters 32 and 33, takes its origin entirely from the earth (for what the sun is for the rest of the planets there, the earth is for the moon in this demonstration). Consider, therefore, how our earth occasions the motion of the moon: while this our earth, and its immaterial *species* along with it, rotates twenty nine and one half times about its axis, at the moon this *species* has the capability of driving it only once around in the same time, in (of course) the same direction in which the earth leads it.

Here, by the way, is a marvel: in any given time the centre of the moon traverses twice as long a line about the centre of the earth as any place on the surface of the earth beneath the great circle of the equator. For if equal spaces were measured out in equal times, the moon ought to return in sixty days, since the size of its orb is sixty times the size of the earth's globe.

This is surely because there is so much force in the immaterial *species* of the earth, while the lunar body is doubtless of great rarity and weak resistance. Thus, to remove your bewilderment, consider that on the principles we have supposed it would necessarily follow that if the moon's material force had no resistance to the motion impressed from outside by the earth, the moon would be carried at
176 exactly the same speed as the earth's immaterial *species*, that is, with the earth itself, and would complete its circuit in 24 hours, in which the earth also completes its circuit. For even if the tenuity of the

earth's *species* is great at the distance of 60 semidiameters, the ratio of one to nothing is still the same as the ratio of sixty to nothing. Hence the immaterial *species* of the earth would win out completely, if the moon did not resist.

What sort of body is the sun?

Here, one might inquire of me, what sort of body I consider the sun to be, from which this motive *species* descends. I would ask him to proceed under the guidance of a further analogy, and urge him to inspect more closely the example of the magnet brought up a little earlier, whose power resides in the entire body of the magnet when it grows in mass, or when by being divided it is diminished. So in the sun the moving power appears so much stronger that it seems likely that its body is of all [those in the world] the most dense.

Likeness of the sun's body to a magnet.

And the power of attracting iron is spread out in an orb from the magnet so that there exists a certain orb within which iron is attracted, but more strongly so as the iron comes nearer into the embrace of that orb. In exactly the same way the power moving the planets is propagated from the sun in an orb, and is weaker in the more remote parts of the orb.

The difference between the solar body and a magnet.

The magnet, however, does not attract with all its parts, but has filaments (so to speak) or straight fibres (seat of the motor power) extended throughout its length, so that if a little strip of iron is placed in a middle position between the heads of the magnet at the side, the magnet does not attract it, but only directs it parallel to its own fibres. Thus it is credible that there is in the sun no force whatever attracting the planets, as there is in the magnet, (for then they would approach the sun until they were quite joined with it), but only a directing force, and consequently that it has circular fibres all set up in the same direction, which are indicated by the zodiac circle.

The principle of motion in the sun and in the magnet is the same.

Therefore, as the sun forever turns itself, the motive force or the outflowing of the *species* from the sun's magnetic fibres, diffused through all the distances of the planets, also rotates in an orb, and does so in the same time as the sun, just as when a magnet is moved about, the magnetic power is also moved, and the iron along with it, following the magnetic force.

By example of the earth, it is proved that there are magnets in the heavens.

The example of the magnet I have hit upon is a very pretty one, and entirely suited to the subject; indeed, it is little short of being the very truth. So why should I speak of the magnet as if it were an example? For, by the demonstration of the Englishman William Gilbert[5], the

[5] William Gilbert. *De magnete magneticisqüe corporibus et de magno Magnete Tellure physiologia nova*. London 1600.

earth itself is a big magnet, and is said by the same author, a defender of Copernicus, to rotate once a day, just as I conjecture about the sun. And because of that rotation, and because it has magnetic fibres intersecting the line of its motion at right angles, those fibres lie in various circles about the poles of the earth parallel to its motion. I am therefore absolutely within my rights to state that the moon is carried along by the rotation of the earth and the motion of its magnetic power, only thirty times slower.

I know that the filaments of the earth, and its motion, indicate the equator, while the circuit of the moon is generally related to the zodiac – on this point there will be more in Chapter 37 and Part V.

177 With this one exception, everything fits: the earth is intimately related to the lunar period, just as the sun is to that of the other planets. And just as the planets are eccentric with respect to the sun, so is the moon with respect to the earth. So it is certain that the earth is looked upon by the moon's mover as its pole star (so to speak), just as the sun is looked upon by the movers belonging to the rest of the planets, for which see Chapter 38. It is therefore plausible, since the earth moves the moon through its *species* and is a magnetic body, while the sun moves the planets similarly through an emitted *species*, that the sun is likewise a magnetic body.

Likeness of the earth and the sun, with respect to the motion impressed upon the planets.

# 35

## Whether the motion from the sun, like its light, is subject to privation in the planets through occultations

This is a good time for me to take up the objections raised in chapter 33, where to the kinship of light and motive power were opposed, first, the mutual occultations of the celestial bodies, and then, the different [manner of] emanation of the *species* of the two.

And concerning the first, it is worthy of consideration whether, just as one opaque body intercepts another's sunlight, mobile bodies similarly impede one another in motion when they lie in line with the sun. If so, light would clearly be the vehicle or instrument of the motive power.

On the causes of latitudes.

It might appear that in order to avoid this as much as possible, God employed the relative inclinations of all the eccentrics, deviations from the ecliptic, and transpositions of the nodes, as well as the proportions of the bodies and the attenuation of shadows in a cone. And since it would not be possible completely to prevent the stars' occasionally lining up with the sun, it is tempting to suppose that the very slow motions of the apsides and nodes (which are, as it were, a kind of aberration of the epicycles from their periodic times) derive their origin thence.

And motions of the apsides.

But it is answered, first, that the analogy between light and motive power is not to be disturbed by rashly confusing their properties. Light is impeded by the opaque, but is not impeded by a body, because light is light, and does not act upon the body but on the surface (or as if on the surface). Power acts upon the body without respect to its opacity. Therefore, since it is not correlated with the opaque, it is likewise not impeded by the opaque.

On this account I would nearly separate light from moving power. unless I were to come upon examples in nature which would leave to the rays of light, even when impeded, a certain effect in those locations where their entry is prohibited. But I am not chiefly concerned here with the association of light with the motive power.

Instead, in order to dissolve this suspicion that the motions are impeded, let us take another example from the magnet. Its power is
178 not at all impeded by the interposition of matter (because, of course, it is immaterial), but passes through sheets of silver, copper, gold, glass, bone, wood, and attracts iron lying beyond these sheets exactly as if no sheets were there. Granted, it is impeded by the interposition of a magnetic plate. But the cause is ready at hand: the plate acts as a counterpart to the magnet. It therefore overcomes, by its strength, anything lying beyond it. And although it is impeded by the interposition of the iron plate, this too belongs to the magnetic nature, and it immediately drinks up the magnet's power and uses it as its own.

Therefore, in order that we may deny that the motion of the celestial bodies is impeded by central conjunctions of two of them, we must say that the nature of the sun differs from the nature of the rest of the celestial bodies more than the nature of the magnet differs from the nature of iron. Also, it must be denied that the planets drink up the power all at once from the sun in the same way as the iron drinks it from the magnet. The question of whether they drink up anything at all, I defer to Chapter 57.

On the motion of the apsides, once more.

In respect to the probable cause of the apogees' motion, it proves nothing concerning this common solar power being impeded by occultation. For the motion of the apogees could be quite different: for instance, it could have an animate origin. You will find an obscure opinion on this point below in Chapter 57.

Further, if the motion of the apogees arises because the motion of the planets around the sun is impeded by the occultation of the motive *species* emanating from the sun, the motion in longitude would be retarded, either by a progressive motion of latitude (by which the apogees would move back) or equally by retardation of the latitudinal motion. Thus, the apogees would stand still, although the observations testify that they move forward.

But the question whether, once the sun is established as the source of motion, the motions proper to the celestial bodies are impeded by occultation, will also be discussed in Chapter 57.

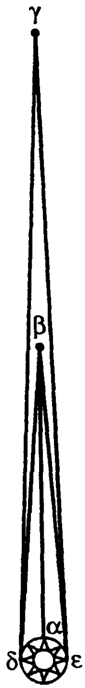
# 36

## By what measure the motive power from the sun is attenuated as it spreads through the world

There follows another, rather more difficult objection, arising from the second argument that was raised in Chapter 33 against the kinship of light and motive power, which seems in concept irreconcilably at odds with our immaterial *species*. This objection wearied me for a long time without offering any prospect of solution.

It was demonstrated in Chapter 32 that the intension and remission of the planets' motion is in simple proportion to the distances. It appears, however, that the power emanating from the sun should be intensified and remitted in the duplicate or triplicate ratio of the distances or lines of efflux. Therefore, the intension and remission of the planets' motion will not be a result of the attenuation of the power emanating from the sun. The logical consequence appears to be proved in the following manner, for light as well as for moving power;
179 however, discussions of light are clearer. The reader should read in 'motive power'. *Initially, let α be any point on the sun's body. It therefore spreads out its rays to every orb, and from a demonstration in optics, as the amplitude of the greater spherical surface γ, considered as an imaginary terminus for those rays, is to the amplitude of the smaller, β, so is the density of light at the smaller orb β to its density at the greater γ.*

*Next, let δε be any luminous great circle on the body of the sun. Thus its individual points, of which there are an infinity, spread out rays to the individual hemispheres β and γ in the same ratio. And the apparent magnitude of the diameter of the circle at the shorter distance (i. e., the angle δβε) is to the apparent magnitude at the greater distance (i. e., the*




*angle* $\delta\gamma\epsilon$*) as the longer distance* $\alpha\gamma$ *from such a circular line (which at a distance appears straight) is to the shorter* $\delta\beta$. *So, since this diameter appears longer from the nearby point* $\beta$ *than from the distant point* $\gamma$, *in the same ratio, while the radiation from any given point is denser at a nearby point* $\beta$ *than at a distant point* $\gamma$, *it is apparent that the density of radiation of the circle at the nearby point* $\beta$ *will have to the density of radiation at the distant point* $\gamma$ *the [inverse of the] duplicate ratio of* $\alpha\beta$ *to* $\alpha\gamma$ .

*Thirdly, let* $\delta\alpha\epsilon$ *be the apparent disc of the sun's body, and since similar surfaces (as are the apparent circular discs here) are in the duplicate ratio of their diameters, while the apparent diameters of the sun are in the simple inverse ratio of the distances* $\alpha\gamma$, $\alpha\beta$, *the circular discs will therefore appear in the duplicate ratio of the distances* $\alpha\gamma$, $\alpha\beta$. *But since the radiation of the circle* $\delta\epsilon$ *at* $\gamma$ *and* $\beta$ *was just proved to be in the duplicate ratio of the distances* $\alpha\beta$, $\alpha\gamma$, *while both are causes of the density [of radiation], it appears that the radiation of the disc, with respect to density or strength, is in the triplicate ratio of the distances* $\alpha\gamma$, $\alpha\beta$.

*For example, if the distances $\alpha\gamma$ to $\alpha\beta$ were as 2 to 1, the radiations of the point at $\alpha\gamma$ and $\alpha\beta$ would be as 1 to 2, with respect to the intensity of light, and the apparent diameters of circles would be 1 at $\gamma$ and 2 at $\beta$.*

*Therefore, the radiation of the diameter of the circle $\delta\epsilon$ would be 1 at $\gamma$ and 4 at $\beta$. But the discs are in the duplicate ratio of the diameters. Therefore the apparent magnitude of the disc at $\gamma$ would be 1, and at $\beta$, 4, as if one were to say that the disc $\delta\alpha\epsilon$ when seen from $\beta$ appears to contain four times as many points as when seen from $\gamma$. Any of these points when seen from $\beta$ illuminates twice as densely as when seen from $\gamma$. Therefore, when the ratios are compounded, the density of radiation of the whole disc $\delta\alpha\epsilon$ at $\gamma$ would have to the density of radiation of the whole disc $\delta\alpha\epsilon$ at $\beta$ the ratio 1 to 8.*

It should not trouble us here that we are reckoning from the sun's apparent disc, although it is a spherical surface. For the numerical ratio of each to the other is the same. Indeed, the spherical surface was demonstrated by Archimedes to be four times the area of the greatest circle described in the sphere. Therefore, most certainly, a body at $\gamma$, twice as far away as at $\beta$, appears to be eight times as obscurely illuminated at $\gamma$ than at $\beta$, not just twice. This, it appears, is responsible for the intensification of rays, by which bodies appear magnified when they approach, as [for example] Venus, when at the perigee of the epicycle, defines a more evident shadow for bodies than at apogee. Therefore, by the force of the analogy which we have instituted between light and motive force, the same should be thought to apply to the motive force.

**180** To this objection, I answer decisively, that in the initial supposition involving the point I made a false assumption. Even though I did indeed say something like that in the *Optics*, you should bear in mind that I was speaking of optics, whose points and lines are not divisible in a simple way. For, as concerns the point, since it has no magnitude, while the rays are amplified with the magnitudes of bodies, it follows that the radiation of a point in itself is nothing, there being no greater or lesser density where there is no radiation. Thus the initial assumption of the ratio of distances $\alpha\beta$ to $\alpha\gamma$ collapses.

We should say instead that any point shines more strongly or weakly to the extent that that point designates to us a greater or lesser quantity.

In the second supposition, concerning the circle, and the third, concerning the disc, there are two false assumptions. The first is that a mathematical circle, lacking breadth, is supposed to shine. For it can

no more shine itself than can the point from whose motion the circle is supposed to be generated. You are no closer to having a surface when you posit a line of three furlongs than you are in positing a line of three feet.

Second, it is supposed that the optical magnification of the diameter or of the disc adds to the strength of the rays, although this is but a deception of the visual faculty, and belongs to the genus of rational thought, of which there is no efficient cause. The physical identity of the circle $\delta\epsilon$, the surface $\delta\alpha\epsilon$ (when light is in question) and the body $\delta\epsilon$ (when power is in question) remains the same whether it is viewed from $\gamma$ or from $\beta$, and will always act the same and have the same effect, spreading the same amount of light in the more diffuse orb $\gamma$ as in the more compact orb $\beta$. Nothing is lost in the journey: the entire *species* carries through to any distance, however remote. It is attenuated only in the extensions of the spheres, so that in the individual points of the spheres, such as $\gamma$ and $\beta$, it is rarer in the former, and denser in the latter, in the inverse ratio of the distances $\alpha\beta$ to $\alpha\gamma$. This is the sole cause of the weakening, not the diminishing of the source $\delta\epsilon$, which in fact does not happen, being but an optical illusion.

Indeed, if it is permitted to argue from Euclid's *Optics* here, less light arrives at the nearer, $\beta$, than at the more remote, for the reason that at $\beta$ a smaller circle bounds the visible hemisphere of the luminous body $\delta\epsilon$ than at $\gamma$. Therefore, not so many particles of the sun $\delta\epsilon$ can be seen from $\beta$ as from $\gamma$. But this is quite imperceptible, hard to express [even] in enormous numbers.

And I, after giving this answer to myself, am having a good laugh at my pathetic worrying over so much hot air.

But the objection can rebound in the opposite direction, thus. If there is the same amount of light spread out in a large sphere as there is gathered together in a small sphere, there will not be the same amount of power in both places, for power is not considered orbicularly in a sphere as light is, but in the circle in which the planet proceeds. Thus, the magnetic filaments of the sun were supposed, above, to be set up only in longitude, not towards the poles or in other directions.

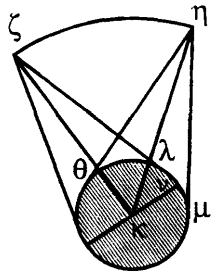
The answer is that the cause of light and motive power is exactly the same, and there is a deception in the reasoning. For in light, the rays
181 do not flow out from the individual points and circles of the body to only the corresponding points and circles of the sphere. Thus, the rays from $\gamma$ do not come only from $\alpha$ (by which arrangement it would

be impossible to ascribe a density to light in the spheres, since it would have no quantity in its origin, arising as it does from a point). Instead, the rays flow out from the whole hemisphere of the lucid body to the individual points of the imaginary spherical surface: thus, the rays from both δ and ϵ flow out to γ. The same thing takes place likewise with power. Even though the magnetic filaments of the solar body are ordered according to zodiacal longitude, and even though there is but a single great circle of the sun's body beneath the zodiac or ecliptic, and roughly beneath the orbit of the planet, and (finally) even though the other smaller circles (which are compacted to the size of points beneath the poles) are subordinated to their corresponding circles in the sphere of the planet, nevertheless, the rays from all the filaments of the solar body (arising from one hemisphere of the body) flow down and converge on the individual points of the path of any of the planets as well as on those poles which are directed toward the poles of the sun's body, and the body of the planet is transported according to the measure of the density of this entire *species* compounded of all the filaments.

Why the planets always stay close to the zodiac.

It does not, however, follow that just as the sun shines equally in all directions, the planets, too, as you might fear, are moved indiscriminately in all directions. For the magnetic filaments of the sun, considered in themselves, do not move, but only inasmuch as the sun, rotating very rapidly in its space, carries the filaments around, and with them the moving *species* spreading abroad with them. Therefore the planet will not go backwards, because the sun always rotates forwards. The planet will not go to the poles (even though there might be some of the *species* from the sun's body at those points) because the filaments of the solar body are not extended in the direction of the poles, nor does the sun rotate in that direction, preferring the direction in which its filaments urge it.

On these suppositions, the tendency of the planets to be carried towards the poles is so small that there is the single region of the zodiac, the mean between the poles, through which all planets of necessity would move in longitude without deflection, if they were to cease their own proper motions (of which see Chapter 38 below). *For the* species *of the solar hemisphere which takes up a post at some point of the zodiac, such as the point ζ in the present diagram, is the total of the filaments of the semicircle all acting as one on that point, as, from [the region from] θ through κ, from λ through μ, etc. But when you move off towards the poles of the world, as at η, then by [the action of]*




*both the other pole – that of the sun* ν *– and of the entire circles of filaments* λμ *which encircle the pole* ν, *appropriated within the purview of* ημ, *the* species *will be compounded from filaments tending in contrary directions, for the opposite parts* λ *and* μ *of the circles move in opposite directions. Therefore that* species θημ *descending near the poles is less well adapted to the motion carrying the planets along.*

182 # 37

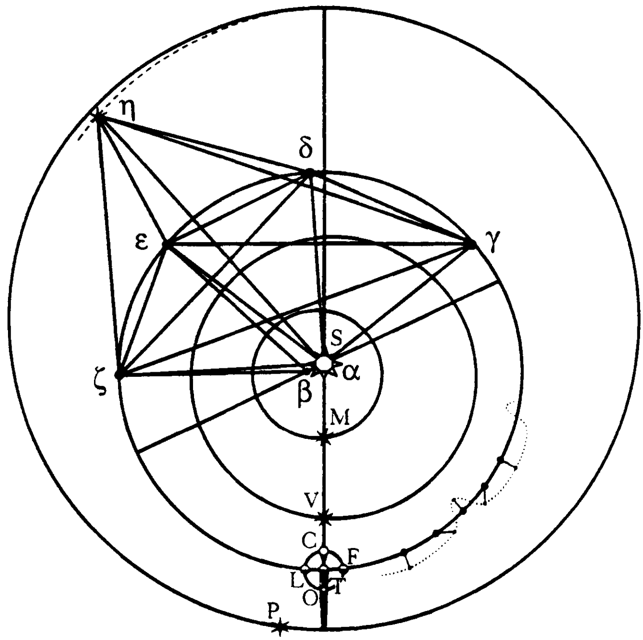
## How the power moving the moon functions

Since in Chapter 34, in passing, I made mention of the motion of the moon, it would be appropriate to treat the matter in greater detail, so that no reservation introduced by the moon might deflect the reader into withholding his ready assent from me on the entire treatise; so that it might instead be marvellously confirmed, by an eminently trustworthy consideration of the lunar motion; and finally, to ensure that the physical aspects of astronomy are treated fully in this book. For even though there are a few things in the theory of the moon which must be deferred, as they are to be treated differently or explained in more detail, they nevertheless have their origin here.

After long and painstaking observations of the moon in every position in relation to the sun, Tycho Brahe expressed the opinion that in the moon, besides the anomaly of the epicycle, and besides that monthly anomaly which was also known to Ptolemy, the mean motion itself (defining it in relation to these two inequalities) is not yet quite 'mean'. That is, it intensifies at the conjunctions and oppositions with the sun, and is remitted at the quadratures. Thus, even if it were undisturbed by epicycles, the moon itself, even though moving on a circle concentric with the earth, would move around nonuniformly[1].

183 *Let* S *be the body of the sun,* M *the orb of Mercury,* V *of Venus,* T *of the earth,* P *of Mars, and so on. And let all of them always move from*

[1] What is being described here is the 'variation', discovered by Brahe. The monthly inequality mentioned earlier, already known to Ptolemy, is the 'evection'. (Note from KGW 3 p. 469.)




*right to left when in the upper part of the diagram. Now let* CLOF *be the orb of the moon,* O *the moon at opposition,* C *at conjunction,* L, F *at quadratures, and let* CLOF *remain for now a concentric described about the earth at* T, *and let it move in the direction* OFCL. *The question is, by what cause is the moon made to move more swiftly about* T *at* C, O *than at* F, L, *since we have just now mentally removed the eccentricity and the epicycles. Here, I know, the reader will expect me to say that it is swifter at* O *because at that place its motion is in the same direction as the motion of all the planets. But this is not the true cause. For in that case, what would happen at* C *is what in fact happens, that the moon, in its compounded motion, is slowest, since its proper motion* FCL *resists considerably this common rightward motion*[2]. *For it should be noted that the moon, from point* C *on its orb, is borne less in the leftward direction* L *than the earth is borne to the right on its orb. Therefore, the moon, by a motion compounded of its proper motion and that which it has in common with the earth, also always moves to the left when above, as when the earth is at* δ, *but here,*

[2] Here Kepler clearly has gotten 'left' and 'right' confused in the text, which error is accordingly corrected in the translation.

*when the earth is down low at* T, *it is carried to the right, but slowly at* C *and swiftly at* O. *A motion of this sort is expressed approximately by the spiral lines set out here.*

*But perhaps you are expecting me to say instead that this phenomenon arises from the sun's motive power being weaker at* O *and stronger at* C*? Much less will I say this. For I would thus make it slow at* O *and* C, *and fast at* F, L, *which is contrary to what is desired. For if it is propelled weakly at* O, *it therefore is slow, and if it is held back more strongly at* C, *so that its tendency to move in the opposite direction from* C *to* L *is less, it will therefore again move slowly from* C *to* L. Thus it was not correct to free the moon from the earth and commit it to the sun. This would cause it at length to wander away from the earth, just as the apogees wander from their places. It would be preferable to attribute to the earth a force that retains the moon, like a sort of chain, which would be there even if the moon did not circle the earth at all. On this supposition, the moon is, as it were, in the same boat with the earth, in the same power of the sun, and now, as if freed from this motion from the sun, it is rotated separately by the earth.

Therefore, I consider the cause of the swiftness at O and C to be none other than this, that the earth T derives its power of moving the moon from the sun S, and preserves it by the continuity of the line TS. Thus SCTO can properly be called the 'diameter of power' since these two (T and S) are the sources of all motion.

On this supposition, that monthly inequality known to Ptolemy also follows. *For if the power coming from the same source* T *is stronger at* C, O *than at* F, L, *then if the apogee is near* C, O, *there is a greater loss of speed than if the apogee were at* F, L. *Thus when going from the apogee at* O *or* C *the equations at* F, L *are greater than when going from the apogee at* F, L *to the conjunctions and oppositions* O, C.

You see, then, that these physical speculations are so ordered that they can even account for the phenomena of the moon. The moon is not driven primarily by the sun in its circling of the earth, but by a power lying hidden in the earth itself, and casting forth its immaterial *species* to the body of the moon, but more strongly along the line that connects the centres of the sun (the primary source) and the earth.

It is, however, hard to explain more clearly how that diameter comes to be more powerful. For neither the sun's power nor the
184 earth's, going out to the moon, is any swifter when the moon falls on

that diameter. That the rotations of these bodies (and therefore also of the *species*) are uniform and forever constant, is a highly reasonable presumption. There remains only the possibility described here, that the power is indeed not swifter, but more robust, when it emanates from the earth in the directions nearer to the line ST, for the reason that it was originally drawn off from the sun to the earth along that line.

Nevertheless, it is the sun which, either immediately or through that faculty which governs the earth's annual motion, is the principal director of the motion which the earth confers upon the moon. This is demonstrated chiefly by the moon's making its circuit beneath the zodiac, like the annual circuit of the centre of the earth, although the earth's diurnal motion, which confers the monthly motion upon the moon, takes place beneath the equator.

# 38

## Besides the common motive force of the sun, the planets are endowed with an inherent force [vis insita], and the motion of each of them is compounded of the two causes

I have spoken of the origin of the motion which rotates the planets around the sun or the moon around the earth; that is, I have spoken of the natural causes of the circle which in the theories of the planets is called either eccentric or concentric, according to the various intentions of the authors. Now, something must be said of the natural cause of the eccentricity, or in the particular hypothesis of Copernicus, of the epicycle on a concentric. For the moving power from the sun has hitherto been uniform, having different degrees only at various amplitudes of the circles. Its innate quality is such that if a planet were to remain at the same distance from the sun it would be carried around perfectly uniformly, and no intension or remission of the solar motion would be perceived. The modicum of nonuniformity perceived in the working of this power arises from the planet's being transposed from one distance from the sun to another, so that it encounters one or another degree of strength of this power from the sun. The question therefore arises, if, as Brahe demonstrated, there are no solid orbs, how does the planet come to ascend from and descend towards the sun? Can this, too, come from the sun? My answer is that it is to some extent from the sun, and to some extent not from the sun.

First argument, from the eccentricity.

Examples of natural things, and the kinship of celestial things for these terrestrial ones which has hitherto been gathered to exist, cry out that in a simple body the operations which are more general are simpler, while the variables, if any (such as, in the motion of the

planets, the varying distance from the sun, or the eccentricity), arise from the concurrence of extrinsic causes.

185 Thus, in a river, the simple property of water is to descend towards the centre of the earth. But because its path is not direct, it flows in to those places where it finds a lower bed, stagnates where it meets with level ground, and is carried along with a roar where it comes upon steeper slopes; and there is a whirlpool where it dashes headlong into projecting rocks. Where water itself, by its inherent force, endeavours to do nothing but descend towards the centre of the earth, a simple task for a simple property, flow and stagnation and waves and whirlpools and all the variety [of phenomena] arise from the causes described, which are extraneous and accidental.

Particularly happy and better accommodated to our inquiry are the phenomena exhibited by the propulsion of boats. Imagine a cable or rope hanging high up across a river, suspended from both banks, and a pulley running along the rope, holding, by another rope, a skiff[1] floating in the river. If the ferryman in the skiff, otherwise at rest, fastens his rudder or oar in the right manner, the skiff, carried crosswise by the simple force of the downward-moving river, is transported from one bank to the other, as the pulley runs along the cable above. On broader rivers they make the skiffs go in circles, send them hither and thither, and play a thousand tricks, without touching the bottom or the banks, but by the use of the oar alone, directing the unified and most simple flow of the river to their own ends.

In very much the same manner, the power moving out into the world through the *species* is a kind of rapid torrent, which sweeps along all the planets, as well as, perhaps, the entire aethereal air, from west to east. It is not itself suited to attracting bodies to the sun or driving them further from it, which would be an infinitely troublesome task. It is therefore necessary that the planets themselves, rather like the skiffs, have their own motive powers, as if they had riders [*vectores*] or ferrymen, by whose forethought they accomplish not only the approach to the sun and recession from the sun, but also (and this should be called the second argument) the declinations of latitudes; and as if from one bank to the other, travel across this river (which itself only follows the course of the ecliptic) from north to south and back.

Second argument, from latitudes.

[1] It may be significant that '*cymba*' (here translated as 'skiff') usually denotes specifically the boat in which Charon ferried the dead across the river Styx. Further, '*portitor*', here translated 'ferryman', is primarily a name for Charon. The verb used to describe the attachment of the oar or rudder is *religare*, cognate with the English word 'religion'.

It is certain from what has been said that the power which comes from the sun is simple. But now, the eccentrics of the planets do not just decline from the ecliptic, but go in various directions, intersecting one another and the ecliptic. Therefore, other causes are conjoined with the motive power from the sun.

# 39

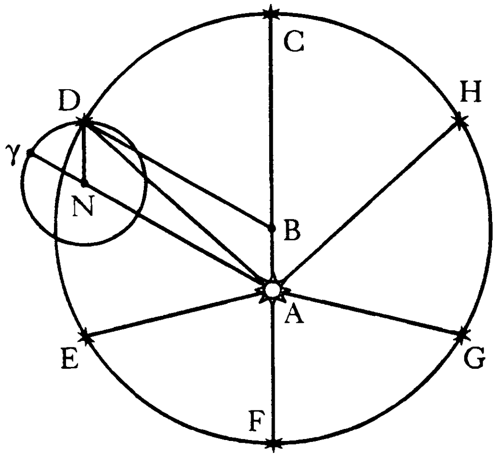
## By what path and by what means do the powers seated in the planets need to move them in order to produce a planetary orbit through the aethereal air that is circular, as it is commonly thought to be

186 And so, in what has been demonstrated, let us take these axioms, which are of great certainty. First, that the body of a planet is inclined by nature to rest in every place where it is put by itself. Second, that it is transported from one longitudinal position to another by that power which originates in the sun. Third, if the distance of the planet from the sun were not altered, a circular path would result from this motion. Fourth, supposing the same planet to be in turn at two distances from the sun, remaining there for one whole circuit, the periodic times will be in the duplicate ratio of the distances or magnitudes of the circle. Fifth, the bare and solitary power residing in the body of a planet itself is not sufficient for transporting its body from place to place, since it lacks feet, wings, and feathers by which it might press upon the aethereal air. And nevertheless, sixth, the approach and recession of a planet to and from the sun arises from that power which is proper to the planet. All these axioms are agreeable to nature in themselves, and have been demonstrated previously.

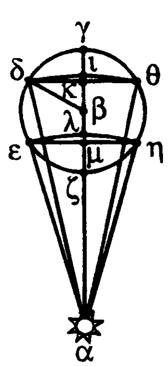
Axioms for theories of the celestial motions.

Now let us work with geometrical figures in order to see what laws will be required to represent any desired planetary orbit. Let the orbit of the planet be a circle, as has been believed until now, and let it be eccentric with respect to the sun, the source of power. *Let that eccentric* CD *be described about centre* B *with radius* BC, *and on it let* BC *be the line of apsides,* A *the sun, and* BA *the eccentricity. Let the eccentric be divided into any number of equal parts, beginning from the line of apsides at* C, *and let the ends of these parts be connected with*

I. What the planet shall do by the motion of its own body when its composite path is made to be a perfect circle. That is, how shall it vary its distances from the sun?




A. *Therefore,* CA, DA, EA, FA, GA, HA, *will be the distances of the end points of the equal parts from the source of power. Now with centre* β, *radius* βγ, *equal to* AB, *let the epicycle* γδ *be described, divided into as many equal parts as the eccentric, beginning from* γ, *and let the line* γβ *be extended so as to make* βα *equal to* BC, *and let the point* α *be connected with the end points of the equal parts of the epicycle, by the lines* γα, δα, εα, ζα, ηα, θα. *These lines will be respectively equal to the distances drawn to the eccentric from* A, *this having been demonstrated above in chapter 2. Next, with centre* α *and radius* δα, *let the arc*



*$\delta\iota\theta$ be described, intersecting the diameter $\gamma\zeta$ at $\iota$, and about the same centre $\alpha$, with radius $\alpha\epsilon$, let the arc $\epsilon\lambda\eta$ be described, intersecting the diameter $\gamma\zeta$ at $\lambda$, and let the end points of the parts equidistant from the aphelion of the epicycle $\gamma$ be connected by the lines $\delta\theta$, $\epsilon\eta$, which will intersect the diameter at $\kappa$, $\mu$, so that $\alpha\delta$ or $\alpha\iota$ is longer than $\alpha\kappa$, and $\alpha\epsilon$ or $\alpha\lambda$ longer than $\alpha\mu$.*

First way: the planet itself moves on an epicycle. Chapter 49 depends primarily upon this.

If it were possible for the planet to move on a perfect epicycle by its inherent force[1], and for its orbit at the same time to be a perfect circle, then we would have to consider similar arcs to be swept out in the same times, on the eccentric and on the epicycle. Consequently, it would now immediately become clear by what means, and by what
187 measure, the distance $\alpha\iota$ would be made equal to AD. For since $\alpha\iota$ and $\alpha\theta$ are equal, when the planet moves from $\gamma$ to $\theta$ the distance $\alpha\theta$ necessarily and without any special contrivance comes out right, equal to AD.

Absurdities of this account.

1. But in addition to the apparent conflict with the fifth axiom created by anyone who claims that the planet moves locally from $\gamma$ to $\theta$ by its inherent force, many additional absurdities are also involved.

*For let* AN *be drawn parallel to* BD, *and let* AN *be equal to* BD, *and about centre* N *let an epicycle be described which shall go through* D. *Now since* CD *is a perfect circle, the same angles are swept out by the planet* D *about the centre of the eccentric* B *as by the centre of the epicycle* N *about the centre of the sun* A *(through the equivalence demonstrated in chapter 2), as long as the diameter of the epicycle* ND *with the planet at* D *always remains parallel to* AB *with respect to its position in the world. Therefore, the speed of the centre of the epicycle* N *about the sun* A *would be made the same as the speed of the planet* D *about the centre of the eccentric* B, *so that those motions would be intensified at the same time and remitted at the same time. And since intensification and remission depends upon greater or less distance of*
2. *the body of the planet from the sun, therefore the centre of the epicycle, remaining at the same distance, is made to move more slowly or more swiftly on account of the planet's being farther from or nearer to the sun.*

*And although the power driving the planets is faster than any of*
3. *them, as is shown in chapter 34, we are here to suppose in our imagination a single ray of power* AN *coming from the sun, as if there were a line upon which the centre of the epicycle* N *would always*

[1] *Vis insita.*

4. *remain. And this line, with that same centre* N, *would be now swifter, now slower, again contrary to what was said above, that the power*
5. *always produces the same speed at the same distance. Moreover, we are required to suppose that the planet has its own rotatory motion away from this imaginary ray* AN *in the opposite direction, traversing unequal distances in equal times, according as this ray itself is swifter or slower.* In adopting this account, we would indeed approach more closely the geometrical suppositions of the ancients, but we would stray very far from physical theory, as is shown in ch. 2. Neither have my thoughts on the matter sufficed to discover a way in which these things can happen naturally.

This last is avoided below in ch. 49, though the other absurdities remain.

A second way in which the planet might produce the eccentric.

Now all this would be conceived more simply were we to consider the diameter of the epicycle ND as always remaining parallel to itself. For then the planet would carry out its motion according to the imaginary device, not of the epicycle, but of the centre of the eccentric B, keeping itself always at the same distance from that centre.

Absurdities

1. But at the beginning of this work, in chapter 2, it was said to be most absurd that a planet (even if you give it a mind) may imagine for itself a centre, and from it a distance, where there is no particular body in that centre for the planet to be aware of.

Now you might say that the planet observes the sun A, and already knows beforehand, through memory, the order of distances from the sun upon which a perfect eccentric would be contingent. But first, this is more indirect, and depends upon something intermediate connecting the effect of the perfectly circular path with the indication of the waxing and waning of the sun's diameter, and this too in some mind. But that intermediate can be nothing but the position of the centre of the eccentric B at a certain distance from the sun, which, as has just now been said, cannot be known by an unassisted mind.

**188** I do not deny that a centre can be conceived, and about it a circle. But I do say this, that if the centre is established by thought alone, without respect to time and without any external indication, it is impossible to set up about it, in reality, the perfectly circular path of some movable body.

2. Besides, if the planet were to derive its correct distances from the sun, according to the rule of the circle, from memory, it could also derive from memory the equal arcs of the eccentric which are to be traversed in unequal times, and which are to be traversed by an extrinsic force originating in the sun, as if it were obtaining the values

right from the Prutenic or Alphonsine tables. Thus it would know from memory beforehand what this extrinsic and mindless power originating in the sun was going to do. All of these are absurd: –

3. Chiefly since, as Aristotle affirms, there is no knowledge of the infinite, while the infinite is involved in this intension and remission.

But it is well established below in chapter 44 that even the observations themselves will not allow CD to be a perfect circle, so these theoretical arguments, weak though they are thought to be, do not stand alone, and are that much the less vulnerable to scornful rejection.

A third way: that the planet, by its inherent force, reciprocates on the diameter of the epicycle.

It is consequently more fitting that the planet itself require no assistance, whether of epicycle or eccentric, but rather that the task which it either performs by itself or has a part in performing is a reciprocating path along the diameter $\gamma\zeta$ directed towards the sun $\alpha$.

The question now is, what is the measure by which the planet metes out the correct distance for any given time?

The measure of the reciprocation is not derived by the planet from a real epicycle:

Now to us the measure is clear from geometry and the diagram. *For whenever the solar power moves the planet forward to the line* DA, *we then find the angle* CBD *and make* $\gamma\beta\delta$ *equal to it; and thus we say that* $\alpha\delta$, *or* $\alpha\iota$ *which is its equal, is the correct distance from* A *to the planet at* D. But this measure which we have proposed for humans, we have just now denied the planet when we removed it from the circumference of the epicycle and restricted it to the straits of the diameter $\gamma\zeta$.

Nor from arcs traversed on the eccentric:

Indeed, in this enquiry it is easier to say what is not the case than to say what is. *This is because at the moments when the sun has placed the planet on the lines drawn from* A *through* C, D, E, F, G, H, *the planet itself is presumed to have arrived at the distances* $\gamma\alpha$, $\iota\alpha$, $\lambda\alpha$, $\zeta\alpha$, $\lambda\alpha$, $\iota\alpha$, *respectively*. Consequently, if the path of the planet is a perfect circle, then to equal parts of the eccentric CD, DE, EF, correspond unequal descents of the planet along the diameter, namely, $\gamma\iota$, $\iota\lambda$, $\lambda\zeta$. Moreover, the order is perturbed: the highest is not the smallest, nor the lowest the greatest, but the middle parts $\iota\lambda$ are greatest, and the
189 extremes $\gamma\iota$ and $\lambda\zeta$ are smaller, and the highest $\gamma\iota$ are a little smaller than the corresponding lowest, $\lambda\zeta$. For $\gamma\kappa$ and $\mu\zeta$ are equal, and $\gamma\iota$ is smaller than $\gamma\kappa$, while $\lambda\zeta$ is greater than $\mu\zeta$.

Nor from the elapsed time; nor from the angle at the sun, or equated anomaly:

Furthermore, this same cause prevents $\gamma\iota$, $\iota\lambda$, $\lambda\zeta$, being made proportional to either the times of the equal arcs traversed CD, DE, EF, or to the angles at the sun CAD, DAE, EAF. For the time or sojourn of the planet on equal parts of the eccentric CD, DE, EF, is continuously diminished from the highest to the lowest points, and

the angles at the sun are continuously increased, but the reciprocations $\gamma\iota$ are increased in the middle regions, such as $\iota\lambda$.

Therefore, if the path of the planet is a perfect circle, the measure of the planet's descent on the diameter $\gamma\zeta$ is neither time, nor distance traversed on the eccentric, nor the angle at the sun.

And physical theories, too, also decisively repudiate these measures.

Nor from an imaginary epicycle or eccentric.

What, then, if we should say this: although the motion of the planet is not physically on an epicycle, this reciprocation is measured out in such a way that distances from the sun are arrived at which are similar to those which exist in an epicycle actually traversed?

First, this would be to attribute to the power belonging to the planet a knowledge of the imaginary epicycle and of its effects in setting out distances from the sun; and further, it would attribute knowledge of the future speed or slowness which the common motion from the sun is going to cause. For it is necessary to suppose here an imaginary intension and remission of motion on the imaginary epicycle that is the same as that of the motion on the real eccentric. This is more incredible than the previous accounts, where the motion of the body was combined with knowledge of the epicycle or eccentric. Therefore, the objections raised there should be understood as applying here as well, the judgements being nearly identical.

In chapter 57 below, the measure of this reciprocation will be made plain.

Nevertheless, for want of a better opinion, we must at present put up with this one. And as for its involving many absurdities, a certain physicist will allow, in chapter 57 below, that on the testimony of the observations the path of the planet is not a circle.

II.
By what means or measure may a planet grasp its distance from the sun?

So far, the discussion has concerned the measure relating to the form of this reciprocation. It now remains for us to find the measure of this measure; that is, the quantity of local motion. For it is not enough for the planet to know how far it should be from the sun: it also has to know what to do in order to be at the correct distance.

To what extent is a perception of the size of the solar body to be attributed to the planets?

Now anyone who is so attracted to the supposition of a perfectly circular orbit as to associate a mind with the planet which could preside over the reciprocation, can say only this: that this planetary mind observes the increasing and decreasing size of the solar diameter, and understands, using this as an indication, what distances from the sun it should arrive at at any given time. For example, sailors cannot know from the sea itself how far they have travelled over the waters, since the course, viewed in that way, has no distinct limits. Instead, they find this either from the amount of time they have

sailed, if wind and sea remain constant and the ship does not stop, or from the direction of the wind and the changing elevation of the pole, or from all or several of these in conjunction, or, should it please the gods, by a contrivance of a number of wheels, with paddles lowered into the water (for certain conceited mechanics are proposing an
190 instrument of this sort, who ascribe the calm of the continents to the water of the Ocean). In just the same way, the mind of the planet cannot by itself measure its position, or the distance between itself and the sun, since between them there is pure aethereal air, devoid of any means of indication. So it must either make use of the elapsed time, in conjunction with a supposed invariance of forces (which has just been denied above), or of a physical machine, which is ridiculous (for by the example of the sun and moon we suppose the celestial bodies to be round, and it is therefore also probable that the entire field of the aetheral air moves around with the planets[2]), or finally, of some suitable means of indication that varies with the distance of the planet from the sun. And other than the single indication of the sun's apparent diameter, nothing else presents itself. *Thus we humans know that the sun is 229 of its own semidiameters distant from us when its diameter subtends 30′, and 222 semidiameters when it subtends 31′*[3].

Thus the planets become surveyors, measuring their distances from the sun from a single reference point, through the apparent size of the solar body.

If it were indeed certain that this motion of the planet along the diameter of the epicycle could not be carried out by any material and corporeal or magnetic power of the planet, nor by an unassisted animal power, but that it is governed by a planetary mind, nothing absurd would be stated[4]. For that the sun is observed by the planets in other respects as well, the motions in latitude bear witness. For by these motions the planets would depart from the middle and royal road of this solar power, as from the mainstream of a river, and move to the sides, as is said in chapter 38, unless they meanwhile paid attention to the sun, approaching and receding along a line drawn to its centre. It would then describe circles which, seen from the earth or the centre of the world, would appear smaller, parallel to some great

There exists in the planets something like a mind, which pays attention to the body of the sun.

[2] That is, it would be of no use to know the relative velocity of a planet through the aethereal air.

[3] Substituting 'semidiameters' for 'diameters' where the sense requires.

[4] This was the standard view of the day, included in all the introductory university textbooks in natural philosophy. The planetary mind was usually identified with the biblical angels, thus bringing Aristotle into harmony with scripture. However, some theorists believed that the mind is united with the planet as soul is with body. This view was widely regarded as heretical, as it suggested that the Prime Mover is the soul of the world (*anima mundi*), and thus that the universe is somehow God's body.

circle. But all planets describe great circles which intersect the ecliptic at points which are opposite with respect to the sun, as was demonstrated for Mars from observations above in chapters 12, 13, and 14. Therefore, the diameter of reciprocation $\gamma\zeta$ is also directed towards the sun, and the latitudes are related to the sun in every respect. This remains true despite my ascribing the latitudinal motions partly to mind and partly to nature and magnetic faculties, in Part V below.

See the marginal note to ch. 63.

Objections which can be raised against perception of the solar body.

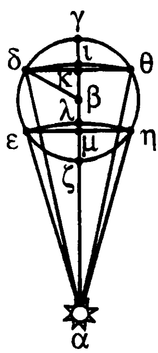
1. Small angle subtended.

Now one cannot say in reply to me that the solar diameter and its variation is far too small to be used as a standard. For it is certain that there is no planet for which it entirely vanishes. Since on earth it is thirty minutes, on Mars it will be twenty, on Jupiter seven, and on Saturn three, while on Venus it will be forty, and on Mercury at least eighty and sometimes as high as one hundred and twenty. The query should not concern the smallness of the body, but rather the unapt coarseness of human perception, which cannot be stretched to sense such small things.

One should on the contrary note that this body, however small, is, nonetheless capable of moving such distant bodies in a circle, as is demonstrated in the preceding chapters. The illumination of the world by such a tiny corpuscle is known to all. And so it is credible that if the movers are endowed with some faculty of observing its diameter, this faculty is as much more acute than our eyes as its work, and the perpetual motion, is more constant than our own troubled and confused schemes.

2. Defect in the means of perception.

191 So then, Kepler, would you give each of the planets a pair of eyes? By no means, nor is this necessary, no more than that they need feet or wings in order to move. But Brahe has recently eliminated solid



orbs. Now our theorizing has not emptied nature's treasure house: we still cannot establish, through our own knowledge, how many senses there are to be. There are even examples at hand worthy of our admiration. For tell us, in physical terms: with what eyes shall the animal faculties of sublunar bodies look upon the positions of the stars in the zodiac, so that when a harmonic arrangement (which we call 'aspects') is found among them, the bodies leap up and display it in their actions? Was it with her eyes that my mother noted the positions of the stars in order to know that she was born with Saturn, Jupiter, Mars, Venus, and Mercury, all in sextiles and trines? And could it have been by the same means that her children, and especially I her first born, came to see the light chiefly on those days when as many as possible of the same aspects, especially of Saturn and Jupiter, recurred, or when they possessed as many pristine positions as possible, with squares, oppositions, and conjunctions? I have observed those things in all cases whatever that have occurred to this very day. But what is to be made[5] of these examples, just as absurd as the previous ones? The answer will have to await those who work harder in their study of nature than is usual today.

So the hypothetical person who says that the planet's path is a perfect circle will say this: that the planet performs its reciprocation so as to make the diameter of the sun, at the end points of equal arcs of the eccentric, appear very nearly* inversely proportional to the lines $\delta\alpha$, $\epsilon\alpha$, $\zeta\alpha$, or to $\iota\alpha$, $\lambda\alpha$, $\zeta\alpha$, which are equal to them, taken with respect to the longest line $\gamma\alpha$; and that through this consideration of the diameter of the sun at the chosen moments of time, come the distances of $\iota$, $\lambda$, $\zeta$ from $\gamma$·

*For in ch. 57 the ratio will be somewhat different.

It should be known, however, that the increases of the diameter of the sun and the arcs of the epicycle do not square with each other well, and so the motive mind will have to have a very good memory in order to adjust the unequal versed sines of the arcs on the epicycle to the equal increases of the solar diameter. For this, see ch. 56 and 57 below.

III. By what animal faculty the mind might obtain the means of determining the distance of its body from the sun.

That shall be enough concerning the means of indicating distances. There remains a third topic to which I wish briefly to draw attention: the nature of the animal faculty which carries the planet about. Anyone who says that the body of a planet is moved by an inherent force is just plain wrong. This we proved at the beginning. But it is

[5] Reading '*ergo*' for '*ego*'.

likewise impossible simply to ascribe this force to the sun instead. For the same force that attracts the planet also repels it in turn, and this is inconsistent with the simplicity of the solar body. But anyone who by some unique argument reduces this motion to the bodies of the sun and the planet in concerted action gives a new cast to the material of this entire chapter, and is consequently referred to this particular topic in chapter 57 below.

You see, my thoughtful and intelligent reader, that the opinion of a perfect eccentric circle for the path of a planet drags many incredible things into physical theories. This is not, indeed, because it makes the solar diameter an indicator for the planetary mind, for this opinion will perhaps turn out to be closest to the truth, but because it ascribes incredible faculties to the mover, both mental and animal.

192 Although our theories are not yet complete and perfect, they are nearly so, and in particular are suitable for the motion of the sun, so we shall pass on to quantitative consideration. And when at last we approach a more exact discovery of the truth, reserved for chapter 57, it will be seen that we laid the groundwork for it here.

# 40

## An imperfect method for computing the equations from the physical hypothesis, which nonetheless suffices for the theory of the sun or earth

Such a long-winded discussion was necessary to prepare a way for a natural form for the equations, on which I am going to be very busy in Part IV. Now we must return to the equations of the sun's eccentric in particular, which is the main subject of this third part, and for the sake of which the general discussion of the last eight chapters has been presented.

My first error was to suppose that the path of the planet is a perfect circle, a supposition that was all the more noxious a thief of time the more it was endowed with the authority of all philosophers, and the more convenient it was for metaphysics in particular. Accordingly, let the path of the planet be a perfect eccentric, for in the theory of the sun the amount by which it differs from the oval path is imperceptible. Those things that are going to be needed for the other planets, on account of this deviation, follow below in ch. 59 and 60.

Through elongations of the planet from the centre of the sun, to find the physical part of the equation.

Since, therefore, the times of a planet over equal parts of the eccentric are to one another as the distances of those parts, and since the individual points of the entire semicircle of the eccentric are all at different distances, it was no easy task I set myself when I sought to find how the sums of the individual distances may be obtained. For unless we can find the sum of all of them (and they are infinite in number) we cannot say how much time has elapsed for any one of them. Thus the equation will not be known. For the whole sum of the distances is to the whole periodic time as any partial sum of the distances is to its corresponding time.

I consequently began by dividing the eccentric into 360 parts, as if

these were least particles, and supposed that within one such part the distance does not change. I then found the distances at the beginnings of the parts or degrees by the method of chapter 29, and added them all up. Next, I assigned an artificial round number to the periodic time; although it is in fact 365 days and 6 hours, I set it equal to 360 degrees, or a full circle, which for the astronomers is the mean anomaly. As a result, I have so arranged it that as the sum of the distances is to the sum of the time, so is any given distance to its time. Finally, I added the times over the individual degrees and compared

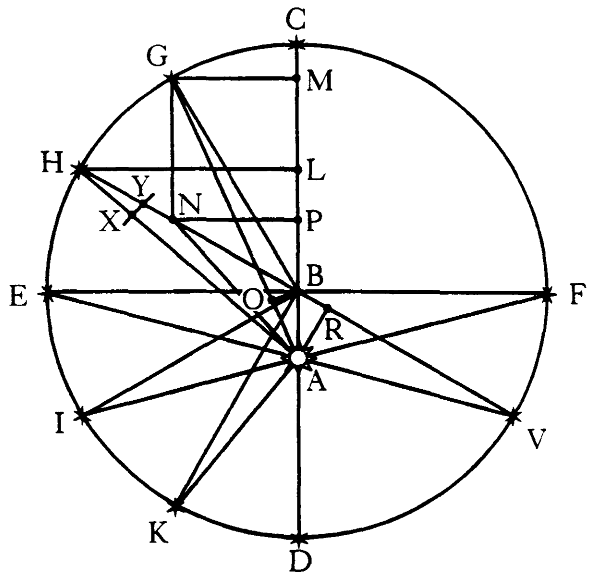
Term: What is the mean anomaly?

193 these times, or degrees of mean anomaly, with the degrees of the eccentric anomaly, or the number of parts whose distance was sought. This furnished the physical equation, to which the optical equation, found by the method of chapter 29 with those same distances, was to be added in order to have the whole.

Through the areas to find the physical part of the equation.

However, since this procedure is mechanical and tedious, and since it is impossible to compute the equation given the ratio for one individual degree without the others, I looked around for other means. And since I knew that the points of the eccentric are infinite, and their distances are infinite, it struck me that all these distances are contained in the plane of the eccentric. For I had remembered that Archimedes, in seeking the ratio of the circumference to the diameter, once thus divided a circle into an infinity of triangles – this being the hidden force of his *reductio ad absurdum*. Accordingly, instead of dividing the circumference, as before, I now cut the plane of the eccentric into 360 parts by lines drawn from the point whence the eccentricity is reckoned.

*Let* AB *be the line of apsides,* A *the sun (or earth, for Ptolemy);* B *the centre of the eccentric* CD, *whose semicircle* CD *shall be divided into any number of equal parts* CG, GH, HE, EI, IK, KD, *and let the points* A *and* B *be connected with the points of division. Therefore,* AC *will be the greatest distance,* AD *the least, and the others, in order, are* AG, AH, AE, AI, AK. *And since triangles under equal altitudes are as their bases, and the sectors, or triangles,* CBG, GBH, *and so on (standing upon least parts of the circumference and therefore not differing from straight lines) all have the same altitude, the equal sides* BC, BG, BH, *they are therefore all equal. But all the triangles are contained in the area* CDE, *and all the arcs or bases are contained in the circumference* CED. *Therefore, by composition, as the area* CDE *is to the arc* CED *so is the area* CBG *to the arc* CG, *and alternately, as arc* CED *is to* CG, CH, *and the rest in order, so is the area* CDE *to the*




*areas* CBG, CBH, *and the rest in order. Therefore, no error is introduced if the areas be taken for the arcs in this way, and substituting the areas* CGB, CHB *for the angles of eccentric anomaly* CBG, CBH.

*Further, just as the straight lines from* B *to the infinite parts of the circumference are all contained in the area of the semicircle* CDE, *and the straight lines from* B *to the infinite parts of the arc* CH *are all contained in the area* CBH, *therefore also the straight lines from* A *to the same infinite parts of the circumference or arc make up the same thing. And finally, since those drawn from* A *and* B *both fill up one and the same semicircle* CDE, *while those from* A *are the very distances whose sum is sought, it therefore seemed to me I could conclude that by computing the area* CAH *or* CAE *I would have the sum of the infinite distances in* CH *or* CE, *not because the infinite can be traversed, but because I thought that the measure of the faculty by which the collected distances mete out the times is contained in this area, so that we would be able to obtain it by knowing the area without an enumeration of least parts.*

194 *Therefore, from the above, as the area* CDE *is to half the periodic time, which we have proclaimed to be 180°, so are the areas* CAG, CAH *to the elapsed times on* CG *and* CH. *Thus the area* CGA

*becomes a measure of the time or mean anomaly corresponding to the arc of the eccentric* CG, *since the mean anomaly measures the time.*

*Earlier, however, the part* CGB *of this area* CAG *was the measure of the eccentric anomaly, whose optical equation is the angle* BGA. *Therefore, the remaining area, that of the triangle* BGA, *is the excess (for this place) of the mean anomaly over the eccentric anomaly, and the angle* BGA *of that triangle is the excess of the eccentric anomaly* CBG *over the equated anomaly* CAG. Thus the knowledge of this one triangle provides both parts of the equation corresponding to the equated anomaly GAC.

The reason why, in the third method in ch. 31 above, part of the equation was simply doubled to give the whole equation.

And hence it is manifest why in chapters 30 and 31 above I said that in the theory of the sun the parts of the equation are very nearly equal. *That is, any given arc, and the angle at the centre which it subtends (as* CG *and* CBG *in the figure), are measured by its area, which is called the 'sector', as area* CBG. *Therefore, with one arm of the compass at* G, *with radius* GB, *let an arc of the circumference be described intersecting* GA *at* O. *Hence, as the area* GBC *is to the angle* GBC, *so is the area* BGO *to the angle* BGO. *But the angle* BGO *is the optical part of the equation. Thus the area* GOB, *through the doubling of the part of the equation, measures the optical part of the equation, for in our account presented earlier the whole area* GBA *was to be consulted for the physical part of the equation.*

*Now clearly the genuine measure of the physical part of the equation* AGB *exceeds the proposed measure of the optical part* OGB *by the small space or area* OAB *(while near the perigee the latter in turn exceeds the former by a small area). Nevertheless, if the eccentricity is small, as is that of the sun or earth, with which we are concerned in this third part, this is not perceptible. For the nearer it approaches to the line of apsides, the narrower becomes the whole triangle* AGB *and consequently also its little part* AOB, *however much its altitude* AO *increases at the same time. On the other hand, in the middle longitudes, the angle* BEA *with its sector is at a certain point directly measured by the area* BEA, *and the excess begins to turn into a defect.*

Therefore, the greatest difference that can occur is that accumulated at the octants, or locations intermediate between the apsides and the quadrants. How great that difference is, will now be shown.

Since for some time now I have used the same form of computation by means of areas in the theory of Mars, I could not ignore this difference on account of the planet's great eccentricity, nor did the doubling of the optical equation avoid all perceptible error. It was

therefore necessary to investigate the area of the triangle of the equation [*triangulum aequatorium*]. This can be done in various ways, but I shall go on to state the easiest.

**195** Easy way of investigating the area of the triangle of the equation.

It is well known that triangles with equal altitudes are proportional to their bases. I say also, that triangles on equal bases are proportional to their altitudes.

*Let* AGB, AHB *stand upon the same base* AB *extended to* C. *From* G *let the line* GN *be drawn parallel to the common base* AB *and intersecting* HB *at* N, *and let* N *be connected with* A. *From the three vertices* G, H, N, *of the triangles let* GM, HL, MP *be drawn perpendicular to the base, determining the altitudes of the triangles. Therefore, since* GN *and* MP *are parallel, and* GM, NP *are perpendicular [to them],* GM *and* NP *will be equal. But* GM *is the altitude of triangle* AGB, *and* NP *is the altitude of triangle* ANB. *Therefore triangles* ANB *and* AGB *have equal altitudes, and since they are both on the same base* AB, *they are equal. And since* ANB *is part of* AHB, *and the base line* HB *is common, together with the vertex* A, *the triangles* NAB *and* HAB *have equal altitudes. Therefore, as the base* NB *is to* BH, *so is* NAB *to* HAB. *But* NAB *and* GAB *were proved equal. Therefore, as* NB *is to* BH, *so is* GAB *to* HAB. *But as* BN *is to* BH, *so is* NP *to* HL, *because* NBP *and* HBL *are similar triangles. Therefore also, as* NP *is to* HL, *so is* GAB *to* HAB. *But* NP *and* GM *are equal. Therefore, as* GM *is to* HL, *altitude to altitude, so is area* GAB *to area* HAB. *Q. E. D.*

Value of the triangle at 90° of eccentric anomaly.

Adrian Romanus

*Now let* BE *be perpendicular to* CD, *and let the triangle* BEA *have a right angle at* B. BE *will be the altitude, and* BA *the base. Therefore, by Euclid I. 42, 900, or half the base* BA *(which for the sun is 1800), multiplied by the altitude* BE, *100,000, which is the radius of the circle, gives the area of the triangle* BEA, *that is, 90,000,000. But the area of a circle of radius 100,000 (from the most recent investigations of Adrian Romanus, a most expert geometer*[1]*) is 31,415,926,536, with no error in even the last digit. And as this the area of the circle is to the 360° of mean anomaly or time (that is, 21,600′ or 1,296,000″), so is the area of the triangle, 90,000,000, to 3713″; that is, 1° 1′ 53″. So the area* BEA *has a value of 1° 1′ 53″. But in chapters 29 and 30, the angle* BEA *was*

[1] Adrian Romanus, or Romain, or van Roomen (1561–1615), Flemish physician, ecclesiastic, and mathematician, author of numerous mathematical and astronomical works. The computation of $\pi$ appears in his *Idea mathematicae pars prima* . . ., (Louvain 1593): he divided the circle into $2^{30}$ triangles to attain an effective accuracy of fifteen decimal places.

*also 1° 1′ 53″*. Therefore, both parts of the equation are equal at this place, that is, near 90°.

At other degrees of eccentric anomaly, we proceed as follows. *Since* BEA *is 3713″, as its altitude* EB *is to* HL *or* GM, *the altitudes of the other triangles – that is, as the whole sine is to the sines of the eccentric anomaly* HBC, GBC, *– so is 3713″ to the areas of the remaining triangles. So 3713″ will be multiplied by the sines of the angles at* B, *and, striking out the last five digits, the remainder will be the physical part of the equation expressed in seconds, corresponding to the angle at* B. *For example, let* HBC *be 45° 43′ 46″, as it was above in ch. 31. Therefore, its sine, 71605, multiplied by 3713″, with the last five digits struck, gives 2659″, that is, 44′ 19″. In the table above, we assumed this to be equal to the optical part of the equation, 43′ 46″.*

Therefore, this little area ABO at its greatest does not exceed 33″.

And this is that fourth procedure for computing the eccentric equations, of which I began to speak above near the end of chapter 34, which closely expresses the very nature of things and the foregoing theories of chapters 32 and 33.

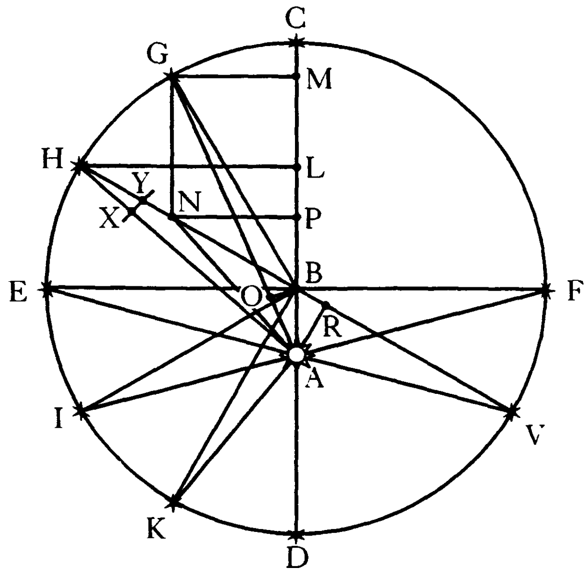
**196** Defect of this operation with the area of the triangle, arising from the supposition of a circular orbit.

Nevertheless, my argument contains a paralogism, not, indeed, of great moment. It arises from this: that while Archimedes did indeed divide the circle into an infinity of triangles, they stood upon the circumference at right angles, so that their vertices were at the centre of the circle B. But one cannot proceed in the same way with triangles standing upon the circumference with their vertices at A, because the circumference is intersected obliquely by the straight lines from A in all places other than C and D.

You could have found this error empirically, as I myself did, by taking all the distances AC, AG, AH, at the individual whole degrees of the angle CBG, GBH, and adding them all up. (These distances, though they are presented in the table in chapter 30, correspond in position to individual whole degrees of the angle at A, and consequently to angles at B cut minutewise. Nevertheless, one could easily find, by interpolation, the distance from A corresponding to any angle of an integral number of degrees about B.) Now the sum comes out to be greater than 36,000,000, although 360 of the distances from B add up to exactly 36,000,000. But, on the contrary, if both sums were measured by the same area of the circle, the sums ought to have been equal.

By 'angles cut minutewise' I mean angles expressed in degrees and some odd minutes.

A demonstration of the error, on the other hand, is as follows. *Through* B *let any straight line other than* CD *be drawn, intersecting*




*the circumference, and let it be* EF*; and let the points of intersection* E *and* F *be connected with* A. *Now since the point* A *does not lie on the line* EF, EAF *is a figure, a triangle. Therefore,* EA, AF *together are longer than* EF, *by Euclid I. 22. But the area of the circle contains the sum of all lines* EF, *and therefore it contains a sum which is less than all the lines* EA, AF, *since any two opposite points on the eccentric, together with* A, *determine such a triangle, with the exception of* C, D, A, *where instead of a triangle there is a straight line.*

I would add, in passing, that it is also proved, in the same way, that the distances from A corresponding to all of the 360 integral degrees of the angle at A (which are in the table in chapter 30 above), added up in one sum, are less than 36,000,000. *For through the point* A *let any straight line other than* CD *be drawn (let it be* EV*), and let* E *and* V *be connected with* B. *In the triangle* EBV, *the straight lines* EB, BV *together will be longer than* EA, AV, *two opposite distances. But all 360 of* EB, BV *taken together are 36,000,000. Therefore, all 360 of* EA, AV *taken together are less than 36,000,000.*

To return to what I was saying, this method of finding the equations is not only very easy indeed, and based upon the natural causes of the motion explained above, but also agrees most precisely with the

Given an elliptical orbit, this method is not in error: therefore, bear it in mind.

observations in the theory of the sun or earth. Nevertheless, it errs in two respects. First, it supposes that the orbit of the planet is a perfect circle, which, as will be demonstrated below in ch. 44, is not true. Second, it uses a plane which does not exactly measure the distances
197 of all points from the sun. Nevertheless, as if by a miracle, each of these exactly cancels the effect of the other, as is demonstrated below in chapter 59[2].

The problem of finding the quadrature of a space between conchoids is proposed to geometers.

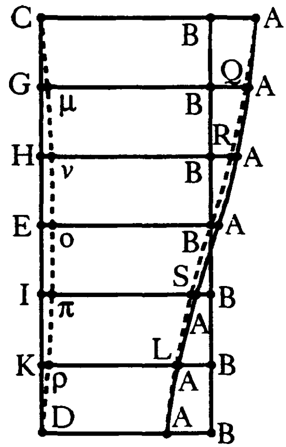
And because at the present time there are first rate geometers who on occasion labour endlessly on matters whose usefulness is not so evident, I call upon one and all to help me here to find some plane figure equal to the sum of the distances. I have indeed myself found it geometrically – in a broad sense – but let them show me how to express numerically what I have delineated geometrically; that is, let them show how to square the figure I have found. *Let the circumference* CED *be unfolded into a straight line and divided into as many parts as before at the points* G, H, E, I, K, *and let perpendiculars be set up at the points of division equal to the radius* CB, *and let the parallelogram be completed. This will be double the Archimedean triangle, by which he measured the area of the semicircle. Now if you were to make individual parallelograms in this manner in the individual sectors, then the whole parallelogram divided into parts will be equivalent to the whole area of the semicircle; that is, the ratio 2:1 holds everywhere.*

*By 'conchoid' I mean, not that of Nicostratus, which is infinite and is so named by him because it is like a conch, but one which is like the conchoid of Nicostratus, just as by 'rhomboid' we mean something that is like a rhombus.

*Now, in the same manner, let the distances* CA, GA, *and so on, be set up, and the points* A *be connected by the conchoid** A, A, A, A, *drawn through the individual points (which are potentially infinite): the figure* AACD *will be equivalent to all the distances from* A. *For similarly the individual lines* AG, AH *have approximately made up the one parallelogram, except that the conchoid is not parallel to* CD, *but inclined to the radii* GA, HA, EA, *exactly as the distances are inclined to the circumference in the circle itself. Hence, there is nothing wrong with the conchoid's being made longer than the semicircle* CD.

*But* EA *is longer than* EB, *so that, if* CA, GQ, HR, EB, IS, KL, DA, *be taken, equal to lines determined by perpendiculars drawn from*

2 Readers who hope to find a clear explanation in ch. 59 will be disappointed: propositions 13–15 of that chapter, where he raises this question again, are no more than qualitative gropings towards a solution. A clear solution required separation of the radial and circumsolar components of the planet's motion, and it was not until the *Epitome* of 1621 that this was done. For a good treatment of this perplexing matter, see Eric Aiton, 'Infinitesimals and the Area Law', in F. Kraft, K. Meyer, and B. Sticker, eds., *Internationales Kepler-Symposium, Weil der Stadt, 1971* (Hildesheim, 1973), pp. 285–305.




A *to the lines between the points and* B *(as, in the circular diagram, the perpendicular* AR *is dropped to* HB *extended, defining* HR *which is shorter than* HA*), the figure between the conchoid* AQRBSLA *and* CD *would be quite equal to the figure* CBBD. *For the conchoid would intersect* BB *at the line* EA, *and because* BA *at the top and bottom are equal, and* BQ *is equal to* LB, BR *to* SB, *and so on, the figures* BBRQA *and* BBALS *would therefore be congruent. One of these is added to, and the other subtracted from, the equal figures* CBBE *and* EBBD, *and therefore, the whole figure between* AQRBSLA *and* CD *is equal to that between* BB *and* CD. *Hence, the small area between the two conchoids* AQRBSLA *and* AAAAAA *is the measure of the excesses of the distances from* A *over the distances from* B – *and the standard of measure is the same as that by which the parallelogram is set equal to all the distances from* B.

The area between the conchoids is of unequal breadth at places equally removed from the centre.

It should also be noted that this area is not of the same breadth at places equally removed from the line EA, but wider below. *For in the circular diagram let* HBR *be extended to* V, *so that* AH, AV *correspond respectively to the upper angle* HBE *and the lower angle* FBV, *which are equal and are equally removed from the middle points* E *and* F. *And about centre* A *with radius* AV *let the arc* XY *be drawn through* AH *and* BH. *Now if you connect* A *and* Y, AYR *will be exactly*
198 *congruent to the triangle* AVR, *for* AV *and* AY *and* AX *are equal, by construction, and are the longer sides, while* VR, RY *are equal and are*

*the shorter. But from the point* H *outside the circumference* XY *two lines are drawn:* HX *through the centre* A, *and* HY *not through the centre. Therefore,* HY *is longer than* HX, *and therefore the greater* AV *or* AX *is increased by the shorter* XH, *and the lesser* VR *or* RY *is increased by the longer* YH – *and nevertheless, the whole* RH *remains shorter than the whole* AH. *Therefore, the difference between* RH *and* AH *is less than the difference between* RY *and* AX, *that is, between* VR *and* VA. *And consequently in the conchoid* SA *is greater, and* RA *less, although* IE, EH *are equal.* Therefore, the area between the two conchoids is not bisected by EA. However, it appears to be bisected by BB, which some geometer should investigate, who should at the same time show how to square the area between the conchoids, so that it may be expressible in numbers. In chapter 43 below you will find a rough estimate of this area.

Granted, these general considerations of the computation of the physical equation are not yet well enough supported by the geometrical apparatus. Nevertheless, I wished to give them a preliminary treatment here, so that when all the planetary inequalities are determined (as, in particular, we presupposed that the course of the sun or earth is a perfect eccentric, which will be denied concerning Mars below in ch. 44 and 53), this operation will not be so sharply divorced from its basis in physical theory. For, touching the theory of the sun, with which we have been concerned to this point, we introduce no discrepancy either by misjudging the area of the conchoid, which we have taken to be less than it really is, or the assumption of a perfect eccentric, in which we appear to be erring in excess – to what extent cannot yet be said, since all has not been presented. But the very things which have been rejected in this chapter as paralogisms will be taken up again below, when we shall have come to a perfectly correct way of expressing the equations, when the thing that gave rise to the paralogism will have been eliminated from that hypothesis.

I have described within a hair's breadth, through most certain observations and proofs, the cause and measure of the second inequality, which makes the planets appear stationary, direct, and retrograde. It has been shown that this second inequality itself shares something in common with the first inequality, and that the theory of the sun or earth (for Copernicus) or of the epicycle (for Ptolemy) is like the theory of the other planets. Also, the physical causes of this inequality have been found, and have been adapted to the calculation

in the theory of the sun. Therefore, it is fitting that I now bring to a close this third part, as a morning task followed by lunch, with the master of the *Metamorphoses* adding his voice to mine:

**Part remains of what has been begun, part of the work is finished:**
**The anchor is cast; here let the craft lie.**[3]

[3] Ovid, *Ars amatoria*, I. 771 *et seq*. (Citation from KGW 3 p. 470.)

## PART IV

# Investigation of the true measure of the first inequality

# From physical causes and the author's own ideas

200 Those things demonstrated in the third part pertain to all the planets, whence they can be called, not unjustly, the 'key to a deeper astronomy'. We should rejoice all the more at this discovery as it becomes clearer that there could have been no way of investigating them other than through observations of the star Mars. Ptolemy did take notice of this bisection of the sun's eccentricity in Venus as well as Mercury, and for that reason introduced the eccentrics on the eccentrics, or, what is the same thing, the gyrations of the centre of the epicycle: this demonstration is reserved for the treatments of the individual planets. Nevertheless, the circumstances of the observations themselves, and Venus's small departures from the sun which allow it only to be observed when low down at night, would have introduced the greatest of impediments to a methodical investigation of this subject were it based upon anything other than Mars. With Mercury, the attempt was even more absurd because it very rarely emerges from the sun's rays, while it is farther from earth than Mars and Venus when they are observed at their nearest. The truth therefore would have come to us, as with Ptolemy, through a search of the broadest plains, and, as it were, by feeling about with our hands in the deepest shadows.

But now, using the star Mars as an example, we shall state how much of the first inequality, which is occasioned by the eccentric, and which is different for each planet, we owe to this common second inequality, which has been found in Part III.

201

# 41

## A tentative examination of the apsides and eccentricity, and of the ratio of the orbs, using the observations recently employed, made at locations other than opposition with the sun, with, however, a false assumption

In the second part, above, I tried to find the aphelion and eccentricity, as well as the distances of the star Mars from the sun on the entire circle, using acronychal observations in imitation of the ancients. And indeed, the eccentric equations corresponded closely to other observations made elsewhere than at opposition with the sun. However, the eccentricity and the distances from the sun were repudiated by the annual parallax of longitude and latitude. Therefore, in order that the distances of the star from the centre of the sun could be found throughout the circumference of the eccentric, the second inequality (epicyclic, for Ptolemy, or belonging to the annual orb for Tycho and Copernicus) had to be explored in Part III. But now, if the planet's path were a perfect circle, the planet's first inequality, which exists by reason of the eccentric, could be investigated immediately. For in chapter 25 above we presented a method with which, given the distances of three points of the circumference from some point within the circumference, and the angles at that point, one could find the position and size of the circle with respect to that point, the centre and eccentricity, along with the apsides.

In chapter 26, the distance of Mars from the centre of the sun was found to be 147,750 in 14° 21′ 7″ Taurus, at the node, on 1595 October 25. Again, in chapter 27, the distance of Mars was found to be somewhat less than 163,100 at 5° 25′ 20″ Libra, and that was on 1590 December 31. And because Mars was 41 degrees from the node, multiplying the sine of 41° by the sine of the greatest inclination, found in ch. 13, yields an inclination at that point of 1° 12′ 40″. The

secant of this exceeds the radius by 22 parts in one hundred thousand, which, in our dimensions, is 34 units. Therefore, the corrected distance at this place would be somewhat less than 163,134. Let it remain 163,100. But the secant of this inclination divided into the secant of 41° gives the secant of an arc 50″ longer. Therefore, 50″ must be subtracted from the position of Mars, so as to make it 5° 24′ 30″ Libra.

Thirdly, in chapter 28 the distance of Mars was found to be 166,180 at 8° 19′ 20″ Virgo, on 1590 October 31, 68 degrees from the node. So the inclination at that place is 1° 42′ 40″. The secant of this is 45 units larger [than the radius], or 75 in our dimensions. Therefore, the corrected distance is 166,255. The subtraction from Mars's position, to reduce it to the ecliptic, is 16″.

These three positions, referred to the same year, 1590, and the month of October, through corrections for the precession of the equinoxes, are:

| | | | | |
|---|---|---|---|---|
| 147,750 | 14° | 16′ | 52″ | Taurus |
| 163,100 | 5 | 24 | 21 | Libra |
| 166,255 | 8 | 19 | 4 | Virgo[1] |

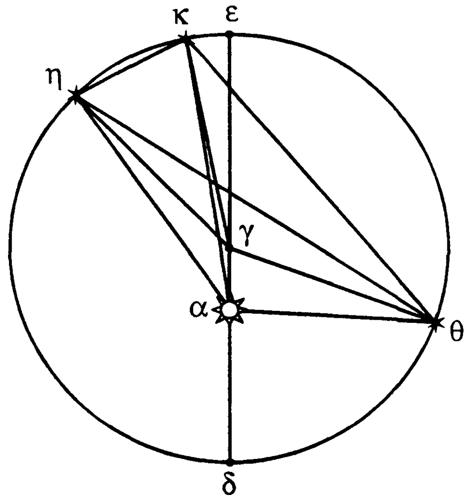
202 It appears that the aphelion is nearer to the eighth degree of Virgo than to the others, because its distance is longer. *So, following [the pattern of] the demonstration in chapter 25, let* $\alpha$ *be the centre of the solar body. From it let* $\alpha\theta$, $\alpha\eta$, $\alpha\kappa$ *be drawn in such a proportion as to produce the above numerical distances, and let all the points be joined. And let the angle* $\kappa\alpha\theta$ *be 114° 2′ 12″, which is the angle from 14° Taurus to 8° Virgo. Similarly, let* $\kappa\alpha\eta$ *be 27° 5′ 17″, which is the angle from 8° Virgo to 5° Libra. And* $\eta\alpha\theta$ *is the sum of the two. For the sun is assumed to be the centre of the zodiac.*

We need now to investigate the circle which passes through $\eta\kappa\theta$, so that $\eta$, $\kappa$, $\theta$ may be three positions of the planet.

In the Ptolemaic form, $\alpha$ will be the earth, the centre of the zodiac, and $\eta$, $\kappa$, $\theta$, three positions of the point of attachment of the epicycle. everything else stays the same.

*So, in triangle* $\eta\alpha\theta$, *the angle with its sides being given, the angle* $\alpha\theta\eta$ *is found to be 20° 26′ 13″. Likewise, in* $\kappa\alpha\theta$, *angle* $\alpha\theta\kappa$ *is given as 35° 10′ 17″. Subtracting* $\alpha\theta\eta$ *from this leaves* $\eta\theta\kappa$, *14° 44′ 4″. Let* $\gamma$ *be the*

[1] There a several minor errors in the computations leading to these figures; none of them, however, effects an error of more than 20″ in the results.




*centre of the circle in question. Let αγ be drawn, and extended to the aphelion ϵ and perihelion δ; and let η and κ be joined to γ.*

*Now, since ηθκ stands on the circumference, and ηγκ on the centre, subtending the same arc ηκ, ηγκ will therefore be twice the angle ηθκ, or 29° 28′ 8″. And where ηγ is 100,000, κη will be 50,868, which is double the sine of half ηγκ.*

*Now, in triangle ηακ, the angle with its sides again being given, κηα is found to be 78° 44′ 1″, and through this, κη [is found to be] 77,187 where ηα is 163,100. Therefore, in the units of which κη formerly was 50,868 and ηγ was 100,000, ηα becomes 107,486. And since ηγκ is 29° 28′ 8″, κηγ will be half the supplement, because ηγ, κγ are equal. Therefore, κηγ is 75° 15′ 56″. Subtract κηα from this. The remainder is γηα·*

*Thus, in triangle γηα, the angle with its sides is given. Therefore, ηαγ is known to be 38° 15′ 45″. And consequently (since αη is in 5° 24′ 21″ Libra) the line of apsides αγ will be in 27° 8′ 36″ Leo. But through the angle ηαγ the eccentricity αγ, 9768, is also found, in units of which ηγ is 100,000. Finally, in the units of which αη is 163,100, ηγ will be 151,740. But in the same units, the semidiameter of the annual orb was also 100,000. Therefore, the ratio of the orbs is that which 100,000 has to 151,740.*

All this is completely erroneous, as you can tell by substituting other distances, belonging to other eccentric positions, for one or

more of the distances used (αθ, αη, ακ). Every time you try this, you will find, by an equally certain and irrefutable argument, that all those numbers come out differently.

And in the following chapter, it will be found that the most certain ratio is approximately 100,000 to 152,640, eccentricity 9264, where the radius is 100,000. The aphelion for 1590 October 31 was found in ch. 16 above to be at 28′ 53″ Leo, which will be confirmed, in the next chapter, to be within 11′ of the truth.

203

# 42

## Through several observations at places other than the acronychal position, with Mars near aphelion, and again several others with Mars near perihelion, to find the exact location of the aphelion, the correction of the mean motion, the true eccentricity, and the ratio of the orbs

You have just seen, reader, that we have to start anew. For you can perceive that three eccentric positions of Mars and the same number of distances from the sun, when the law of the circle is applied to them, reject the aphelion found above (with little uncertainty). This is the source of our suspicion that the planet's path is not a circle. On this supposition, one could not use three distances to learn the others. Therefore, the distance at any particular place has to be deduced from its own observations, and especially those at aphelion and perihelion, through the comparison of which we will learn the true eccentricity.

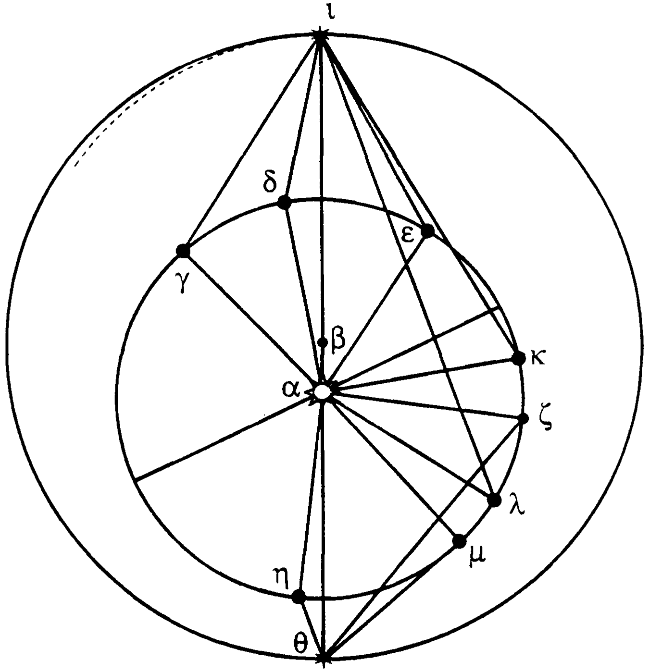
*Let α be the centre of the world, αβ the line of apsides, and ιθ the eccentric upon centre β, with ι the aphelion and θ the perihelion. From chapter 41, or better, from chapter 16, we understand that the observations in which Mars is nearest to ι are these.*

I. On 1585 February 17 at $10^h$, the planet was seen at 15° $12\frac{1}{2}'$ Leo, with latitude 4° 16′ North.

II. On 1586 December 27 at $4^h$ in the morning, at 29° $42\frac{2}{3}'$ Virgo, lat. 2° $46\frac{3}{5}'$ N.

And on 1587 January 1 at $7^h$ $8^m$ in the morning, at 1° 4′ 36″ Libra, Lat. 2° 54′ N. And on January 9 in the morning, at 2° $51\frac{1}{2}'$ Libra, lat. 3° 6′ N.

On 1588 November 10, at $6^h$ $30^m$ in the morning there was 31° 27′ between Mars and Cor Leonis. Mars's declination was 3° $16\frac{1}{4}'$ North.




204 Therefore, Mars was at 25° 31′ Virgo, latitude 1° 36′ 45″ N.[1] On December 5 at $6^h$ in the morning there were 45° 17′ between Mars and Cor Leonis. The declination was 2° 5′ south. Therefore Mars was at 9° 19⅖′ Libra, latitude 1° 53½′ north. These observations were not, however, confirmed by fixed stars on the other side of Mars.

On 1590 October 6, at $4^h\,45^m$ in the morning, Mars was observed at an altitude of 12½ degrees, [and distances taken] from the Tail of Leo[2] and the Heart of Hydra[3], with its declination. But since neither of the fixed stars was lined up directly with Mars in longitude,

[1] It has often been remarked that Kepler's computational errors have a way of cancelling each other or otherwise vanishing, and this is a good example. The longitude given here is 9′ greater than the data show it to be. However, the longitude for December 5 is 14′ less than that required by the data, and since the position given in the table below is the result of an interpolation between the two, the longitude given there is only about 3½′ off. From there onward, the nature of the computations tends to reduce the error even further, so that in the computation of the position of the line αι it is less than one minute. For the record, κι should be at 2° 39′ 10″ Libra, angle ακι should be 68° 16′, and αι as computed from this position should be at 29° 21′ 35″ Leo.

[2] Denebola. However, Kepler probably meant 'Cor Leonis': he surely could not have been referring to Denebola, whose longitude *was* the same as Mars's, and which was brought in later for that very reason, as he notes below.

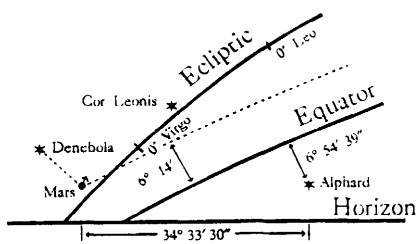
[3] Alphard, α Hydrae.

it happened that the two right ascensions, when compared with the given declination, resulted in a discrepancy of 6′. This could easily happen if the declination were only a tiny bit too small. Indeed, they appear not to have had much confidence in this, with the result that they measured Mars from the Tail of Leo, which is at the same longitude, all the distance being latitudinal, with the aim of knowing Mars's latitude with greater certainty from this rather than from the declination[4]. But let the declination of 6° 14′ stand, as well as the distance from the Heart of Hydra of 34° 33½′. Its right ascension would thus be 168° 56¼′[5]. Therefore, its position would be 17° 16¾′ Virgo, latitude 1° 16⅔′ north. The table of refraction for the fixed stars shows 4 minutes at this altitude, while the table for the sun shows more, Also, Virgo is rising steeply. Therefore Mars has to be put forward (eastward) about 3 minutes or (using the solar refractions) a little more, whence it was subtracted by refraction. The parallax was quite small, so it hardly removes anything from the refraction. Mars would have been at 17° 20′ Virgo[6].

On 1600 March 5/15 at $8\frac{1}{2}^h$ pm Mars was at 29° 12½′ Cancer, latitude 3° 23′ N. And on March 6/16 at $8\frac{1}{2}^h$ at 29° 18′ Cancer, lat. 3° 19¾′ N.

Now the times that correspond to Mars's return to the same place on the eccentric are:[7]

[4] Kepler's meaning here is obscure. The following diagram, showing the relative positions of the stars and the horizon, may make matters clearer:




The position of Mars is primarily determined by its measured distance from Alphard. It can be seen from the diagram how natural it would have been to have taken the distance from Cor Leonis also, as confirmation. As Kepler suggests, the distance from Denebola was probably taken in order to check Mars's latitude, since the two stars were nearly lined up in longitude.

[5] Here an error occurs that is not compensated. On the basis of Kepler's data, Mars's right ascension should be 169° 9′ 9″, and its longitude, 17° 35′ 17″ Virgo, latitude 1° 25′ 32″ north. The effects of this error will be traced through the computations and noted in the appropriate places.

[6] 17° 38′ Virgo.

[7] Adjustments are made upon all observations other than the first, so that the times may be 686 days, 23 hours, and 31 minutes apart.

| | | | Observed position of Mars | And of the sun | And distances of the sun from earth from ch. 30 |
|---|---|---|---|---|---|
| 1585 | 17 Feb. | 10. 0 pm | 15° 12′ 30″ Leo | 9° 22′ 37″ Pisces | 99,170 |
| 1587 | 5 Jan. | 9.31 pm | 2° 8′ 30″ Libra | 25° 21′ 16″ Capr. | 98,300 |
| 1588 | 22 Nov. | $9.02\frac{1}{2}$ pm | 2° 35′ 40″ Libra | 10° 55′ 8″ Sagit. | 98,355 |
| 1590 | 10 Oct. | 8.35 pm | 20° 13′ 30″ Virgo[8] | 26° 58′ 46″ Libra | 99,300 |
| 1600 | 6 Mar. | $6.17\frac{1}{2}$ pm | 29° 18′ 30″ Cancer | 26° 31′ 36″ Pisces | 99,667 |

The procedure for referring the observations to the appropriate times is this. Since in 1587 the diurnal motions of Mars are decreasing, as is apparent both in Magini and in the observations on the three days, I have assumed the following diurnal motions: 17, 16, 16, 16, 15, 15, 14, 14, 13, 13, 13, 12, 12.

On 1588 November 10 the observation is 39 minutes less than the midday position of Magini. On December 5 it is 33 minutes less. And our moment is between these. Therefore, we have also taken the intermediate difference of 36′.

In 1590 the observation is solitary, and, as was seen, was itself not well made. Nevertheless, the diurnal motion in Magini is a constant 37′ over many days[9].

Now to the point: and while I have so far presented many methods of finding or testing the eccentric positions and distances, I nevertheless here follow yet another one, it being the easiest. *Let* $\delta$, $\epsilon$, $\kappa$, $\lambda$, $\gamma$ *be positions of the earth, with* $\delta$, $\gamma$ *on the left and* $\epsilon$, $\kappa$, $\lambda$ *on the right side of the eccentric. And since the lines* $\alpha\delta$, $\alpha\epsilon$, $\alpha\kappa$, $\alpha\lambda$, $\alpha\gamma$ *are given, and*
205 *also the angles* $\alpha\delta\iota$, $\alpha\epsilon\iota$, $\alpha\kappa\iota$, $\alpha\lambda\iota$, $\alpha\gamma\iota$, *I shall take a third element common to all the triangles, namely the side* $\alpha\iota$, *which is one of the magnitudes sought, and using this side I shall find the angles at* $\iota$ *and see whether they place the line* $\alpha\iota$ *at the same zodiacal position (except to the extent that it is moved forward in the later times by the precession of the equinoxes). From this I am going to know whether the value assumed for* $\alpha\iota$ *was any good.*

*The basis of the method is this: that as* $\alpha\iota$ *is to [the sines of] the angles* $\delta$, $\epsilon$, $\kappa$, $\lambda$, $\gamma$, *so are* $\alpha\delta$, $\alpha\epsilon$, $\alpha\kappa$, $\alpha\lambda$, $\alpha\gamma$ *to [the sines of] the angles at* $\iota$.

[8] Should be 20° 30′ 42″ Virgo. Note that Kepler has introduced yet another error in referring the observation to the required time (see the next footnote).

[9] From October 6 at $4^h 45^m$ to October 10 at $8^h 35^m$ is $4^d 15^h 50^m$, which, at 37′ per day, amounts to 2° 52′ 25″. Kepler's figures show that he added 2° 53′ 30″.

| | | |
|---|---|---|
| $\gamma\alpha$ | 26° 31′ 36″ | Pisces |
| $\gamma\iota$ | 29° 18′ 30″ | Cancer |
| $\alpha\gamma\iota$ | 122° 46′ 54″ | |
| $\delta\alpha$ | 9° 22′ 37″ | Pisces |
| $\delta\iota$ | 15° 12′ 30″ | Leo |
| $\alpha\delta\iota$ | 155° 49′ 53″ | |
| $\epsilon\alpha$ | 25° 21′ 16″ | Capricorn |
| $\epsilon\iota$ | 2° 8′ 30″ | Libra |
| $\alpha\epsilon\iota$ | 113° 12′ 46″ | |
| $\kappa\alpha$ | 10° 55′ 8″ | Sagittarius |
| $\kappa\iota$ | 2° 35′ 40″ | Libra |
| $\alpha\kappa\iota$ | 68° 19′ 28″ | |
| $\lambda\alpha$ | 26° 58′ 46″ | Libra |
| $\lambda\iota$ | 20° 13′ 30″ | Virgo[10] |
| $\alpha\lambda\iota$ | 36° 45′ 16″[11] | |

*The sines of these, multiplied by the earth-sun distance, and divided by the magnitude assumed for $\alpha\iota$, 166,700, yields the sines of the angles which, added to the observed positions of Mars at $\gamma$, $\delta$, and subtracted at $\epsilon$, $\kappa$, $\lambda$, put the line $\alpha\iota$ at the following positions:*

| $\gamma$ | $\delta$ | $\epsilon$ | $\kappa$ | $\lambda$ |
|---|---|---|---|---|
| 29° 28′ 44″ Leo | 29° 18′ 19″ Leo | 29° 19′ 21″ Leo | 29° 20′ 40″ Leo | 29° 20′ 30″ Leo[12] |
| | | *Should have been at* | | |
| 29 30 51 | 29 18 0 | 29 19 36 | 29 21 12 | 29 22 48 |
| | | *or at* | | |
| 29 29 51 | 29 17 0 | 29 18 36 | 29 20 12 | 29 21 48 |

[10] Should be 20° 30′ 42″ Virgo.
[11] Should be 36° 28′ 4″.
[12] Should be 29° 46′ 34″ Leo.

*That is, the five positions ought to have differed by no more than the amount occasioned by the precession of the equinoxes.*

*You can see from the diagram that if, other things remaining the same, you had taken* αι *to be shorter, it would be moved forward at* γ, δ *and back at* ε, κ, λ, *but not by an equal distance for all of them. And at the same time, you would have made matters worse at* δ, κ, λ, *and better at* γ, ε. *The opposite would have happened if you had lengthened*
206 αι. *But it is fitting to have these small errors distributed among all the positions. Therefore, the distance* αι *is not to be changed at all, and the planet, at the prescribed times, is at the positions last mentioned.*

If I may be allowed, I would like to seek confirmation using the method of chapter 28. *The points* δ, ε *being joined, you will find* δε *to be 74,058,* δεα *68° 36′ 0″* εδα *67° 21′ 3″. Therefore* εδι *is 88° 28′ 50″, and* δει *is 44° 36′ 46″, and* ειδ *is 46° 54′ 24″. Therefore,* ιε *is 101,380, and* εαι *is 33° 58′ 33″. Therefore, in 1587* αι *was at 29° 19′ 49″ Leo (we just now chose 29° 18′ 36″, the difference of one minute keeping it in agreement with other positions). Finally,* αι *is 166,725, and the position of* κ *is in agreement [with the former one].*

Since 166,666⅔ is the sesquialter of the radius, 100,000[13], it is credible that this is the ratio of the mean distance of the earth from the sun to the greatest distance of Mars from the sun. But at present I shall base nothing upon conjecture.

And since the plane of the eccentric is inclined to the ecliptic here at an angle of 1° 48′, whose secant is 49 units above [the radius], or 82 of our present units, the most correct distance of Mars and the sun will be 166,780, as far as can be told from these observations, which, you will recall, were deduced from ones that were rather distant instead of being optimally obtained on the very days in question.

Let us now proceed to the perigee, where the catalog of observations, and the approximate knowledge of the mean motion, show the following to be the nearest observations:

I. On 1589 Nov. 1 at $6\frac{1}{6}^h$ in the evening, Mars was at 20° $59\frac{1}{4}'$ Capricorn, with latitude 1° 36′ south.

II. On 1591 Sept. 26 at $7^h\ 10^m$ at 18° 36′ Capricorn, lat. 2° $49\frac{1}{5}'$ south.

III. On 1593 July 31 at $1\frac{3}{4}^h$ am at 17° $39\frac{1}{2}'$ Pisces. lat. 6° $6\frac{1}{4}'$ south, and August 11 at $1\frac{3}{4}^h$ am at 16° $7\frac{1}{2}'$ Pisces, Lat. 6° $18\frac{5}{6}'$ south.

The times correspond thus:

13 'Sesquialter' means 'in the ratio of 3 to 2', which these numbers clearly are not.

| | | | Mars | Sun | Dist. of sun and earth |
|---|---|---|---|---|---|
| 1589 | November | 1 $6\frac{1}{6}^h$ pm | 20° $59\frac{1}{4}'$ Capricorn | 19° 13′ 56″ Scorpio | 98.730 |
| 1591 | September | 19 $5^h\ 42^m$ | 14° $18\frac{1}{2}'$ Capricorn | 5° 47′ 3″ Libra | 99.946 |
| 1593 | August | 6 $5^h\ 14^m$ | 16° 56′ Pisces | 23° 26′ 13″ Leo | 101.183 |

For 1591 we need to take it on faith that the diurnal motions are the same as those of Magini, since the observation is solitary. And since in Magini it moved 4° 16′ in 7 days, on September 19 at $7\frac{1}{6}^h$ Mars would have been at 14° 20′ Capricorn, and at $6\frac{1}{6}^h$ it would have been at
207 14° $18\frac{1}{2}'$ Capricorn. About the station on July 16 or 17, Mars was about 1° 16′ farther forward in the calculation than in Magini. Now, on September 26, it is still 0° 53′ farther forward. Therefore, over 70 days the difference has been diminished by about 23 minutes. So if we interpolate, this difference will be about 2 minutes greater on September 19. We shall therefore believe that at our given moment Mars is at 14° 20′ Capricorn.

In 1593 Mars was leaving its station. And on midnight of July 30 the position of Mars disagreed with Magini's midday position by 3° $25\frac{1}{2}'$, and on August 10 by 3° $59\frac{1}{2}'$, so that the difference was increased, but gradually less so. Therefore, I have assumed a difference of 3° 46′ for August 6, so that at $1\frac{3}{4}$ hours after midnight it would be at 16° 52′ Pisces. And the diurnal motion was 10′. This is 8 hours 30 minutes past our time, which would account for about 4′ of Mars's retrograde motion. Therefore, at our time it was at 16° 56′ Pisces. It is certain that (on this point at least) we are no more than one minute high or low.

It was not observed more frequently at perigee. For in 1595 its arrival at perigee fell in the middle of summer, when twilight lasts all night in Denmark. In 1597 Tycho Brahe was travelling. And when it is near the sun in its winter semicircle it lingers long, since its speed is not much less than the sun's.

*In the diagram, let Mars's eccentric position be* $\theta$, *the positions of the earth,* $\zeta$, $\mu$, $\eta$; *and let*

| | | |
|---|---|---|
| $\zeta\alpha$ be 19° 13′ 56″ Scorpio | $\mu\alpha$ 5° 47′ 3″ Libra | $\eta\alpha$ 23° 26′ 13″ Leo |
| $\zeta\theta$ 20° 59′ 15″ Capricorn | $\mu\theta$ 14° 18′ 30″ Capricorn *or* 20′ | $\eta\theta$ 16° 56′ 0″ Pisces |

*Therefore,*

| | | |
|---|---|---|
| $\alpha\zeta\theta$ is 61° 45′ 19″ | $\alpha\mu\theta$ 98° 31′ 27″ *or* 32′ 57″ | $\alpha\eta\theta$ 156° 30′ 13″ |

*When the length of the common side $\alpha\theta$ is assumed to be 138,400, its position comes out thus:*

Through $\zeta$ 29° 55′ 20″ Aquarius
$\mu$ 29° 53′ 6″ (or 54′ 36″) Aquarius
$\eta$ 29° 59′ 10″ Aquarius

*But if it was 55′ 20″ at $\zeta$, it should have been 56′ 56″ at $\mu$, and 58′ 32″ at $\eta$, for that is the amount of the precession of the equinoxes. It can thus be seen from the diagram that the line $\alpha\theta$ determined through $\eta$ goes too far forward, and through $\mu$, $\zeta$, too far back, in relation to that through $\eta$. Other things remaining unchanged, this happened because I assumed too small a value for $\alpha\theta$. Therefore, if I make it a hundred parts longer, the following positions come out:*

From $\zeta$ 29° 57′ 10″ Aquarius;
from $\mu$ 29° 55′ 36″ Aquarius (or 29° 57′ 6″ Aquarius);
from $\eta$ 29° 58′ 17″ Aquarius[14]

*So now the positions of $\alpha\theta$ have been made to be too close to one another*[15]*, and more so now in closeness than before in remoteness. Therefore, the most correct length of $\alpha\theta$ will be about 138,430.*

At this point the plane is inclined 1° 48′ (as it was before at the opposite position), and the secant is 49 units greater than the radius. But as 100,000 is to 138,430, so is this 49 to 68. Therefore, the correct length of the radius is approximately 138,500, at least from these observations involving long interpolations.

### Investigation of the Apsides, from the Above

With all three observations taken into account, let the position of the line $\alpha\theta$ on 1589 November 1 at $6\frac{1}{6}^h$ pm be taken as 29° 54′ 53″ Aquarius, so that in 1591 it would be 29° 56′ 30″, and in 1593, 29° 58′

[14] Kepler's reasoning here is faulty. Consider, for example, the triangle $\alpha\zeta\theta$. The line $\alpha\zeta$ is fixed in position and magnitude, and the angle $\alpha\zeta\theta$ is given. Therefore, $\zeta\theta$ is given in position. Kepler found that $\theta$ as determined from point $\zeta$ was too far forward: that is, that the point $\theta$ was placed to the left of its true position. This means that $\alpha\theta$ was too long, for when $\theta$ is moved to the right along $\zeta\theta$, $\alpha\theta$ becomes shorter. Recomputation of Kepler's figures confirms this conclusion: they are way off. Correct values are: from $\zeta$, 29° 53′ 18″ Aquarius; from $\mu$, 29° 50′ 31″(or 29° 51′ 47″) Aquarius; and from $\eta$, 29° 59′ 58″ Aquarius. Figures in agreement with Kepler's are obtained by decreasing $\alpha\theta$ by 100.

[15] Here may be seen the probable cause of Kepler's error (see the preceding footnote): he seems to have thought of $\alpha\theta$ as fixed in position, while the points $\zeta$, $\mu$, and $\eta$ are moved around as the length of $\alpha\theta$ is changed. Actually, $\zeta$, $\mu$, and $\eta$ are fixed, and the three different positions of $\theta$ determined by them are moved around as the length of $\alpha\theta$ is changed.

6″ Aquarius. The vicarious hypotheses of chapter 16 shows it to be at 29° 52′ 55″ for the first of the times.

208 But previously and in like manner we took αι on 1588 November 22 at $9^h 2\frac{1}{2}^m$ to be 29° 20′ 12″ Leo.

From 1588 November 22 at $9^h 2\frac{1}{2}^m$ to 1589 November 1 at $6^h 10^m$ are 344 days diminished by $2^h 52\frac{1}{2}^m$, while a whole revolution to the same fixed star has 687 days diminished by 0 h. 28 min. Therefore, our interval appears to exceed half the periodic time by a few hours. Consider:

| | | | |
|---|---|---|---|
| 343 Days | 11 Hours | 46 Min. | Half the period |
| 343 | 21 | 52½ | Our interval |
| Excess | 10 | 6½ | |

And from the position at the earlier time, 29° 20′ 12″ Leo, to the position which Mars held at the later time, 29° 54′ 53″ Aquarius, is 180° 34′ 41″, or 180° 33′ 53″ with the precession of 48″ subtracted. Therefore, if the excess of 33′ 53″ beyond the semicircle is responsible for the 10 hours 6½ minutes of Mars's diurnal motion, the aphelion would consequently be understood to be at 29° 20′ 12″ Leo.

But we know the diurnal motions of Mars on the eccentric near apogee and perigee from the distances just found and from the demonstrations of chapter 32. For the diurnal motions are approximately in the [inverse] duplicate ratio of the distances. At apogee the diurnal motion is about 26′ 13″, at perigee 38′ 2″ since the mean diurnal motion is 31′ 27″.

Consider, then: if Mars, in moving from its apogee point, expends half its periodic time, at the end of this time, having traversed exactly 180 degrees, it is going to be at the perigee point. But now if it begins this space of time one day after it was at apogee, it will begin its course 26′ 13″ beyond apogee and will end it at 180° 38′ 2″. Therefore, in half the time it will traverse 11′ 49″ more than half the path. The opposite will happen if it begins one day before apogee.

209 Therefore, since our time, too, shows an arc greater [than a semicircle], our aphelion also should be moved forward. First, we shall credit half of our hours to the time before aphelion, and half after perihelion. The planet would then begin from 5′ 16″ before aphelion, which would thus be put at 29° 25′ 28″ Leo, and it would come to 8′ 1″ after perihelion, the amount of travel being 13′ 17″ beyond 180°. But its path was seen to be 33′ 53″ beyond 180°.

Therefore, it is still faster by 20′ 36″. Therefore, since to increase the path by 11′ 49″, one day, or the promotion of the planet to 26′ 13″ beyond aphelion, is needed, how much will the planet be promoted from aphelion to increase the path by 20′ 36″?

The rule of proportion shows it to be 1 day $17^h$ $54^m$, or a distance from aphelion of 45′ 42″. Therefore, the aphelion is to be moved forward again 45′ 42″ from the position we just gave it, 29° 25′ 28″ Leo. So it will fall at

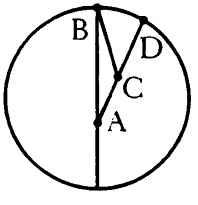
| | |
|---|---|
| | 28° 39′ 46″ Leo |
| On 1588 November 22, above | 28 50 44 Leo |
| Difference | 10′ 58″. |

To which of the investigations of the aphelion one ought to give more trust, is uncertain. For it could easily happen that in the positioning and assuming of the lines $\alpha\iota$, $\alpha\theta$ we have erred by 4 minutes, two for the one and two for the other, owing to difficulties in the observations. And this is all that needs to be accumulated, through the compounding of errors, to change the aphelion by 11 minutes. For the present, however, the operation is good enough for us to trust.

### Correction of the Mean Motion

When the aphelion is changed, the mean motion is changed as well. For at the same time at which in the previous investigation Mars was thought to fall at aphelion, with no equation, it has now passed the aphelion by 11 minutes. Therefore, it has an equation of 4 minutes, subtractive. Thus in its mean motion it has passed that original position by 4′[16].

[16] A diagram may be helpful here. Suppose the aphelion is originally at B, but it is moved back 11′ to D. The corresponding equation, the angle ABC, is 4′. So the line CB, the mean motion, is now 4′ farther forward than its original position along the line AB, at the same time as before.



### Investigation of the Eccentricity

First, the distances found previously should be corrected, if necessary, to the extent that they are some small amount distant from the apsides just found, the aphelia by 40 minutes, perihelia by 75 minutes. But there is no perceptible change so close to the apsides.

| | |
|---|---|
| Therefore, the Aphelial distance is | 166,780 ($\alpha\iota$) |
| Perihelial distance | 138,500 ($\alpha\theta$) |
| Sum | 305,280 ($\iota\theta$) |
| Half | 152,640 (semidiameter $\iota\beta$) |
| Eccentricity | 14,140 ($\alpha\beta$) |

And as 152,640 is to 100,000, so is 14,140 to the eccentricity 9264. But half the eccentricity of the equating point was 9282. The difference of 18 is clearly of no importance. You see how precisely the eccentricity of the equating point is to be bisected in Mars in order to establish the distance between the centres of the eccentric and the world. Above, in chapter 32, I took this to be fundamental, and in the following chapters declined to demonstrate it. Now, however, that obligation is discharged.

# 43

## On the defect in the equations accumulated by bisection of the eccentricity and the use of triangular areas, on the supposition that the planet's orbit is perfectly circular

What was proved in Part III concerning the bisection of the eccentricity in the theory of the sun has now in turn been demonstrated with perfect certainty for Mars. And now that our evidence of this is complete, it would at last be time to proceed to the physical theories of chapters 32 and the following, seeing that they are going to apply to all planets in common, had I not seen fit to present them earlier. I did so because there, in the theory of the sun or earth, the procedure of computing the equations from physical causes produced perfect results, and because I knew that where that method of constructing the equations is to be applied to the theory of Mars, much more difficult physical theories are to follow.

Now when the true configuration of the orbits is found, it must necessarily produce the true eccentric equations, which have hitherto alone been provided by the vicarious hypothesis found in chapter 16. This, therefore, we shall now use as a test.

Therefore, following what was demonstrated in chapter 40 (all of which is to be understood as holding here), let the orbit of the planet, in accord with the well-worn opinion, be circular, even though ch. 41 has just urged us to doubt it. *Therefore, at the eccentric anomaly of 90° the eccentricity 9264 found in ch. 42 will be the tangent, which will give the optical part of the equation, 5° 17′ 34″. And since at the eccentric anomaly of 90° the area of the triangle is right-angled, the radius multiplied by half the eccentricity, 4632, gives the area of the triangle, 463,200,000. Now as the area of the circle, 31,415,926,536, is to 360 degrees or 1,296,000 seconds, so is this area 463,200,000, to 19,108″,*

*or 5° 18′ 28″, the physical part of the equation. Consequently, the whole equation is 10° 36′ 2″, so that to the mean anomaly of 95° 18′ 28″ corresponds the equated anomaly of 84° 42′ 26″.* But according to the method of chapter 18, the vicarious hypothesis, accurate enough for the longitudes, shows us that to the mean anomaly of 95° 18′ 28″ there ought to correspond the equated anomaly of 84° 42′ 2″. The difference is 24″.

Now let our eccentric anomaly be taken as 45° and 135°. *And as the whole sine is to the sine of these angles, so is 19,108″, the area of the greatest triangle of the equation, to the area at this position, 13,512″, or 3° 45′ 12″, so that by addition of this the physical part of the equation to the eccentric anomaly the mean anomalies of 48° 45′ 12″ and 138° 45′ 12″ are constructed. But from the given sides of the given angles, the angles of equated anomaly corresponding to these mean anomalies come out to be 41° 28′ 54″ and 130° 59′ 25″.* But by the vicarious hypothesis, as in chapter 18 of this work, the same simple anomalies of 48° 45′ 12″ and 138° 45′ 12″ being taken, the equated anomaly for the former comes out to be 41° 20′ 33″, less than by the area of the triangle, the excess being 8′ 21″; and for the latter, 131° 7′ 26″, more than by the area of the triangle, the defect being 8′. So, since it is certain that an error of this magnitude cannot be attributed to our
211 vicarious hypothesis, I had to accept that this procedure for finding the equations was still imperfect.

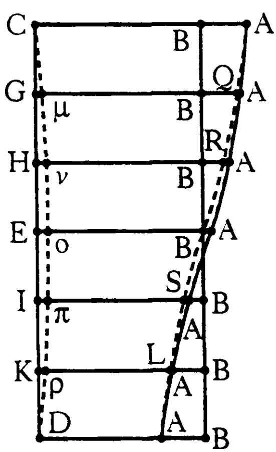
Indeed, in chapter 19 as well, when I tried out the bisection on Mars and computed the equations using a motionless point of the equant in the Ptolemaic manner, a difference was found at about 45° of eccentric anomaly of nearly the same amount, but in the opposite direction. For in the upper quadrant, the planet was closer to the aphelion, and in the lower to the perihelion, than it should have been; while here in the upper quadrant it was farther from the aphelion, and in the lower from the perihelion, than it should be. And so in the upper quadrant it is moving too swiftly away from the aphelion, and the same from the perihelion below. Therefore, it is slower than it should be in the middle longitudes.

Refutation of false causes of this imperfection

I believe it might now occur to the reader that the cause of these errors might perhaps lie in the flaw to which this operation with areas is subject, mentioned in chapter 40: that the areas are not equivalent to the distances that modify the swiftness and slowness. But the present error cannot arise thence. For first of all, the excess of the sum of the distances over the area of the circle is small: just about as

small, that is, as the little space between the conchoids. Then, too, the area makes all the distances a little smaller than they should be, and most of all those that are at the middle longitudes. So if any error arises hence, it will appear in our not having made the planet take enough time in the middle longitudes. But the errors we are now seeing are in the opposite direction, for we have made the planet take too much time in the middle longitudes.

The same can be raised in objection to anyone who might conceive a suspicion that the error arose from our having rejected the double epicycle of Copernicus and Tycho, which make the orbit of the planet oval, in favour of the present Ptolemaic perfect circle. For it was said at the end of chapter 4 that the Copernican orbit does not make an incursion towards the centre, as it would have to for our purposes, but moves outwards from the centre by 246 parts, which would only increase the error on the assumption that the elapsed times are proportional to the distances.




Estimate of the area between the two conchoids.

But to make it clear to the eye that the area of the conchoid of ch. 40 has but a small effect, consider that the secant of the angle 5° 19′ (the maximum optical equation) is 100,432, which is the line EA. So from this excess of 432, which is the small line BA, part of the line EA, we will be able to get an approximate idea of the sum of all these excesses, in this way.

The secant of 89°, and its tangent, taken together, are as great as

A short cut for finding the sum of all the sines at once.

the sines of all degrees of the whole semicircle, as Cardan shows us by hand in the part of *De subtilitate* in which he explains the properties of the circle [1]. A proof of this is given by Justus Byrgius[2].

*Therefore, if all our remaining excesses (other then the greatest, 432) were [to the greatest]*[3] *as [all] the sines in one semicircle are to the semidiameter, then as 100,000 is to the sum of the secant and the tangent of 89° (that is, 11,458,869), so, approximately, would 432 be to*
212 *49,934, the approximate sum of all the excesses at integral degrees of the semicircle*[4]*. For the excesses of the distances in the upper quadrant are longer than those excesses of the secants, to about the same extent that they are shorter in the lower quadrant.*

By what ratio the excesses of the distances of the points on a circle from an eccentric point, or the breadth of the space between the conchoids, may grow.

But nevertheless, it is not true that the excesses QA, RA, SA, and so on, are to one another as the sines of the corresponding number of degrees. Instead, they are approximately in the duplicate ratio of the sines. *As for example, the sine of 90° is twice the sine of 30°. Now the optical equation of 90° is 5° 19′, and half of its sine gives an arc which is likewise about its half, that is, 2° 39′ 15″, for the optical equation at 30° of eccentric anomaly, whose secant is 100,107. And here 107, the excess of the secant over the radius, is about one fourth of the former, 432, while the sine of 30° is half the sine of 90°.* Some geometer should see whether this be demonstrable. For me it suffices at present to answer those very small questions with which I am occupied.

Therefore, to arrive at 432, parts are accumulated that are not proportional to the sines, but are always smaller, and at the 45th degree or thereabouts are but their halves. Before that point they are less than the halves, so that about 30° they are only the fourths, and at length become imperceptible.

1 Girolamo Cardano (1501–1576), physician, mathematician, and philosopher. Although perhaps best known today for his pioneering work on probability theory, his reputation among his contemporaries rested chiefly upon his unorthodox physical theories, presented in *De subtilitate rerum*, (Paris and Nuremberg, 1550, and many other editions), and *De rerum varietate*, (Basel, 1557). Both works are included in Vol. III of his *Opera omnia*, (10 vols., Lyons, 1663). The passage cited is in *De subtilitate* Book XVI (*De scientiis*), ed. 1550 p. 303. (Citation from KGW 3 p. 471).

2 Justus Byrgius, or Jost Buergi (1549 or 1552–1632), Swiss instrument maker and mathematician, who, like Kepler, was at the time employed by Rudolph II. The proof was thus most likely given to Kepler personally: nothing is known of it other than this one mention. (See R. Wolf, *Johannes Kepler und Jost Buergi*, 1872.)

3 Supplied to make sense of the proportion.

4 Here Kepler assumes as a working hypothesis that the small segments QA, RA, SA, and so on, between the conchoids, are to one another as the sines of their eccentric anomalies, and then uses the Cardan/Buergi rule to compute an approximate sum. The hypothesis is, however, false, as he points out in the next paragraph: from a single example, he concludes that the small segments are proportional to the *squares* of the sines of the eccentric anomalies. Caspar's analysis using modern techniques (KGW 3 p. 471) shows this to be very nearly correct.

And so, of the sum of 49,934, we shall retain only one seventh, or about 7000. This is also shown empirically, by computing all the distances degree by degree and adding them up[5].

And because one distance of 100,000 has the value of 60′, this little sum has a value of no more than $4\frac{1}{5}'$[6], which is nonetheless spread all around the circumference, so that about 45° and 135°, where it is greatest, this tiny error turns out to be imperceptible even in Mars.

Consequently, we must seek another occasion for this discrepancy.

[5] It is hard to see how Kepler could have believed that the mysterious figure of 7000 (which he may have hastily supposed to be the square root of 49,000) is confirmed by experience, since he himself computed the sum (p. [48–2] of this translation), and found it to be 37,781.

[6] Should be $22\frac{2}{3}$.

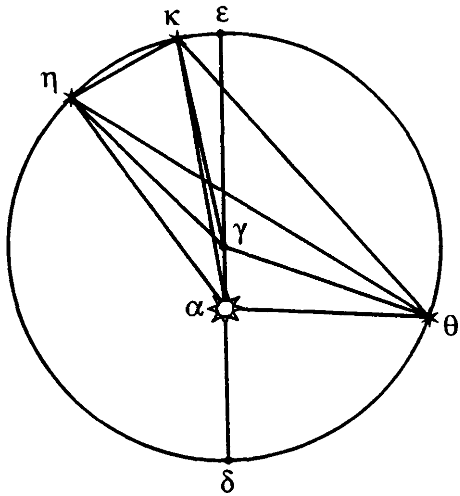
# 44

## The path of the planet through the ethereal air is not a circle, not even with respect to the first inequality alone, even if you mentally remove the Brahean and Ptolemaic complex of spirals resulting from the second inequality in those two authors

With the eccentricity and the ratio of the orbs established with the utmost certainty, it must appear strange to an astronomer that there remains yet another impediment in the way of astronomy's triumph. And me, Christ! – I had triumphed for two full years. Nevertheless, by a comparison of the things which have been established in chapters
213 41, 42, and 43, preceding, it will easily be apparent what we are still lacking. The positions of the aphelia, eccentricity, and the ratio of the orbs, as constituted in the several places, differed greatly from one another. Nor were the computed physical equations in agreement with the observations (which the vicarious hypothesis represents). Let the diagram of chapter 41 be brought back. *And because, in that diagram, in units of which* $\gamma\eta$ *was 151,740,* $\gamma\alpha$ *would be 14,822, when* $\gamma\alpha$ *and* $\gamma\eta$ *or* $\gamma\epsilon$ *are added,* $\alpha\epsilon$ *would be 166,562. In chapter 42 this was found to be 166,780. Likewise, when* $\gamma\alpha$ *is subtracted from* $\gamma\delta$*, the remainder,* $\alpha\delta$*, is 136,918, which in chapter 42 was found to be fully 138,500.*

*Again, the true longitude of the lines* $\gamma\epsilon$, $\gamma\alpha$, $\alpha\epsilon$, *and* $\alpha\delta$ *was found in chapter 42. If, therefore, what was supposed and used in chapter 41 is true, that the path of the planet is a circle, it is not difficult to say how much* $\alpha\kappa$, $\alpha\eta$, $\alpha\theta$ *ought to be. Since in Oct. 1590* $\alpha\epsilon$ *was at 28° 41′ 40″ Leo, and* $\kappa$, $\eta$, $\theta$ *are as given in chapter 41, the angles* $\kappa\alpha\gamma$, $\eta\alpha\gamma$, $\theta\alpha\gamma$ *will be given. Therefore, the optical equation will also be given:* $\alpha\kappa\gamma$ *0° 53′ 13″,* $\alpha\eta\gamma$ *3° 10′ 24″,* $\alpha\theta\gamma$ *5° 8′ 47″*[1]. *And as the sine of these angles*

[1] These values result from the use of an eccentricity of 14,040, which is, of course, different from either the value of 14,822 derived above, or 14,140 mentioned below as the best value.




*is to the best value for the eccentricity* αγ, *14,140, so are the sines of* κγε, ηγε, θγα *to* ακ, αη, αθ.

| | | | |
|---|---|---|---|
| *They come out thus:* | ακ 166,605 | αη 163,883 | αθ 148,539 |
| *But in the observations they were found to be:* | 166,255 | 163,100 | 147,750 |
| *Difference:* | 350 | 783 | 789 |

If anyone wishes to attribute this difference to the slippery luck of observing, he must surely not have felt nor paid attention to the force of the demonstrations used hitherto, and will be shamelessly imputing to me the vilest fraud in corrupting the observations of Brahe. I therefore appeal to the observations of subsequent years, at least those made by experienced observers. For if in any respect I have given free rein to my inclinations in one direction, I only go so much the farther into error on the other side. But there is no need of this. I am addressing this to you who are experienced in matters astronomical, who know that in astronomy there is no tolerance for the sophistical loopholes that beset other disciplines. To you I appeal. You see at κ a small defect from the circle, at η, θ on both sides, a rather large one, enough so that we cannot excuse it by uncertainties in observing (for in chapter 42 I reckon an uncertainty of perhaps 200, or at most 300 units).

What, then, is to be said? Could this be the situation described in chapter 6 above, in which by transposition of the reference point from the sun's mean motion to its apparent motion I set up another eccentric that makes an excursion towards the side of the sun's apogee? By no means. For there, it would approach from the one side by the same amount as it moves away on the other. Here, however, you see that the planet approaches within the circular orbit on both sides. This is confirmed by many other observations, some of which follow below in chapters 51 and 53.

Clearly, then, [what is to be said] is this: the orbit of the planet is not a circle, but comes in gradually on both sides and returns again to
214 the circle's distance at perigee. They are accustomed to call the shape of this sort of path 'oval'.

Second argument.

This same thing is also proved from chapter 43 preceding. There it was supposed that the plane of a perfect eccentric is approximately equivalent to all the distances of the equal parts of the circumference of that eccentric from the source of the motive power, however many they are. Thus, the parts of the plane measure the amounts of time which the planet spends on the parts of the corresponding eccentric circumference. But if that plane about which the planet marks a boundary is not a perfect circle, but is diminished at the sides from the amplitude it has at the apsides, and if nevertheless this plane circumscribed by an irregular orbit still measures the times which the planet takes to traverse the whole and its equal parts, then this diminished plane measures a time equal to that measured by the previous undiminished plane. So the parts of the diminished plane nearest aphelion and perihelion measure a greater time, because in those regions the diminution is narrowest, but the parts at the middle longitudes measure less time than before, because the greatest diminution in the whole plane occurs there. So if we now use the diminished plane in adjusting the equations, the planet will become slower near aphelion and perihelion than it was in the previous faulty form of equation, and swifter near the middle longitudes, because here the distances are lessened. Therefore, the times, when they are abstracted from the plane and adjusted upward and downward, will be accumulated at aphelion and perihelion in much the same manner as, if one were to squeeze a fat-bellied sausage at its middle, he would squeeze and squash the ground meat, with which it is stuffed, outwards from the belly towards the two ends, emerging above and below from beneath his hand.

And indeed, if contraries heal one another, this is plainly the aptest medicine for purging the faults under which, in chapter 43 above, our physical hypothesis was perceived to be labouring. For the planet is going to be swifter at the middle longitudes, where previously it was perceived to be going slower than it should, and it will be slowed down above and below, near the apsides, where previously it did violence to the equations belonging to the eighths of the period through its excessive fleetness.

This, then, is the other argument by which it is proved that the orbit of the planet really is deflected from the established circle, making ingress towards the sides and the centre of the eccentric.

But for all that, this argument still did not have enough effect upon me to let me go beyond it and think about the planet's departure from the orbit. When I had sweated for the longest time trying to reconcile equations of this sort, I was finally discouraged by the absurdity of the measurements, and abandoned the whole enquiry until later, when the distances, found in the way shown in chapter 41, informed me about the departure from the orbit, and I once more took up this problem of the equations.

And from this, what I promised I would prove, in chapters 20 and 23 above, is now done: that the orbit of the planet is not a circle but of an oval shape.

215

# 45

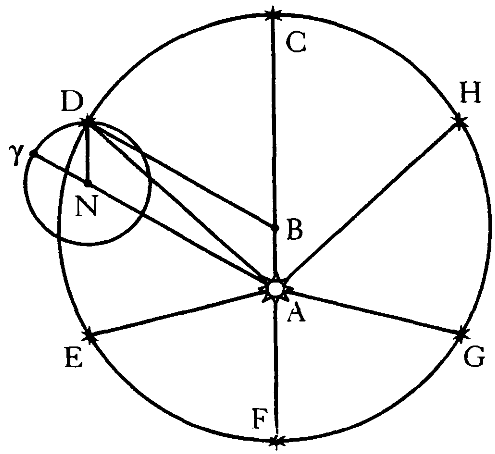
## On the natural causes of this deflection of the planet from the circle: first opinion examined

When I was first informed in this manner by Brahe's most certain observations that the orbit of the planet is not exactly circular but is deficient at the sides, I judged that I also knew the natural cause of the deflection from its footprints. For I had worked very hard on that subject in chapter 39. And I suggest to the reader that he reread that entire chapter carefully before going on. For in that chapter I ascribed the cause of the eccentricity to a certain power which is in the body of the planet. It therefore follows that the cause of this deflecting from the eccentric circle should also be ascribed to the same body of the planet. But then what they say in the proverb, 'A hasty dog bears blind pups', happened to me. For in chapter 39, I worked very energetically on the question of why I could not give a probable enough cause for a perfect circle's resulting from the orbit of the planet, as some absurdities would always have to be attributed to the power which has its seat in the planet's body. Now, having seen from the observations that the planet's orbit is not perfectly circular, I immediately succumbed to this great persuasive impetus, believing that from those things which were called absurd in fabricating the circle in chapter 39, now transmuted into a more probable form, an orbit of the planet which would be both correct and in agreement with the observations would be effected. If I had embarked upon this path a little more thoughtfully, I might have immediately arrived at the truth of the matter. But since I was blind from desire, I did not pay attention to each and every part of chapter 39, staying instead with the first thought to offer itself – a wonderfully probable one, owing to

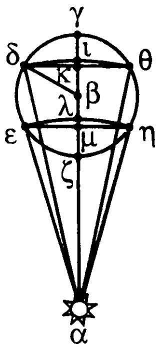
the uniformity of the epicyclic motion – and thus entered into new labyrinths, from which we will have to extract ourselves in this chapter 45 and the following ones all the way to 50.

*Let the diagram from chapter 39 be repeated. Near the end of that chapter the opinion was ventured that, in order to describe a perfect circle, the planet would perform motion on an epicycle by its inherent force, thus disengaging its body from the ray of power from the sun. As, for example, if the ray of power from the sun were* AC, *and moved forward at an unequal pace from* AC *to* A$\gamma$, *while the planet were initially at* C, *and from that time forth, by its inherent force, it were to disengage itself from [the ray]* AC *or* A$\gamma$. *Thus, at the time when* AC *comes to A$\gamma$, the planet from* C *or* $\gamma$ *would come to* D, *and would also do this at a nonuniform pace, more swiftly or slowly in the same proportion as [the length of]* AC. *For in this fashion the line* ND
216 *through the centre of the epicycle and the planet remains ever parallel to the line* AB. *However, I said in chapter 39 that it appeared absurd to me that the planet [in moving] from* $\gamma$ *to* D *at a nonuniform pace disengages itself from the ray of the solar power, and thus accommodates itself by its own force to the extrinsic force from the sun, and has foreknowledge of its speed and the decrease thereof. Therefore, let it be that this absurdity is avoided: let* AC *still go nonuniformly, but let the planet go uniformly from* $\gamma$ *to* D. Let us see whether what follows is anything like what we have proved in the preceding chapter from the observations.

*While the centre of the epicycle* N *and its aphelion [in moving] from the line* AC *to* A$\gamma$ *will be slow from* C *to* $\gamma$, *it being near the eccentric's aphelion* C, *the planet [in moving] from* $\gamma$ *to* D *is supposed not to be slow but to proceed with its mean motion. Consequently, the angle* $\gamma$ND *will be greater than the angle* $\gamma$AC. *So* ND *will not be parallel to* AB *but will be inclined towards* AC. *Thus the planet* D *will not stay on the circle which it began to describe from* C, *the one, that is, which goes through* CF, *but will encroach from the circumference* D *and the parallel* ND *towards* CA. *And this is exactly the testimony of the distances* AD *computed from the observations in the preceding chapter, namely, that they do not reach all the way to the circumference of the circle* CF. *This same thing is also testified by the physical equations constructed through the summation of the distances* AC, AD, *namely, that the planet ought to be faster at the sides of the eccentric; that is, that its distances from the sun ought to be supposed smaller.* Therefore, owing to the considerable persuasive force of this consensus, I




concluded forthwith that the planet's incursion at the sides is the result of this: that the power moving the planet and administering the distances according to the law of the circle anticipates the power of the sun, in that the former makes equal progress in equal times thus sending the planet down uniformly towards the sun according to the law of the epicycle, while the latter, in its varying degrees [of speed obtained] through the varying distances, moves the planet in its care forward nonuniformly, and more slowly when it is high. It thus happens that the distances of equal arcs on the epicycle are accumulated near the aphelion C and the perihelion F, and are more sparsely scattered about the middle longitudes. In this way, all the shorter ones are drawn back upwards from the correct [circular] distances from perihelion to the place of longer ones. That error therefore began to become rooted in me which I had happily begun to refute in chapter 39 above, that it is a property of the planetary power to lead the body of the planet around in the path of an epicycle. Had the diameter of the epicycle ND remained equidistant from AB, I could have shed my erroneous opinion, and could have ascribed (as is perfectly correct) all promotion in zodiacal longitude to the sun, leaving to the planet only the reciprocation on the diameter $\gamma\zeta$, as in part of chapter 39. But because the observations testified that the diameter of the epicycle is inclined in the middle longitudes, this error concerning the motion of the planet on the circumference of the epicycle, whose motion would be regular measured with respect to




217 the line AN$\gamma$ going from the sun A through the centre of the epicycle N, was admirably confirmed in me. Think yourself, reader, and you will feel the force of the argument. It was such that I did not think it possible for the planet's orbit to be rendered oval in any other way.

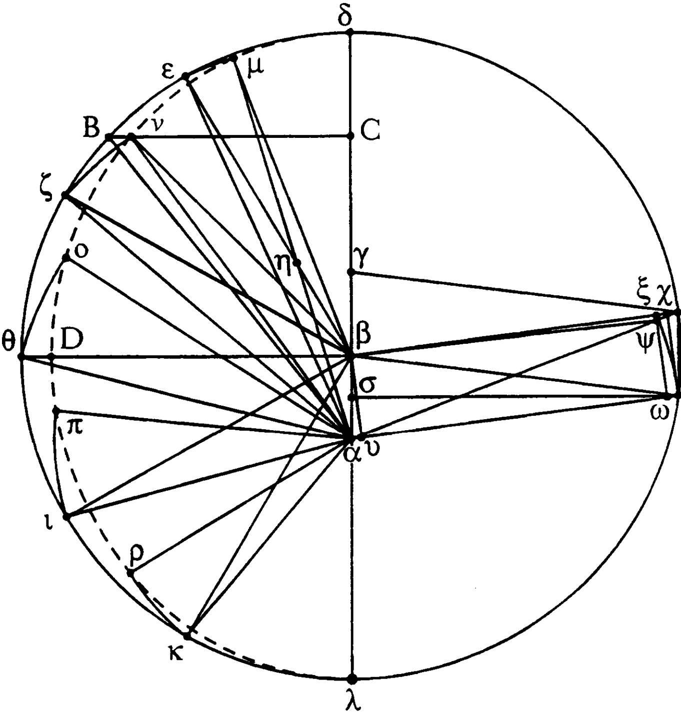
When, therefore, these things occurred to me, quite certain of the quantity of the incursion at the sides (that is, that the numbers would be in agreement), I celebrated another triumph over Mars. Nor did it appear to me any difficulty, if there were some discord among the numbers, to dissipate it through some slight adjustment in the equations of the centre all around, so as to make it imperceptible.

And we, good reader, can fairly indulge in so splendid a triumph for a little while (for the following five chapters, that is), repressing the rumours of renewed rebellion, lest its splendour die before we enjoy it. If anything will be left of it afterwards, we shall go through it in the proper time and order. You are merry indeed now, but I was straining and gnashing my teeth then.

# 46

## How the line of the planet's motion can be described from the opinion of chapter 45, and what its properties are

In the preceding chapter a cause was stated by which it could happen that the planet depart from a circular orbit. However, the geometrical description of the path cannot be carried out using this model. For the epicycle is inclined according to the length of the distances, while the multitude and length of the distances is in turn dependent upon the rotation of the epicycle. *And because the sum of the distances is contained in the plane of the eccentric, as was demonstrated in chapter 40, that sum cannot be found unless the epicycle be transformed into an eccentric. But it was demonstrated in chapter 2, and repeated in chapter 39, and used in chapter 40, that if a concentric be described about centre* $\alpha$ *with semidiameter equal to* $\beta\delta$, *and on it an epicycle with semidiameter* $\alpha\beta$; *and then about centre* $\beta$ *an eccentric* $\delta\lambda$ *with eccentricity* $\alpha\beta$; *and afterwards the circumferences of both the epicycle and the eccentric* $\delta\lambda$ *be divided into similar parts; the distances of the points of division, both of the epicycle and the eccentric, from the chosen point* $\alpha$ *would be respectively equal to one another in length. On this premise, since in chapter 40 we used an eccentric to present a plain and easy demonstration, and a method of computing the distances, here, too, we can examine the distances on the eccentric, even though we are supposing them to be meted out by the uniform motion of the planet's epicycle. This procedure seems to open a way to us to a geometrical description of the planetary path that follows from the hypothesis of chapter 45. Let us therefore say, for the sake of*
218 *understanding, that in the circuit of the epicycle the planet makes digressions from the sun* $\alpha$ *of the same magnitude as if it were on the*




*circumference of a perfect eccentric δλ (which shall be a semicircle defined by the straight line λαβδ) describing equal arcs in equal times, such as δε, εζ, ζθ, θι, ικ, κλ. It does this in such a manner that the angles at β are equal, and β is the point of uniform motion, at least for this position for which the distances are being sought. Let the points of division be connected to α and β. Now this semicircle is purely fictitious, drawn only for computing the sum of a number of distances. If the planet were moved forward with the same degree of power from the sun at both δ and λ, in the same manner as the epicyclical rotation is supposed to be always uniformly set in motion, then it really would traverse these equal parts of the eccentric, from which we have taken the distances, in equal times. Under these conditions, the distances corresponding to the times denoted by the points of division would be these very ones, αδ, αε, αζ, αθ, αι, ακ, αλ, not only in quantity, but also in their identical position. In a word, the path of the planet would be the circle δθλ.*

*The planet does in fact represent quantitatively the reported dis-*

*tances resulting from the uniform rotation of the epicycle, but is itself moved forward unequally in equal times by the sun, less at* $\delta$, *more at* $\lambda$. *Thus in the time signified and measured by* $\delta\beta\epsilon$*, *it does not traverse the space* $\delta\epsilon$, *although it does attain the distance* $\alpha\epsilon$. *And in [that same] time (measured by the angle* $\lambda\beta\kappa$, *equal to* $\epsilon\beta\delta$*) it traverses more space than* $\kappa\lambda$, *although it attains the distance* $\alpha\kappa$. *Therefore, the planet has a distance equal in magnitude to* $\alpha\epsilon$ *before it actually arrives at* $\epsilon$, *and a distance equal to* $\alpha\kappa$ *before it arrives at* $\kappa$*; and inversely, when it arrives at* $\epsilon$ *or* $\kappa$, *it has already been at the distances* $\alpha\epsilon$ *and* $\alpha\kappa$, *and for that reason will now be somewhat nearer. Thus the planet, when at* $\epsilon$, $\kappa$, *and all the other points of this sort, is nearer to the point* $\alpha$ *than are the points* $\epsilon$, $\kappa$ *on the circumference. So the planet moves inward from the established distance of the circle* $\delta\lambda$ *towards the point* $\alpha$ *which is near the centre* $\beta$, *never coinciding with this circle at any points other than* $\delta, \lambda$. *For the manner of the incursion is the same in the opposite semicircle.*

*In this place, when we are computing only the distance $\alpha\epsilon$ (that is, $\alpha\mu$), the angle $\delta\beta\epsilon$ measures the time, the genuine and physical measure of which is the plane surface $\delta\alpha\mu$, as will be made clear below.

*Also, the plane* $\delta\alpha\epsilon$, $\delta\alpha\zeta$, *and so on, contains in itself the sum of the distances of all the points on the arc of the epicycle, which, by chapter 40, is similar to the arc* $\delta\epsilon$. *And yet the planet, in equal times (which are now being measured by* $\delta\epsilon$, $\epsilon\zeta$*), describes unequal arcs on its real path, short when it is far from the sun* $\alpha$, *long when it moves near to the sun, in such a way that the arcs of the planetary path which are traversed in equal times are in the inverse ratio of the distances, by ch. 32. It thus happens that the arc* $\epsilon\delta$ *(which is here the measure of the time) exceeds the arc of the path traversed, which let be* $\mu\delta$, *to about the same extent that the area* $\epsilon\alpha\delta$ *exceeds the sector* $\epsilon\beta\delta$, *whose measure is the angle* $\epsilon\beta\delta$ *or the arc* $\epsilon\delta$.

First attempt at a description of the oval.

*If you declare the entire plane area to be 360° in number, just as the circumference of the circle, and the periodic time as well, then the number of the time, or* $\delta\epsilon$ *(at this position) is approximately the mean, either arithmetic or geometric (for they hardly differ) between the number of the sum of the distances or the area* $\epsilon\alpha\delta$, *and the number of the planetary path or* $\mu\delta$. There occurs here a multiple obstacle to calculation[1]. First, that the plane of the circle is not perfectly equivalent to the sum of the distances, as was demonstrated in chapter 40, even though it was said at the end of chapter 43 that the defect is quite small.

Chapter 48 is devoted to removing these difficulties.

219 Second, that the proportion just described is not exactly geometri-

[1] Αμεχανια.

cal. True, the individual distances are to the individual mean distances in the inverse ratio of the individual arcs of the planetary path to the mean arcs. But the sum of a certain number of distances does not maintain the same ratio to the sum of the same number of mean distances, as the sum of the same number of arcs to the sum of the mean arcs, inversely. As you will see from an example. *Let the two distances be 12 and 11, the mean 10, and let the mean arc be the same. And let it be that, as the distance 12 is to the mean distance 10, so is the mean arc 10 to the arc $8\frac{1}{3}$ belonging to the distance 12. But also, as the distance 11 is to 10, so is 10 to the arc $9\frac{1}{11}$. Compound the distances 12 and 11 into one sum, which will be 23. The sum of the two means is 20, the sum of the two arcs is $17\frac{14}{33}$. Here, 10 was indeed the mean proportional between 12 and $8\frac{1}{3}$, and between 11 and $9\frac{1}{11}$, but now the sum 20 is not the mean proportional between 23 and $17\frac{14}{33}$, but between 23 and $17\frac{19}{23}$, which is greater*[2].

However, this ratio is valid for the arithmetic mean. *For example, let 10 be the arithmetic mean between 12 and 8; likewise, between 11 and 9. Compound 12 and 11: they make 23. Compound 8 and 9: they make 17. Therefore, 20 is again the arithmetic mean between 17 and 23.* And since it was demonstrated in chapter 32 that there is hardly any difference for the present undertaking between the arithmetic and geometric means, what is here denied to be true for all cases will therefore be only a tiny bit different from the truth.

Third, even if the area $\epsilon\beta\delta$ were the exact geometric mean between $\epsilon\alpha\delta$ and $\mu\beta\delta$, nonetheless, it cannot be constructed geometrically. *For the triangle $\alpha\epsilon\beta$ ought to be equal to the sector $\epsilon\beta\mu$. But geometers have yet to devise a method by which a given angle can be cut in a given ratio.*

Term: Properly, a sector is part of a circular plane cut off by two straight lines from the centre. It is thus used improperly of a plane that is not perfectly circular.

Fourth, if none of the above deter us, *the sector $\mu\beta\delta$ of the circle is still not the same as the so-called 'sector' $\mu\beta\delta$ of the oval plane. Therefore, even if the arc $\mu\delta$ were defined as if it were on the circumference of a circle, nevertheless, nothing would follow concerning $\mu\delta$ defined as if it were an arc on the path of the planet, which is not a circle.* Therefore, even though it might be of use to those who want to use numbers, to know that $\epsilon\beta\delta$ is a mean between $\epsilon\alpha\delta$ and $\mu\beta\delta$; nevertheless, for us, who strive after a geometrical way, this passage does not lie open.

[2] This last number should be $17\frac{9}{23}$, which is less than $17\frac{14}{33}$, not greater, as Kepler states. However, it remains true that they are unequal, which is what Kepler is undertaking to prove.

We shall therefore try another way. *And on our fictitious eccentric* $\delta\theta\lambda$ *the measure of the time is* $\delta\epsilon$, $\delta\zeta$ *for finding out the distances* $\alpha\epsilon$, $\alpha\zeta$, *while the ratio of the sectors* $\delta\beta\epsilon$, $\delta\beta\zeta$ *to one another is the same as that of the arcs* $\delta\epsilon$, $\delta\zeta$. *However, on the true path of the planet, the plane between the arc of the path and the sun* $\alpha$ *is likewise the true measure of the time during which the planet is found on the arc lying above it, by chapter 40. Therefore, from the point* $\alpha$ *of the diameter let straight lines be projected enclosing spaces equal to* $\epsilon\beta\delta$, $\zeta\beta\delta$, *so that the space* $\epsilon\eta\mu$, *which is subtracted from the space* $\epsilon\beta\delta$, *is equal to the space* $\eta\alpha\beta$, *which is added to that same space* $\epsilon\beta\delta$. *And let these lines be* $\alpha\mu$, $\alpha\nu$. *And about centre* $\alpha$, *with radii* $\alpha\epsilon$, $\alpha\zeta$, *let arcs* $\epsilon\mu$, $\zeta\nu$ *be drawn intersecting these lines at* $\mu$, $\nu$. *Will the points* $\mu$, $\nu$, $o$, $\pi$, *and so on, constructed in this way, be thus obtained correctly, so that in the times* $\delta\epsilon$, $\delta\zeta$, $\delta\theta$, $\delta\iota$, $\delta\kappa$ *the planet will arrive at them?* This is indeed approximately true, but here, too, three things are wanting. First, as above, the plane is not exactly equivalent to the sum of the distances. Second, there is no geometrical way showing how to cut a given
220 semicircle in a given ratio with a straight line drawn from a given point on the diameter. Third, it is not known whether the shortfall for any of the planes $\mu\alpha\delta$, $\nu\alpha\delta$, and so on, produced by the deflection of $\mu$, $\nu$, from the circumference, is in the same ratio as the rest. Nevertheless, these will still be useful to those who wish, contrary to the geometrical usage, to proceed using least parts, with the aid of numbers.

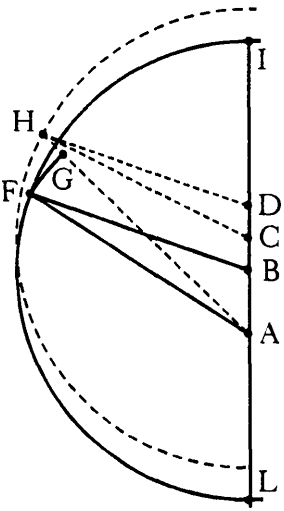
Second attempt at describing our oval.

Since geometry has left us destitute, in order that we may have a description of the line which has been born to us out of the theory of chapter 45, let us go seek the assistance of a contrivance[3] by fetching our vicarious hypothesis from chapter 16, which places the lines $\alpha\mu$, $\alpha\nu$, and so on, on which the planet stands, at the correct zodiacal places at the correct times, combining it with the present fictitious eccentric $\delta\theta\lambda$, from which the theory of chapter 45 has persuaded us that we have derived the correct length of the lines $\alpha\epsilon$, $\alpha\zeta$; that is, $\alpha\mu$, $\alpha\nu$.

Third attempt and method of describing the oval born in chapter 45.

And besides, it is a good idea for the sake of shedding some light to compare the two hypotheses with one another, combined into one diagram. For although both are deceptive on certain points, each is useful for investigating certain truths (to the extent that they can be known at this point). And in this diagram, many things which have been said so far are brought together into a single view.

[3] ᾿Ατεχνια·




*Let* A *be the centre of the earth (or of the sun, for Copernicus),* AI *the line of apsides,* AD *the eccentricity of the point of the equant. And although it was denied in chapter 19 that the point* D *could remain fixed, and* AD *the same, this is to be understood as true only if* DA *is bisected. But if we remain at liberty to divide* DA *as we please, as in chapter 16, then this point can remain fixed. Therefore, let* AD *be divided in the ratio found in chapter 16. Let the point of division be* C, *and let* AC *be 11,332,* CD *7232. And about centre* C, *with radius* CH *equal to 100,000, let the eccentric be described, as sketched out by the dotted line passing through* H. This, then, will be the hypothesis of chapter 16. *For, taking any known angle of mean anomaly, let a straight line,* DH, *be drawn from the centre of the equant* D *bounded by a point on the circumference, containing between itself and the line of apsides the required angle*, which is the measure of the proposed time. And let the point* H *be connected to* A. *The angle* IAH *will thus be the equated anomaly and the true zodiacal position of* AH, *and the planet most certainly will be on the line* AH *at the given time and anomaly, by ch. 16 and 18. But the distance* AH *will be false, and the planet will not be at the point* H, *because the division of* AD *at* C *and the eccentric* H *described about* C *are false by ch. 19, 20, and 42. There*
221 *it was shown that* AD *is to be bisected at* B, *so as to describe a more correct eccentric* IL *about* B, *but it will not be a perfect circle.* Now let

*In the vicarious hypothesis of chapter 16, this is the proper measure of the time, for it is placed at D, the point of the equant, in accord with the opinion of the ancients.

the other hypothesis be delineated. *And let* AD *be bisected at* B, *so that* AB *is 9282 (or, according to the numbers of ch. 42, 9264), and about centre* B *with radius* CH, *let another eccentric* IL *be described, which in this chapter I have also called a fiction*, for computing the correct distances. This is the one which in the last diagram was described as* δθλ, *about centre* β. *And let the mean anomaly, which was previously proposed to us in the form of time, be transferred from* D *to* B, *and the straight line* BF *be drawn from* B *parallel to the former* DH. *And let the point of intersection F of the new eccentric be connected with* A.

*It is true with respect to the shape that this was a fiction, since the path of the planet is not a circle. But with respect to position, and the centre B, it is not a fiction, but true: thus, this fiction described about B is the opposite of the prior fiction about C.

*Therefore, by what was said in this chapter 46,* AF *will be the distance of the planet at* F *from the centre of the sun at* A, *which the hypothesis of chapter 45 requires. But the angle* BAF *is false, and the zodiacal position of* AF *is false. For at the selected time and mean anomaly the planet is not found on* AF. *Before, however, the true line of the planet was* AH, *and the distance* AH *was false. So about centre* A *with distance* AF *let the arc* FG *be described, intersecting* AH *at* G. *Thus the line* AG, *constituted by two manifestly false hypotheses, is nevertheless true in its zodiacal longitude, and its length is consonant with the hypothesis of chapter 45.*

Thus through the vicarious hypothesis of chapter 16, which consists of the points A, C, D, and the eccentric H, we have made up for the defect of geometry, which was unable to show us the position of the line AG (onto which the correct distance AF is to be transferred) which we required of the hypothesis of chapter 45.

A fourth manner of description rejected.

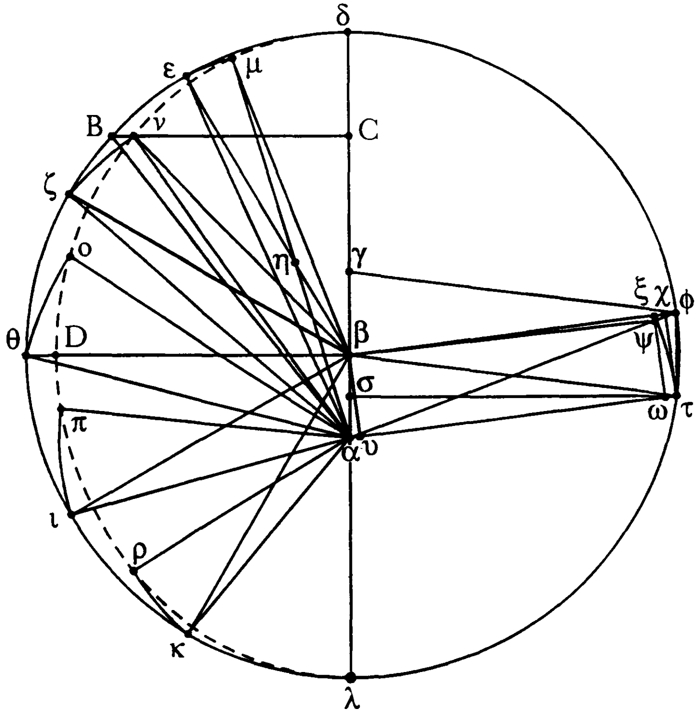
One might ask, 'Couldn't we, in both the former and the latter diagram, *take as given the point* γ *of uniform motion, and from it draw* γμ, γν, γο, γπ, γρ *parallel to* βε, βζ, βθ, βι, βκ, *and draw the arcs* εμ, ζν, θο, ιπ, κρ *intersecting these parallels? And then understand the points of intersection to be the determinate places and positions of the distances?*

The answer is no. For in so doing we shall err considerably in transferring the distances too high up, as is easily seen in the latter diagram. *For in it the line* AH *containing the true distances* AF *is lower than the line* DH *from the point of uniform motion* D *parallel to* BF.

What sort of oval is generated from these descriptions.

Durer

Whichever of the described ways is used for delineating the line possessing the body of the planet, it now follows that this way indicated by the points δ, μ, ν, ο, π, ρ, λ, is truly oval, not the elliptical one to which the mechanics give that name from the egg (*ovum*), contrary to correct usage. For an egg is rotated about two vertices,




one more blunt, the other sharper, and is visibly inclined at the sides. It is such a figure, I say, that we have created. *For the planet is fast at λ, slow at δ, and less fast at the former than it is slow at the latter. For there are more of the long distances exceeding the semidiameter than there are of the short ones (for they are longer up through 92⅔°*[4]*, and then shorter for 87⅓°, as can be demonstrated from the theory presented in chapter 29). But in addition, that greater number of long [distances] is crowded into a narrower arc of the eccentric by being translated upwards, while these fewer [shorter] ones are spread out into a larger arc. So that to a mean anomaly of 92⅔°*, which contains 92⅔° of distance*[5]*, there corresponds an eccentric anomaly of about 87⅓°. The remaining 87⅓° of mean anomaly, with the same amount of distances*
222 *shorter than the radius, is scattered over the remaining angle at the*

*It amounts to this much in the erroneous opinion of chapter 45, with which we are playing around here.

[4] Of mean anomaly.
[5] Because the sum of the distances is the area, which represents the time, which is the same as the mean anomaly.

*centre of the eccentric, 92⅔°. Consequently, the short distances near perihelion are farther from one another than are the longer ones at aphelion. So even if the ratio between two neighboring perihelial distances remained constant, the departure from the circle would nevertheless be more attenuated about ε, μ, δ than about ρ, κ, λ*[6]*. For the short ones are transposed into the position of the longer ones in a shorter space at δ than at λ. But in addition, the distances of the equal parts of the epicycle near to perihelion are in a greater ratio to one another than the distances of the parts near aphelion*[7]*. For it was demonstrated in chapter 40 above that the conchoidal area is wider in its lower part than in its upper. Therefore, the conchoid must be attenuated in greater steps over a shorter space at its lower point than at its upper, and in addition those greater intervals are compared to shorter lines. So on both counts the ratio is increased.* With so many causes concurring, it appears that the part cut off from our eccentric circle is much wider below than above, at an equal distance from the apsides. Anyone can easily explore this using numbers, or by a mechanical delineation, by assuming some appreciable eccentricity.*

*A figure of this kind is to be found in the manuals of spherics and the commentaries of Reinhold on the theories of Peurbach, in the theory of Mercury.

[6] This appears not to follow from the premises. All that has been established is that the distances are more closely packed near aphelion than near perihelion. If 'same ratio' here means a simple linear decrease, then the orbit's shape would be the opposite of what Kepler claims. For the changes of distance at aphelion and perihelion would be the same as they are everywhere else, in contrast with the changes under the epicyclic rule, which would be very small at aphelion and perihelion. This would create an outward-facing point at aphelion, and an inward-facing cusp at perihelion. If, on the other hand, 'same ratio' means 'the same as the ratio of decrease at aphelion' (as Kepler's next argument suggests), then we would only know that a given radial decrement occurs nearer to aphelion than to perihelion. And since the opposite occurs in the circle (as Kepler himself proved in ch. 40), this establishes that the departure from the circle is *greater* at aphelion than at perihelion, which is the opposite of what Kepler wished to prove.

[7] This is true, but its effect is only to counteract the tendency described in the preceding footnote. In fact, the asymmetry of the oval is so slight as to be beyond Kepler's powers of analysis (and beyond any significance). See D.T. Whiteside, 'Keplerian Planetary Eggs, Laid and Unlaid, 1600–1605', *Journal for the History of Astronomy* 5 (1974), pp. 1–21.

# 47

## An attempt is made to find the quadrature of the oval-shaped plane which chapter 45 brought forth, and which we have been busying ourselves to describe in chapter 46; and through the quadrature a method of finding equations

We have accomplished nothing if, from the hypothesis we have taken, and the physical causes of chapter 45, which we follow here as true, we shall not have constructed the correct equations, no less than the distances. But the equation is compounded from the parallaxes of the points on the eccentric and the elapsed time. The former of these I am accustomed to call the optical part of the equation, and the latter, the physical part. Now the elapsed time, even if it is really something different, is certainly measured most easily (if not most perfectly) by the plane area circumscribed by the planet's path. We therefore turn to the measurement of the plane area of the eccentric ovoid, the rules for delineating which have been laid down already. Now there is going to be something lacking in our account that prevents our stating the true measure of this time. For at the circumference of the ovoid

223 the lines that join the parts of its circumference with the source of power are even more inclined than on the circle. This is even true as well of the lines that are drawn from the centre of the eccentric to those same parts of the ovoid, although otherwise the radii from the centre to the circumference of a perfect circle make perfect right angles. But the consequence of this is that the sum of the distances is not exactly measured by the plane surface, nor are the arcs of the ovoid exactly proportional to the distances. All these things will be clear from a rereading of chapters 40 and 32. A guess as to how small this discrepancy is going to turn out to be, however, can be grasped from ch. 43.

Terms: What is the optical part of the equation, and what is the physical part.

And how else can we measure this plane surface, compare it to the

plane surface of a circle, and divide it into prescribed parts, unless we find a square equal to the trimmed-off part, or the lunule cut off? Here we will have to summon up from tragedy a *deus*, or rather a sort of *ratio, ex machina*, which will show us how to manufacture a quadrature of the ovoid, or of its border in the last diagram but one, the lunule δολθ, whose removal from the surface of the circle generated the ovoid δολ. And just as I called upon the geometers before in ch. 40 for the area of the conchoid, and begged their assistance, I do so again now for the ovoid (or, if you prefer, the 'metopoid'[1]).

Term:
An ellipse is the regular figure resulting from the cutting of a cone [by a plane] through its axis. Others say it is an oblong circle.

If our figure were a perfect ellipse, the job would have been done by Archimedes, who demonstrates in his book *On Spheroids*, prop. 6, 7, and 8, that the area of an ellipse is to the area of a circle sharing a common major diameter with the ellipse, as the rectangle contained by the diameters (or the 'figure'[2] of the section) is to the square on the circle's diameter.

But let the figure be a perfect ellipse, for they hardly differ. Let us see what follows.

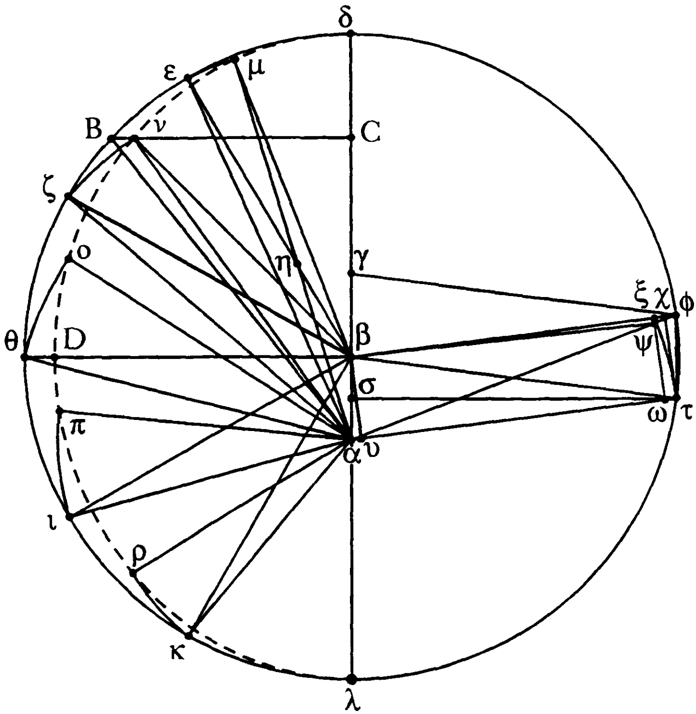
I say, therefore, that the lunule δολθ cut off from the semicircle will turn out to be imperceptibly greater than the small semicircle whose semidiameter is the eccentricity itself, αβ, or 9264. *For let αβ be bisected at σ (as in ch. 29), and from σ let στ go out perpendicular to αβ. Let α and β be connected to τ. Now let γφ be drawn parallel to βτ, and let βφ, αφ be joined. And about centre α with radius ατ let the arc τψ be drawn, intersecting αφ at ψ and βφ at ξ.*

Term:
What the mean longitude was for the Arabs. Today we incorrectly say that the mean longitude is the point on the circumference which has the mean length: that is, which is at the mean distance from the centre of the world.

*Now since the point τ is equally remote from α and β, we are (following the Arabs in using the term most properly) at the mean longitude, that is, at the average distance of the planet τ from the sun α. And because γφ is parallel to βτ, the point ψ on the line αφ (in the diagram of the previous chapter) is the genuine and most true position of the translation of ατ to αψ. Therefore ψ is also the point of the*

1 This is presumably from the Greek *metopon*, meaning 'forehead' or 'face', and would accordingly mean, 'face shaped figure'. Kepler is trying to find a word that means 'egg shaped' without suggesting an ellipse.
In the following treatment, Kepler approximated the oval with an ellipse having the same axes, supposing that the oval's excursions outside the ellipse are about the same as its incursions within it: thus, the areas of the oval and the ellipse would be very nearly equal. The trial ellipse, it should be noted, has an eccentricity greater than that of the true ellipse, and so the breadth of the lunule is greater than for the true ellipse. In fact, it is just twice the breadth of the true lunule, a circumstance which will play an important role in the further course of the investigation.

2 This is the 'area' mentioned in Apollonius's *Conics* I.13 and elsewhere, which characterizes the ellipse.




*planet's average distance. Hence, the little part of the line* $\beta\psi$ *between* $\psi$ *and the circumference is the measure of the breadth of the lunule about the middle longitudes, while the small line* $\xi\phi$ *is greater than this breadth by some imperceptible magnitude.*

*Let a perpendicular be drawn from* $\beta$ *to* $\alpha\tau$, *and let it be* $\beta\upsilon$. *I say that* $\xi\phi$, *a part of the line* $\beta\phi$, *is twice* $\alpha\upsilon$.

*For let* $\tau\phi$ *be drawn, and from* $\tau$ *let* $\tau\chi$ *come out perpendicular to* $\beta\phi$. *Similarly, from* $\xi$ *let* $\xi\omega$ *come out perpendicular to* $\alpha\tau$. *Since the straight line* $\alpha\gamma$ *intersects the parallels* $\gamma\phi$, $\beta\tau$, $\beta\gamma\phi$ *and* $\alpha\beta\tau$ *will be equal. And* $\gamma\beta$ *is equal to* $\alpha\beta$ *by construction. But also,* $\beta\phi$ *is equal to* $\alpha\tau$, *for both are equal to* $\beta\tau$ *by construction. Therefore, triangle* $\gamma\phi\beta$ *is congruent with triangle* $\beta\tau\alpha$. *Thus,* $\gamma\phi$ *will be equal to* $\beta\tau$. *But they are parallel by*
224 *construction. Thus* $\beta\gamma$ *and* $\tau\phi$, *which connect the ends of equal parallels on the same side, will also be parallel and equal. But* $\beta\gamma$ *is also equal to* $\alpha\beta$. *Therefore,* $\alpha\beta$ *and* $\tau\phi$ *are also equal and parallel. Consequently,* $\beta\phi$ *and* $\alpha\tau$ *will also be parallel. And because the angles*

*at $\chi$ and $\upsilon$ are right, and the base $\tau\phi$ is equal to the base $\beta\alpha$, and the angle $\beta\alpha\tau$ or $\beta\alpha\upsilon$ is equal to the angle $\tau\phi\beta$ or $\tau\phi\chi$, $\alpha\upsilon$ and $\chi\phi$ will therefore be equal, as well as $\beta\upsilon$ and $\tau\chi$ perpendicular to them.*

*Again, because the lines $\tau\chi$ and $\xi\omega$ are equal, being parallels between parallels*[3]*, while $\beta\tau$ and $\alpha\xi$ are equal, and the angles at $\chi$ and $\omega$ are right, the remaining sides of the triangles $\beta\chi$ and $\alpha\omega$ will also be equal. But $\beta\xi$ and $\upsilon\omega$ are also equal, for they are parallels between the parallels $\beta\upsilon$, $\xi\omega$. Therefore, when the equals $\beta\xi$ and $\upsilon\omega$ are subtracted [from $\beta\chi$ and $\alpha\omega$], the remainders $\xi\chi$ and $\alpha\upsilon$ will be equal. But before, $\chi\phi$ and $\alpha\upsilon$ were also equal. Therefore, $\xi\phi$ is twice $\alpha\upsilon$.*

With these things demonstrated, we draw nearer to our proposition. *And to the diameter $\phi\beta$ of the circle (which should be understood to be extended to the other circumference), a straight line is drawn perpendicular from a point $\tau$ on the circumference, namely, $\tau\chi$. Therefore, as $\phi\chi$ is to $\chi\tau$, so is $\chi\tau$ to the remainder of the diameter. Therefore, the rectangle contained by $\chi\phi$ and the remaining part of the diameter is equal to the square on $\tau\chi$.*

*And because the square on $\tau\phi$, that is, $\alpha\beta$, is equal to the [sum of the] squares on $\tau\chi$ and $\chi\phi$, when equals are added, the rectangle contained by $\chi\phi$ and the entire diameter is equal to the square on $\alpha\beta$.*

*And because $\phi\xi$ is twice $\phi\chi$, the rectangle contained by $\phi\xi$ (which is imperceptibly greater than the breadth of the lunule $\upsilon\phi$) and the semidiameter $\phi\beta$ is equal to the square on $\alpha\beta$.*

*But the rectangle contained by $\xi\phi$, $\phi\beta$ is the difference of the rectangle contained by $\xi\beta$, $\beta\phi$ and the square on $\beta\phi$. And the lunules are also the difference between the areas of the ellipse and the circle. And as the rectangle contained by $\xi\beta$, $\beta\phi$ is to the square on $\beta\phi$, so is the area of the ellipse to the area of the circle, approximately*. Therefore also, as the square on $\beta\phi$ is to the rectangle $\xi\phi$, $\phi\beta$, that is, the square on $\alpha\beta$, so is the area of the circle to the area of the two lunules, approximately. And by permutation, as the square on $\beta\phi$ is to the area of the circle, so is the square on $\alpha\beta$ to the area of the lunules, approximately.*

*But as the square on $\beta\phi$ is to the area of the circle of which $\beta\phi$ is the radius, so is the square on $\alpha\beta$ to the circle of which $\alpha\beta$ is the radius. Therefore, the area of the circle of which $\alpha\beta$ is the radius imperceptibly exceeds the two lunules $\psi\phi$ cut off. It exactly equals the lunules $\xi\phi$*

*Approximately, I say. For if $\beta\xi$ were the shorter semidiameter of the ellipse, and $\xi\phi$ were the excess of the longer, then the ratio between the areas of the circle and the ellipse would be exactly the same. But $\beta\xi$ is not completely and exactly the shorter semidiameter.

[3] This is not immediately obvious, but is true because each of the lines is perpendicular to one of the pair of parallel lines $\alpha\tau$, $\beta\phi$.

*which are a little wider than they should be, because $\xi\phi$ is imperceptibly longer than $\psi\phi$, as was said at the beginning.*

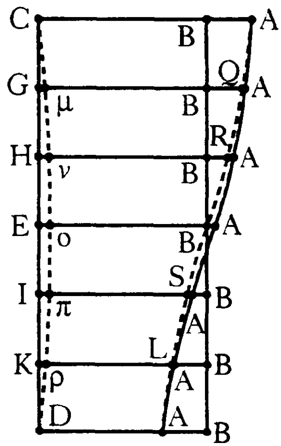
This demonstration also has its use in the true physical hypothesis.

So, granted what we have supposed, namely, that the area of the ellipse differs imperceptibly from the area of our ovoid, as a result of the compensation between the excess of the ovoid over the ellipse in the upper regions, and the defect in the lower regions, – these, as I said, being granted, we have squared our 'new moon' figures, and thus also the ovoidal one. Or, properly speaking, we have 'circled' it. For Archimedes teaches us the ratio of the circle and the square.

225 We shall now put this to use, thus. Since the area of the ovoid is less than the area of the circle by the area of the small circle described by the eccentricity, let the area of this small circle be computed next. *Now the ratio of the areas is the duplicate of the ratio of the diameters. And as $\beta\phi$, 100,000, is to $\beta\alpha$, 9264, so is $\beta\alpha$ to $\xi\phi$, 858. Therefore the ratio between $\beta\phi$ and $\xi\phi$ is also the duplicate of the ratio between $\beta\phi$ and $\beta\alpha$. Therefore, as $\beta\phi$, 100,000, is to $\xi\phi$, 858, so is the area of the circle, 31,415,900,000, to the area of the small circle, 269,500,000.*

Therefore, when the area of the small circle is subtracted, the remainder is the area of the ovoid, 31,146,400,000, equivalent to 360 equal parts of the periodic time.

Those things which have been said so far are entirely consonant with the opinion of chapter 45. Nevertheless, to use them, it is not enough to know the magnitude of the area of the ovoid. Indeed, we need also understand how to divide it, from the centre $\beta$ or the point $\alpha$, in any given ratio. *For example, in the previous diagram, let the point $\theta$ be taken, and let the planet be observed on the line $\alpha\theta$, but let it recede from the circumference $\theta$ towards the sun $\alpha$. Therefore, given the eccentricity $\alpha\beta$ and the angle $\theta\alpha\beta$, and supposing that the planet is at the point $\theta$ of the circumference, the angle $\theta\beta\delta$ will be given, and hence also the sector of the perfect circle, namely, $\theta\delta\beta$, and the area of the triangle $\theta\beta\alpha$, that is, the whole area $\theta\delta\alpha$ which (with the exceptions in chapter 40 above) ought to have been the measure of the time elapsed while the planet moves from $\delta$ to $\theta$, if the planet had gone in a perfect circle. But because it describes an oval inside the circle, not embracing the full area of the perfect circle, so, exactly as just a moment ago we needed knowledge of the area of the whole ovoid, we now need to know what portion of the ovoid is contained between the lines $\delta\alpha$ and $\alpha\theta$, that is, what portion the area of the part $\delta\theta$ of the lunule is of the area that measures the two lunules, namely, the area of the circle on the eccentricity. For when this is subtracted from the portion of the circle*




*cut off by the lines* $\alpha\theta$, $\alpha\delta$, *the remainder is the portion of the ovoid cut off by the same lines* $\alpha\theta$, $\alpha\delta$. *Thus, finally, the whole oviform will be correctly compared to its part* $\delta\alpha\theta$ *in order to find the time, or the retardation of the planet, which occurs between the lines* $\alpha\delta$ *and* $\alpha\theta$.

Once again, now, where is a geometer who will show us how to do this? *Let the last diagram of chapter 40 be presented again, in which* CD *is the semicircle stretched out into a straight line, divided into equal parts, and* DE *is a quadrant. And on the line* EA *from* E *let some line [*Eo*] be extended towards* A *which bears the ratio to the longest line* BA *(the one on the line* CA*) which that* BA *has to* BC. *And let the rest,* Gμ, Hν, Iπ, Kρ, *be set up similarly in the appropriate magnitudes, having the breadth of the lunule for their respective places, so that* Gμ *is a little shorter than* Kρ, *and* Hν *shorter than* Iπ *(although they are the same distance from* C *and* D*), in accordance with what was demonstrated in ch. 46. Thus the lunule will be unfolded partwise onto a straight line, and so delineated as to show the abbreviation of the distances.*

*And because the whole space between* CD *and* AA *is twice the area of the stretched-out semicircle* CD, *the geometer should consider whether the small space between the curve* CμνοπρD *and the straight line* CED *is also going to be twice the lunule cut off from the area of the circle.*

226 *Nothing appears to contradict the possibility of this being so. For when the lunule really is a lunule,* CD *is then curved inwards while*

*retaining its same length. But* Cμνοπ$\rho$D, *which was just constructed longer than* CED, *is then much shorter. Therefore, the lunule then contains a much smaller area than now.* But, O geometers, to say this is not to demonstrate it. Therefore, you will assist me. And if this turns out to be true, you will teach me a method by which may be known the magnitude, not only of the whole small area between the straight line CED and the curve CoD, which I have so far said is equal to the small circle on the eccentricity (for two lunules are equal to the small circle, and this small area is now supposed to be twice one of the lunules), but also of any part of it, at any given length of the parts CG, CH; and by which this may be compared to the area between CD and BB.

So once again, as before in ch. 46, since there is no way out through geometry, we shall be content with a contrivance. And no wonder, for the opinion born in ch. 45, which threw us into these difficulties, is false.

*Therefore, let the previous diagram from chapter 46 be considered again. If the area* δολ, *which is an ovoid, were a perfect ellipse, when the ellipse* δολ *and the circular area* δθλ *are described on the common longer diameter* δλ, *and the planes of the two figures are divided by lines* BC *applied ordinatewise (that is, perpendicularly to the longer diameter* δλ*) from one side of the longer diameter, the portions of the ellipse* νδC *would always remain in the same ratio to the portions of the circle* BδC. *This is demonstrated by the authors who wrote on conics, and Archimedes takes it over in* On Spheroids *prop. 5. If this were so, there would indeed be no need to know the oviform area. For then we would substitute the area of the circle for the area of the ellipse, and the parts of the circle for the similar parts of the ellipse.*

*Let it be that* δολ *is a perfect ellipse, for it is but slightly different from one. And from any point on the ellipse,* ν *say, let a perpendicular be dropped to* δλ, *which let be* νC, *and let it be extended until it intersects the circle at* B. *And let* B *and* ν *be joined to* α. *Now as* βφ *is to* βξ *so is* CB *to* Cν, *from the assumption of a perfect ellipse and prop. 5 of* On Spheroids, *and also as* BC *is to* Cν *so is the area* BδC *to the area* νδC. *But also as* BC *is to* Cν, *so is the area* BαC *to the area* ναC. *Therefore, as* βφ *is to* βξ, *so is the area* αBδ *to the area* ανδ.

Where the time is to be numbered for finding the distance of the planet from the sun.

*First, let it be, at the planet's proposed time of departure from* δ, *that as the periodic time is to four right angles, so is the proposed time to the angle about* β (δβζ, *say), and let the distance* αζ, *to which* αν *is equal, be computed.*

*Again, let it be that as half the periodic time is to the known area of the semicircle* δθλ, *so is the proposed time (whose measure we have just*
227 *now said is something else,* δζ, *when the distance* αζ *was computed) to the area* αBδ. *Thus the area is given. Now a value for angle* Bβδ *must be found, such that its sine* BC *multiplied by half* αβ *(that is, the area of the triangle* αBβ*), together with the sector* Bβδ, *will add up to equal the area just found from the time. Here one has to proceed by trial and error [*conjectatione et regula falsi opus est*]. When you have obtained the angle* Bβδ, *in triangle* Bβα, *from the angle* β *and the known sides* αβ, βB, *the angle* Bαδ *next becomes known. And because the ratio* Bν *to* BC *is known,* Bαν *will also be known, and when it is subtracted, there will remain* ναδ, *the correct equated angle for the time selected.*

Where the time is to be numbered for finding the eccentric equation.

*For example, as in ch. 43, let the mean anomaly, that is, the artificial or astronomical numbering of time, be 95° 18′ 28″. And because 360° is equivalent to the area of the perfect circle, 31,415,926,536, 95° 18′ 28″ will therefore be equivalent to the area 8,317,172,671. Let this be* θαδ. *Now, if the eccentric anomaly* δθ *is 90°, as I suppose conjecturally, its sector* θβδ *would be 7,853,981,670. And the sine* θβ *of 90° is 100,000. This multiplied by half the eccentricity* αβ, *4632, gives 463,200,000 as the area* θβα. *The sum of the areas is 8,317,181,670, which is* θαδ, *and which exceeds what it should be by some small amount. We therefore guessed well that the angle or anomaly of the eccentric* δβθ *is 90°. And because its sine is 100,000, the lunule* θD *cut off at* θ *will be 858. Therefore, the shorter semidiameter* Dβ *will be 99,142, which is to 100,000 as 9264 is to 9344. This is the tangent of the angle* αDβ, *5° 20′ 18″, making the equated anomaly* Dαδ *84° 39′ 42″. The vicarious hypothesis shows this to be 84° 42′ 2″, the difference being 2′ 20″.*

Term: What the 'mean anomaly' is.

The method of equating used here should be noted. For in the end we are going to follow it when it is established that the path of the planet is a perfect ellipse, but nearer by half to the circle. The distance alone will be found by another method.

The investigation of the eccentricity in ch. 42 depends upon aphelial and perihelial distances, and in these there can be some slight error which is increased tenfold in the establishing of the eccentricity. Therefore, it should be noted in passing that if a perfectly reliable way of equating through the physical causes is finally found, a perfectly true eccentricity can afterwards be established, and through it the aphelial and perihelial distances can be entirely corrected. *For example, provided we can trust the vicarious hypothesis for the planet's zodiacal longitude, and suppose that everything we have assumed here and in ch. 45 is true, the equation has been made 2′ 20″ too large here, while the optical and physical effects upon the equation at the middle longitudes are equal, as here. So bisect the error, the half being 1′ 10″. When this is subtracted from the angle last found, 5° 20′ 18″, making it*

A method of correcting the eccentricity, inserted in passing.

*5° 19′ 8″, this shows a tangent of 9310. Before, it was 9344. The difference of 34 subtracted from the eccentricity 9264 leaves a corrected eccentricity of 9230.* But we do not follow this now, since our assumptions are wrong on the very small quantities. It should be enough for us to advise ourselves of its future usefulness in the chapters following next.

Let us also see what promise this form of computing equations holds at the eighths of the periodic time. Let the mean anomaly be 48° 45′ 12″, as in ch. 43. And since the numerical measure by which the areas are expressed is a matter of indifference, we shall retain the number 360° for the area of the circle, and 19,108″ for the area of the greatest triangle (for just now, in the other system of numbering, it was 463,200,000). *Let us guess that the anomaly of the eccentric, or* $B\beta\delta$ *in the diagram, is 45°. Therefore, the sine,* BC, *is 70,711. This, multiplied by the greatest triangle 19,108″ and with the noughts dropped, gives the triangle* $B\alpha\beta$ *as 13,512″, or 3° 45′ 12″ for this location. This, added to the sector* $B\beta\delta$, *45°, gives 48° 45′ 12″ for the area* $B\alpha\delta$, *which is also the mean anomaly we assumed. We therefore*
228 *guessed the angle* $\beta$ *well. Now, as the radius* $\beta\phi$ *is to* $\beta\xi$, *99,142*[4], *so is* BC, *70,711, to* $Cv$, *70,104. And because* BC *is 70,711,* $C\beta$, *the sine of its complement, will also be 70,711 at this location. Therefore,* $C\alpha$ *is 79,975. But as this is to 100,000, so is* $Cv$ *to the tangent of the required angle* $v\alpha C$, *41° 14′ 9″. The vicarious hypothesis shows 41° 20′ 33″.*

The same things are also easily investigated in the lower octant. *Let the mean anomaly be 138° 45′ 12″, and let the area, whose angle at* $\alpha$ *is sought, be expressed in the same units. We will find that the sine of an angle of 135° at* $\beta$, *which is 70,711, makes the sum of the sector and the area of the triangle this much. And because, as before, the sine 70,711 is shortened in order to constitute the lines of the ellipse that are applied ordinatewise, becoming 70,104, this is now to be combined with the sine of the complement of 135°, which is 70,711, now decreased rather than increased by the eccentricity* $\alpha\beta$, *making it 61,447. Thus, as this is to 100,000, so is 70,104 to the tangent of the required angle, 48° 45′ 55″, or its supplement 131° 14′ 5″. The vicarious hypothesis shows 131° 7′ 26″.* Compare this with ch. 43, and with other methods, using this table.

So, of the two physical hypotheses for computing the eccentric

[4] Since $\beta\xi = \beta\phi - \xi\phi$, and $\xi\phi$ was found earlier to be 858.

| Common mean anomalies | Through a simple eccentricity | Through bisection of the eccentricity and the doubling of the upper part of the equation | Through bisection of the eccentricity and a stable equant point, in the Ptolemaic manner | The vicarious hypothesis using a free division, practically in agreement with truth | Physical hypothesis, through the assumption of a perfect circle | Physical hypothesis through the supposition of the opinion of chapter 45 and of a perfect ellipse |
|---|---|---|---|---|---|---|
| | The various corresponding equated anomalies | | | | | |
| 48° 45′ 12″ | 41° 40′ 14″ | 40° 45′ 52″ | 41° 15′ 31″ | 41° 20′ 33″ | 41° 28′ 54″ | 41° 14′ 9″ |
| 95 18 28 | 84 40 44 | 84 37 48 | 84 41 22 | 84 42 2 | 84 42 26 | 84 39 42 |
| 138 45 12 | 130 40 46 | 131 45 0 | 131 15 31 | 131 7 26 | 130 59 25 | 131 14 5 |
| | Ch. 20 and 29 | The excess and defect go in the opposite direction if the lower part is doubled. Ch. 29 | Ch. 19 | Ch. 16 and 29 | Ch. 43 and 29 | The present Ch. 47 |

You will note that the truth is exactly in the middle between these.

By these indications we are assured that we are on the track which will carry us through at last to the natural and most true causes of the equations, as well as of the celestial motions.

equations, that one shows equations nearer the truth, which previously, in ch. 45, also gave truer distances, namely, the last one. And, what may seem strange, by a slight increase in the eccentricity it becomes equivalent to the Ptolemaic method, using a stable equant point and a bisected eccentricity.

And since we convicted this Ptolemaic method of error above, the physical method, which is in effect the same as the Ptolemaic, must also be somewhat askew of the truth. And it does indeed make the planet slow near the apsides, and too swift about the middle longitudes. This is the first argument by which it is proved that either the opinion of chapter 45 is erroneous, or it has been transposed into numbers by an erroneous method.

But because the plane of the circle is not equivalent to the aggregate of all the distances, nor is the oval figure that, according to the opinion of ch. 45, Mars describes, a perfect ellipse, as we have
229 assumed, the causes for the departure from the truth are still hidden. For in addition to these two concurring errors in the calculation, there can be a third, relating to its foundation, the opinion expressed in ch. 45. So we have not yet set up equations according to the law of the opinion of ch. 45, and have not yet done justice to the hypothesis we took up there, because we have been abandoned by geometry. Therefore, we still cannot charge it with being erroneous. For the computation which is going to do this will declare for itself the rule by which its innocence will be judged.

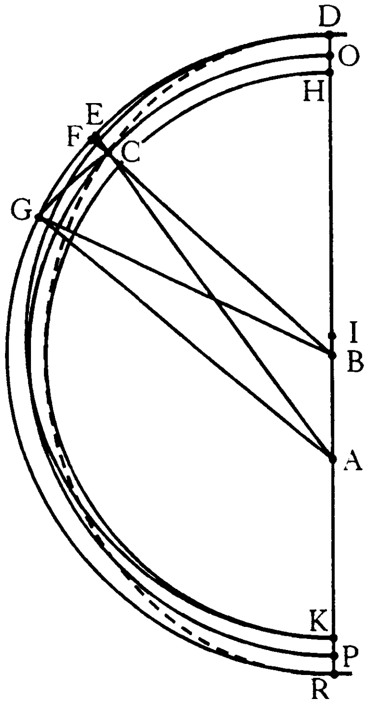
# 48

## A method of computing the eccentric equations by a numerical measure and division of the circumference of the ovoid described in chapter 46

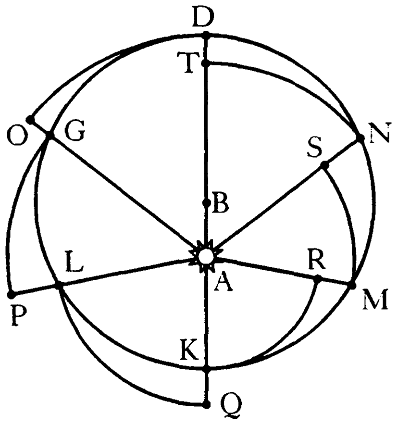
So, since the calculation taken up in the preceding chapter was abandoned by geometry on so many counts, so as to become suspect of being responsible for the excesses and defects which we noted in the eccentric equations in that chapter, I finally sought refuge in the numberings of arithmetic, by which I attempted to avoid the obstacles which stood in the way of our describing the path of the planet in chapter 46. For first, because the plane was not the perfect measure of the sum of the distances, I dismissed the plane and computed the distances of individual parts of the equally divided circumference. Second, since the ratio did not remain the same when any given number of terms of the geometrical proportion were added, I therefore determined each individual proportion separately for each distance in relation to its minimal arc. Third, since the sum of any particular number of distances in ch. 46 could not be established geometrically, I established it arithmetically. For there was no difficulty in that. Fourth, in my procedures I had no dealings with sectors, whether circular or oval, and therefore it could obviously no longer be a hindrance to me that these sectors differ from one another.

And so, with renewed preparations, I settled down again upon the eggs, in order to know at least at the end whether the equations shown to us by the vicarious hypothesis also follow from the hypothesis under consideration, which gives the correct distance (that is, the one following the opinion of chapter 45.)

I approached the matter thus. *About centre* B, *with radius* BD, *let*




*the circle* DGR *be described, in which the line of apsides is* DR, *and* A *is the source of power or the centre of the sun. On the circle* DG *let the point* G *be chosen, and joined to B* and A. *And at first, let the angle* GBD *be the measure of the time, for computing the distance. Consequently,* GA *will be the true distance of the planet from* A, *although the planet has not moved all the way from* D *to* G. *For this method of computing or demonstrating the distance was one of the presuppositions of ch. 45. But let* DG *be a small part of the circle, such as 1° out of*
230 *the 360°. And since all the distances* AG *of this sort at the ends* D *and* G *of all the degrees* DG *can be computed in this way, by what was demonstrated in chapter 29, I gathered all the 360 distances* AG *into one sum, in a very long addition. This was found to be 36,075,562 (with an eccentricity of 9165), corresponding to the entire oval path of Mars. Now about centre* A, *with radius* AG, *let an arc,* GC, *be described towards* D. *And because the greater the distance the shorter the planet's path, when the distance of an arc of the circle* DG *is given (which arc is now, when we are computing the distance* GA, *nothing*




*but the measure of the time), the length of the oval path* DC *will also be given, which the planet describes in the time* DG *under consideration (which is a simple anomaly of 1°). For as the length of the whole of the oval circumference is to the sum of all the distances, so is the distance of the arc* DC *(found by the arc* DG*) to the length of its oval arc* DC. *For it was proved in ch. 33 above, and used in chapter 46 (where the foundations for this operation were laid) that the arcs traversed are inversely as the distances*[1]. *However, I applied this precaution: that* AD *and* AG, *the distances of the ends* C *and* D *of the arc from* A, *be added, and the mean of the sum be taken as the genuine distance of the whole arc* DC. *For let some eccentric circle* DK *described about centre* B, *be divided into any number of parts, say, at* D, G, L, K, M, N, *and from the beginnings of the parts let arcs be drawn about the centre of the world* A *until they intersect the lines drawn from* A *to the ends of the arcs, as* DO, GP, LQ, KR, MS, NT. *The areas on the left semicircle,* ADO, AGP, ALQ, *will be greater than they should be, while the arcs on the right,* ANT, AMS, AKR, *will be less than they should be. So, when least arcs are in question, the one is compensated by the other, so that* TNA *and* ODA *are approximately equal to the area* GDNA.

Thus, the length DC in the previous diagram being given, which corresponds to the given time DG and to the distance GA or CA, the

[1] Since the distances are *inversely* proportional to the arcs, there is nothing constant about the ratio of any arc to its distance, nor about the ratio of their sums. For instance, near aphelion the arcs are small and the distances large; therefore, the ratio of their sums will be less than the ratio of the sums nearer perihelion, where the arcs become longer and the distances shorter. So Kepler is wrong, both here and on p. 485, near the bottom.

angle of equated anomaly CAD should now also be found. *Let* C *be joined to* B, *and let* AC *be extended to* E, *where it intersects the circle, and* BC *to the intersection* F. *It was therefore not enough to know the length* DC. *The angle* CBD *ought to have been investigated as well. For because* CD *is shorter than* FD, CD *does not measure the angle* FBD, *or* CBD. *And again, even though* CD *is shorter than* FD, *if you placed an observer at* B *it would appear from* B *to be just as much a measure of the angle* CBD *as would* FD. *But according to the demonstrations of chapter 32, it is true within all limits of sense perception that to the extent that* FD *is farther from* B *than is* CD, FD *is also longer than* CD. *Also, it is true within the same limits of sense perception (for the present purposes it does not matter how acute) that* CE *and* CF *are equal (though in the truth of the matter* CE *is longer than* CF, *which is drawn through the centre, by Euclid Book III prop. 7). I have therefore supposed, first, that* CD *and* FD *are equal, and*
231 *both are a measure of the angle* CBD *or* FBD, or also EBD, *as if the arc* EF *were imperceptible. Thus, from knowing* CD, *the angle* EBD *was given. Therefore, in triangle* EBA, *from the angle* EBA *and the sides* EB, BA, *I sought the length of* AE, *whence I subtracted* AC *or* AG *computed earlier, and the remainder was* CE *or* CF, *the amount that the other end of* CD *approached the centre* B. *So, when* CE *is bisected (for this is allowed when perceptibles are in question) the approach of* CD *towards* B *was known, if it were to approach equally at all its points. But from the amount of approach, the optical parallax, or the apparent size of* CD, *was also given: that is, the angle* CBD *now corrected, which was previously assumed to be a little smaller, though in our figures there is no error. Therefore, given the angle* CBD, *now corrected, which is the supplement of angle* CBA, *and the side* CA, *and the eccentricity* BA, *the equated anomaly* CAD, *which was sought, was given.*

It was impossible to use this method to establish independently any equation other than the first, at a mean anomaly of 1°. All the rest, all the way to the 180th, always presupposed that the equation immediately preceding was known. I can't imagine anyone reading this not being overcome by the tedium of it even in the reading. So the reader may well judge how much vexation we (my calculator and I) derived hence, as we thrice followed this method through the 180° of anomaly, changing the eccentricity each time.

But the foundation of this calculation has not yet been laid out. For I said that I presupposed knowledge of the length of the whole oval.

Whence, then, is this known? As for me, once I had descended to this clumsy numbering procedure, I did not manage to avoid clumsily presupposing the length, and then, when the whole thing was complete, seeing whether in the 180th operation I came out with an apparent position of more than 180°, or less. For if it came out at exactly 180°, I knew that the length I had assumed for the oval was good, but if it was less, I had assumed it to be too small, and if more, greater.

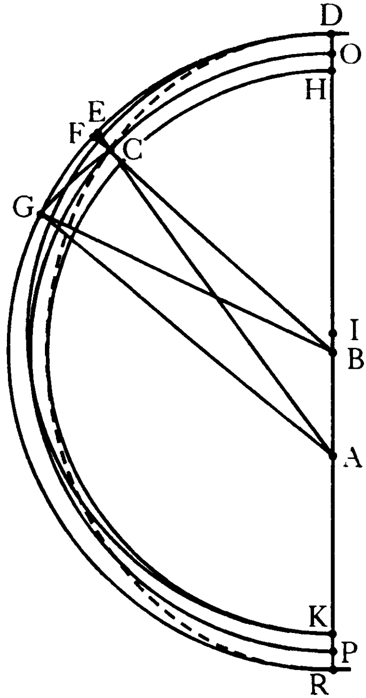
Nevertheless, we are not destitute: we have a kind of geometrical helping hand for making a good guess as to the oval's length. *For as* BD *is to* BA *let* BA *be to* DH, *extended from* D *towards* B. *By what was demonstrated in chapter 46, the rectangle contained by the breadth of the lunule and the semidiameter of the circle is nearly equal to the square on the eccentricity. Therefore, by Euclid VI. 17, the eccentricity is the mean proportional between the breadth of the lunule and the semidiameter. But this is how the diagram is set up. Therefore,* DH *is the breadth of the lunule.*

*Let the half of* DH *also be taken, and extended from* B *towards* D, *and let it be* BI. *And about centre* I, *with radius* ID, *let the circle* DK *be described, tangent to the eccentric at* D. *And also, about centre* B, *with radius* BH, *let the circle* HK *be described, tangent to the previous circle at* K. *It is obvious that the circle* HK *is smaller than* DK, *and the circle* DGR *is greater than* DK. *And because circular circumferences are to one another as their semidiameters, as* BD *is to* DI *and* BH, *so is the greater circle* DG *to the smaller circles* DK *and* KH. *But* DI *is the arithmetic mean between* DB *and* HB, *because* BI *is half of* HD. *Therefore, the circle* DK, *tangent to both the smaller and larger circles described about the same centre* B, *is also the arithmetic mean between those circles to which it is tangent.*

*If the oval path is continued, by hypothesis it will be tangent to the greater circle at aphelion* D *and perihelion* R, *and to the smaller circle*
232 HK *at the middle longitudes. Thus, it is greater than the smaller circle* HK, *and smaller than the greater circle* DR. *It is therefore likely that the oval circumference is not much different from the length of the circular circumference* DK.

The following demonstration, however, makes this more credible.

*Let the mean proportional between* BH *and* BD *be taken, and let it be* BO, *and about centre* B *with radius* BO *let the circle* OP *be described. Thus, by Archimedes,* On Spheroids *5, the area of this circle* OP *will be equal to the area of an ellipse whose longer semidia-*




*meter is* BD, *and whose shorter is* BH. *And because the greatest of all figures of equal perimeter is the circle, conversely (in the common significance of the term), of figures with equal area, the one with the shortest perimeter will be the circle. Therefore, since the proposed figures, namely, the ellipse which has semidiameters* DB, BH, *and the circle* OP, *are equal in area, from the evidence just presented, the circumference of the ellipse will be longer than the circumference of the circle* OP. *But* BO *is imperceptibly less than* ID, *since* BO *is taken as the geometrical mean, and* ID *the arithmetic mean, between the same terms. For by the theory presented in Euclid's Book V, since* BO *is the mean proportional between* HB, BD, *as the lesser,* HB, *is to the greater,* BD, *so is* HO, *the excess of the mean, to the defect,* OD[2]. *Therefore, since* HB *is less than* BD, HO *will also be less than* OD. *But* BI *is equal to the half of* HD. *Therefore,* BI *is greater than* HO, *and less than* OD. *Therefore, to the common semidiameter* HB *of the least*

[2] This proportion is incorrect. It does not follow from HB:BO::BO:BD that HO:OD::HB:BD. But from HB<BD it follows correctly that HD<OD, and this inequality is what Kepler needs.

*circle* HK *unequals are added, namely, less than the half of* DH *to make* BO, *and half of* DH *to make* DI. *Therefore,* DI *is greater than* BO. *Thus the circle* DK *is greater than* OP. *But it is only imperceptibly greater, since* DH *is less than a hundredth of* DB. *Therefore, on the supposition that these circles are practically equal, and on the supposition that the oval is a perfect ellipse, the circumference of the oval will be a little longer than the circle* DK, *and certainly longer than the circle* OP. *And because in ch. 47 above* DH *was 858 where* DB *was 100,000, let half of* DH, *429, be subtracted from* DB, *100,000. The remainder will be 99,571. Next, as 100,000 is to 99,571, so approximately will the circumference of the circle be to the circumference of the oval, which is sought. And because the circumference of the circle has 360 degrees or 21,600′, or 1,296,000″, a small part will be removed which has 5560″ or 92′ 40″. And for the semi-circumference of the oval, 46′ 20″ are to be subtracted, or even less, if the oval exceeds the circle* DK *at the place considered in the measuring.* As for me, I made no use of any demonstration, but by a most laborious and dogged calculation found the defect of the oval semicircle to be 45′ 45″, so that where the perfect semicircle is 180°, the oval is 179° 14′ 15″.

Now this shortening of the oval circumference is necessarily equal to the opposite optical lengthening (for although this oval is shorter, it nonetheless appears contained within two right angles, or exactly 180 degrees, and is judged to be that long). Hence the reader may with good reason doubt whether in this process it was necessary first
233 to shorten the entire oval, and afterwards to lengthen it again by parts optically. For from the diagram it would seem that the shortening is at its greatest where the approach towards the centre B is greatest, and vice versa.

If in fact these variations did happen to be equal, the following method would be set up by which we could compute the equations.

*At first, let the mean anomaly be* GBD, *from which the distance* GA *may be computed, which, added to* AD, *the distance of the other end of the preceding from* GD *(which is always 1°), and the sum being halved, gives the uniform distance of the arc* CD *(the same for all its points). And we would then say that as the length of the semicircle is to the sum of all the distances on the semicircle, so is this distance of the arc* GD *to the length of* FD, *which is the apparent size of* CD *seen from* B. *Now from* FD, *as if it were a measure of the angle* CBD, *and from* AC, AB, *we would find the equated anomaly* CAD *for a shorter path than before.*

But the reader should know that these two variations do not walk at the same pace. For the optical amplification which arises from the approach of the path DC to the centre B, happens chiefly about the middle longitudes, and hardly at all at aphelion and perihelion. Contrariwise, the shortening of the oval path, which arises from the incursion of the planet towards the centre, is about the same everywhere. For two opposite distances at the middle longitudes of the eccentric add up to the sum of two near the line of apsides, one near aphelion and the other near perihelion. But the arcs of the oval circumference are in the inverse ratio of the distances. Therefore, two of these arcs, at the middle longitudes, will be equal to two other arcs, one near aphelion and the other near perihelion. If these arcs of the oval path are equal, the diminution of the arcs at all four places will be approximately equal also. This is confirmed by experiment. For if the defect of the oval semicircle is 45′ 15″, the defect of the 180th part of the oval would be about 14″ about aphelion. And the amplification from the approach of the oval does not equal one second at aphelion.

So, as concerns the proposed ocular estimation of the diagram, what was said in the objection is not simply true, that the shortening of the oval and its optical amplification compensate one another. This would indeed be so if all the arcs of the oval path were presented directly when seen from centre B. But this happens only at the middle longitudes. Near the apsides, on the other hand, these arcs are not at the same distance at both ends. Therefore, they are not made to be as much greater in their appearance, by approaching, as they are made smaller by being shortened.

So, following this method, I constructed equations for Mars at all degrees of the eccentric, and I did it three times. For the first time I took an eccentricity, 9165, which was not great enough, thinking that I would thus be more certain in my treatment of the areas. And then I used more than 180° in the figuring, when I should have used less.

So, when this last operation showed more than 180°, which was
234 absurd, for the second trial I assumed half the oval to be 179° 14′ 15″. At a mean anomaly of 45°, the result was:

| | | | |
|---|---|---|---|
| equated anomaly . . . . . . . . . . . . . | 38° | 5′ | 33″ |
| while the vicarious hypothesis of chapter 16 said . . | 38 | 4 | 54 |
| Difference | | | 39 |

| | | |
|---|---|---|
| At anomaly 90 – equated anomaly . . . . . . . | | 79° 31′ 31″ |
| The vicarious, index of truth . . . . . . . . . . | | 79 27 41 |
| | Difference | 3 50 |

| | |
|---|---|
| At anomaly 135. Equated . . . . . . . . . . . . | 127 0 1 |
| The veracious vicarious . . . . . . . . . . . . . | 126 51 9 |
| Difference | 8 52 |

And from this, chiefly from the anomaly of 90°, I realized that the eccentricity of 9165 was too small. This I corrected, using the method presented in passing in the preceding chapter. For seeing that at the middle longitudes we have 3′ 50″ too much in the greatest equations, half of it, 1′ 55″, was given to the optical part, and the rest to the physical. And since 9165 subtends 5° 15′ 30″, you take 5° 17′ 25″, which yields 9227. And so, with a new eccentricity of 9230 (which is hardly different from the 9264 which I found in ch. 42, nor is it much farther from 9282, which is half of the eccentricity of the equant in ch. 16) I went through the whole job again. *First, the distances* GA *or* CA *were constructed at the individual whole numbers of degrees of equated distance anomaly* GAD. *Next, these were brought over to* GD *or* GBD, *the whole number of degrees of mean distance anomaly. Third, adjacent pairs, such as* GA, AD, *were added. Fourth, division by those divisors was carried out one hundred eighty times. The sum is 358° 28′ 30″, which is the length of the oval path. Fifth, the individual arcs of the oval path were added to one another in order. Sixth, the optical amplifications were borrowed from the previous unsuccessful operation, since I saw that over two computations they differed hardly at all. So these, too were added to the above sum, in order. Seventh, the sums of the arcs were increased by the sums of the optical amplifications.* Eighth, from the angle CBD thus found about the centre of the eccentric B, and the distance CA as opposite side, and the eccentricity AB as the third side, I sought out the 180 angles of optical equation ACB, whence the total equations and the equated anomalies were derived. The resulting anomalies were:

| Mean | Equated | Vicarious hypothesis | Difference |
|---|---|---|---|
| 45 | 38° 2′ 24″ | 38° 4′ 54″ | 2′ 30″ |
| 90 | 79 26 49 | 79 27 41 | 0 52 |
| 135 | 126 56 25 | 126 52 0 | 4 25 |

So the eccentricity still can be increased, and up above, [moving] from aphelion, the planet is made to be slightly slower than it should be, and the same near perihelion, and therefore it is swifter than it
235 should be at the middle longitudes, as was also found before in chapter 47. So too many of the distances seem to be collected near the apsides, and not as many as required, or not as long as required, about the middle longitudes. But a consideration of this follows in its proper place.

So, when I saw that as the physical causes introduced in chapter 45 were the more skilfully, and the more fittingly advocated, for the implementation of the theoretical foundations of the calculation, I came ever nearer to the true equations furnished by the vicarious hypothesis of ch. 16, I greatly congratulated myself, and was confirmed in the opinion of chapter 45.

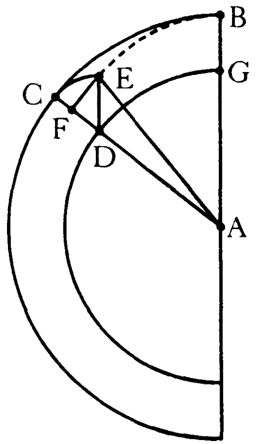
On the other hand, since I disliked the many contrivances with which I contended in this chapter, I did not rest until I had established a more certain and direct way, and at the same time I began to suspect that the calculation had not accomplished everything which the opinion of chapter 45 had required.

# 49

## A critical examination of the previous method for the equations, and a more concise method, based upon the principles constituting the oval in the opinion of chapter 45

So, in order to see the cause of the contrivance in this method, now fully presented, consider upon what foundations it rests. The planet is supposed to move uniformly on the epicycle, and to be swept around by the sun nonuniformly, according to the distance. From these two principles of motion, the oval path arises. But this method does not allow one to know what portion of the oval path corresponds to a given time, even if the distance of that portion be known, unless the length of the whole oval were known from the beginning. And the length of the oval cannot be known, except through the measure of the incursion of the planet from the circumference of the circle at the side. But further, the measure of this incursion cannot be known before it is known what portion of the oval path is traversed in any given time. This, as you see, begs the question, and in our operation we presupposed what was being sought, namely, the length of the oval. This is not just a fault in our understanding, but is uttertly alien to the primeval ordainer of the planetary courses: we have hitherto found no anticipation of such lack of geometry in the rest of his works. Therefore a different approach must be taken for calling the opinion of chapter 45 to the calculations, or if this cannot be done, the opinion will totter owing to its being suspect of circularity of argument.

From this consideration the implication has occurred to us that in using the uniform measure of time, we divided the composite oval
236 path into unequal parts, and thus measured out the parts of this composite oval, unequal but equated again by the compensation of




the distances, by the equal elapsed times of the planet on all of them. And indeed, we had among our presuppositions just one power, that from the sun, that intensifies according to distance, the power proper to the planet doing so not at all. But now, in this undertaking, we have in a way encumbered both forces with this relation to the distances, because we have given the common work of the two, the oval, to the planet, according to the measure of the distances to be traversed.

Therefore, although we have approached rather close to the truth in the effect of this method, we have nothing in which to glory that the opinion of chapter 45 has been expressed if we abandon the reasoning behind it. We would therefore have appeared to be going about our business more correctly if, dismissing the composite oval and the quadrature of its area, the subject matter of chapters 46, 47, and 48, we were to convert the calculation to the principles themselves of the oval path, assumed in chapter 45. Let chapter 45 be taken up again, *and about* A, *the centre of the sun's body, with radius* AD, *let the circle* DG *of the centre of the epicycle be described; and another, with centre* A, *and radius* AB, *the circle of the aphelion: in which* AGB *is the line of apsides, and let the planet, when it is at aphelion, be at* B. *Now let some time have passed from when the planet was at* B, *and let its measure be* CDE, *the angle on the epicycle, in order that as the aphelion of the epicycle* B *is translated to* C, *and the centre of the epicycle* G *to* D, *the planet will have moved on the epicycle from* C *to*

E. *Therefore, in order to know the angle* DAB *at the time* CDE, *consider that the planet passes across from* B *to* E *by two powers. One makes it move nearer the sun, and at the same time also draws it away from the line* AC *or* AD, *on which it was previously, when* AC *was at* AB. *The other moves the planet forward along with the centre of the epicycle, so that the centre* D *of the epicycle is on the line* AC, *although it was formerly on* AB. *Now the power that drives the centre of the epicycle around in a time designated by 360°, moves through 360°, or four right angles, about* A, *owing to the sum of 360 distances. Therefore, the sum of any number of distances being given from the time* CDE, *as before, the angle* DAB *will also be given. For the impression which the sun makes upon the body of the planet through the mediation of the distances* AB, AE, *is also presumed to make the same impression upon the centre of the epicycle* GD. *This is because if the planet did not disengage itself from the ray of power* AB *or* AC *and move towards* B, *but only descended towards the sun, it would then still be at the point* F* *on* AC, *on which line the centre of the epicycle* D *also lies. And it has disengaged itself by the law of the epicycle, at the distance* DE *and the angle* CDE *(for this is prescribed by the opinion of chapter 45, under which we are operating here). Thus by a kind of fiction for itself it places the centre of its epicycle at* D. *For we have said in ch. 39 how it is to be imagined that the power or fictitious radii of power* AB, AC, *and so on, act to position the planet. Now, though, the ratio of the arcs* BE *of the oval path to the whole oval is not quite the same as the ratio of the corresponding arcs* GD *of the perfect circle to the whole circle. But neither is it true that as* BC *is to the whole perimeter of the circle* BC *so is the arc* BF *of the oval to the whole oval. But this should not stand in our way, for* BE, *and* BF *too, are*
237 *composed of two powers, and if anything is disturbed in the proportion this comes from the planet's making its own descent on the circumference of the epicycle (following the opinion of chapter 45). For if the planet were to remain at the highest point of its epicycle, and were subject to the same motive force from the sun, adumbrated by* AB, AE, *which is nonuniform (which indeed cannot happen simultaneously, for when the distance of the planet from the sun remains the same, the motive strength from the sun remains the same), then a perfect arc of a greater circle* BC *would be described, whose ratio to the whole* BC *is the same which the arc* GD *has to the whole* GD.

*Under certain special conditions, this is true, namely, if the rays of power from the sun act as the planet's location, like a cart in which the planet is carried, as we suppose here. In itself, however, it is not true. On this point see the 'first way' in chapter 39. For among the five absurdities on which that one was rejected, we here omit only one, the last, and keep the other four.

*I am indeed aware that if the planet is supposed to be on a smaller perimeter, that of the centre of the epicycle* DG, *it will go much faster.*

*But that is not a reason for assigning a greater speed to the centre of the epicycle. For, by supposition, the centre of the epicycle moves, not in its own right, since it is not a body, but because of the planet. It is thus presupposed that the planet moves its own body away from the solar rays according to the law of the epicycle, and makes use of certain rays of power from the sun for its position (ideas which were indeed rejected in ch. 39, but taken up again in ch. 45 in somewhat altered form, and which are retained here in order to explain my attempts). On this basis, the foundation of the subsequent calculation is sound, whatever its result might be. For the oval is present here no less than before, since* DE *and* AB *do not remain parallel. For to the extent that the long distances* AB, AE *exceed the medium distances* AG, AD, *the arc* DG, *or the angle* DAG, *is made less than the angle* CDE, *the measure of the time. Thus* DE *inclines towards* B, *and* E *consequently makes an incursion from the circumference of the circle towards* BA. *For by chapter 2, if* DE *remained parallel to* AB, *then* E *would be on the circumference.*

This gives rise to the following method. The distances are found for each degree of mean anomaly. The method you have in ch. 39 above, and I also used it above in ch. 47 and 48. First, distances are found for nonintegral degrees of mean anomaly, or CE. Then, by interpolation, they are carried over to integral degrees of CE. If you find this meandering route annoying, and if the greater labour of the direct way pleases you, and if, further, you want to have the whole thing presented in one overview, proceed thus.

*Measure the time, or the artificial units expressing time, which is the astronomers' mean anomaly, on the epicycle* CE, *from its aphelion* C, *in a direction opposite the series of signs*[1]. *Thus the angle* ADE, *or its supplement* CDE, *is given as a whole number of degrees of mean anomaly. The radius* AD *is also given as 100,000, and the radius of the epicycle* DE *is 9264. Therefore, part of the equation,* DAE, *will be given, and the distance* AE. *Put both of these into a catalog, entered alongside of the mean anomaly* CE, *for future use. In this manner, let all the distances* AE *be gathered and added, and the sum will be found to be about 36,075,562. For this sum was found using an eccentricity only slightly different from our present one, which is 9264. The 360th part of this has the value 100,210, and the same fraction of four right angles is one degree. Therefore, as each of the distances, in order, is to the distance 100,210, so is the arc of this distance 100,210 (60′) to the*

[1] That is, in the direction of decreasing celestial longitude.

*arcs belonging to the other distances. For, as was frequently announced in ch. 39, 47, and 48, the ratio is inverse. Next, multiplying 60 minutes, or 3600 seconds, by 100,210, and dividing the product by*
238 *each of the distances in the semicircle, 180 times, or better, by half the sum of adjacent pairs of distances (following the advice given in ch. 48), will yield the angles of the centre of the epicycle,* DAG. *Next, beginning from the two least values of* DAG, *add them, and to the sum add the third, and again add the fourth to the sum of the three preceding, and so on, until you have accumulated all the 180 degrees. And if your final sum comes out to be exactly 180° it will prove to you that you did everything right, never departing from the instructions. And these sums of yours, which are the angles* DAG, *should again be inscribed in a catalog with the corresponding mean anomalies in the margin, for ready reference.*

*So, since an integral equation is to be computed, that is, the equated anomaly for a given mean anomaly, first, with the mean anomaly* CDE *measured on the epicycle, you extract the angle* DAG *or* CAB *from the latter catalog, the one with the sum of the angles. And with the same mean anomaly, you also extract the part of the equation* CAE *from the previous catalog. And when this is subtracted from the angle* DAB, *the remainder is the equated anomaly* EAB. *The variations in the other semicircle are known.*

| | |
|---|---|
| *Let the mean anomaly be 45°.* | |
| *The sum of its distances gives* DAG *as* . . . . . . | *41° 26′ 0″* |
| *By the same anomaly, the part* CAE | |
| *of the equation is given as* . . . . . . . . . . | *3° 30′ 17″* |
| *Therefore, the equated anomaly* EAB *is* . . . . . . | *37° 56′ 43″* |
| *Our vicarious hypothesis said* . . . . . . . . . . | *38° 4′ 54″* |
| *Difference* | *8′* |

In this manner, at

| Mean anomalies | We found an equated | | | While in the truer vicarious | | Difference |
|---|---|---|---|---|---|---|
| 45 | 37 | 56 | 43 | 38 | 5 | 8− |
| 90 | 79 | 26 | 35 | 79 | 27 | 0 |
| 120 | 110 | 28 | 8 | 110 | 18½ | 9½+ |
| 150 | 144 | 16 | 49 | 144[2] | 8 | 9+ |

[2] In KGW 3 p. 313, the number of degrees in the two equated anomalies is 114, while in the first edition they are 114 and 144, respectively. They should obviously both be 144°.

Near the apsides the planet is made to be slower than it should be, and near the middle longitudes swifter than it should be.

You will say that we have come out worse, since in ch. 48 we came nearer the truth in our results. But, my good man, if I were concerned with results, I could have avoided all this work, being content with the vicarious hypothesis. Be it known, therefore, that these errors are going to be our path to the truth. Meanwhile, let us be assured that at last we have brought the physical causes, which we supposed in chapter 45, at least once to a calculation entirely free of error. At the same time, moreover, the calculation of chapter 47, above, is confirmed, since this one is equivalent; and it is certain that wherever we entertained suspicions on the grounds of a lack of geometry, we were not inconvenienced in any perceptible way. Thus if there remains any discrepancy between these results and the truth, it is to be attributed, not to the method of applying numbers, but to the opinion of ch. 45, whence these numbers flow. This is to say, not that the opinion itself has immediately become totally false, but just that we have been hasty. For instead of waiting for the plenary judgement of the observations, when we understood the planet's path to be oval we
239 immediately seized upon a certain quantity for that oval, solely on account of the elegance of the physical causes and the graceful uniformity of the epicyclic motion, which was falsely given credence.

Now the manner in which the ultimate and truest opinion is to be brought to a calculation, and made to conform closely to these chapters, will be told in its place*. I am now going to continue with the unfolding of my remaining trials.

* Ch. 56, 58, 59, 60.

# 50

## On six other ways by which an attempt was made to construct the eccentric equations

How small a heap of grain we have gathered from this threshing! But you also see what a huge cloud of husks there is now. They ought to have been hauled back to the beginning of ch. 48, since before I investigated the arcs of the oval path I would have dealt with them. But for the sake of bringing light, they ought to be winnowed. Besides, we might end up finding a few useful grains.

**In the first and second methods, the procedure was this**

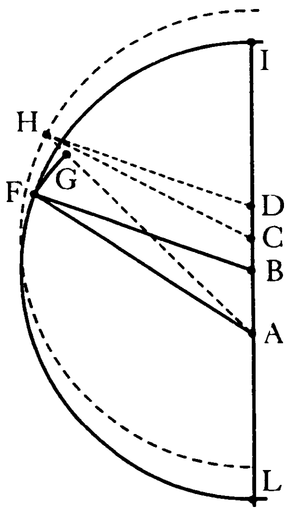
Term: What is the 'anomaly of distance'?

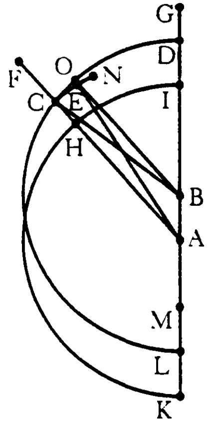
*Although it is quantitatively a mean with respect to the others, you should nonetheless beware of calling it 'mean', for in its proper usage, 'mean' denotes the [anomaly of] time.

First, with eccentricity 9165, which is a little less than the correct value, I sought out all the distances, according to the procedure shown in chapter 29. These corresponded to integral degrees of an anomaly occupying a middle position between the mean anomaly and the true equated anomaly, Although I am calling it 'equated' for the time being, I nevertheless add a condition, that it be used only for the distances. I therefore name it 'anomaly of distance'*. *In the second diagram of chapter 46 it is the angle FAB, and in the following one, CAD.*

Second, I sought out the third proportional lines, each of which is to its distance as this distance is to the radius, 100,000.

Third and fourth, I added the lines so found according to their kinds, and the sum of the distances was 35,924,252, less than 36,000,000. The cause of this you have in chapter 40. But the sum of the proportional lines was found to be 36,000,000, which holds me in wonder. And because it is delightful, I wish some geometer would




A problem proposed to geometers. Since elsewhere there are three anomalies, of which 1. is called the

prove it to be necessary[1]. *About centres* A, B *let two equal circles* IH *and* DC *be described, and let the centres* A, B *be joined, and* AB *extended so as to intersect the circle about* A *at* I *and* K *and the circle about* B *at* D *and* L. *Then let the circle about* A *be divided into any*

[1] The problem Kepler is proposing amounts to the straightforward (for modern mathematicians) evaluation of an integral, and the value does indeed turn out to be 360 times the radius. The procedure is sketched out by Caspar in KGW 3 p. 474.

mean, 2. the eccentric, and 3. the equated.

240 in this diagram and in this particular attempt, in order to avoid confusion, let us understand the first to be the arc CD or the angle CBD, the second to be the angle CAD or the arc ED, and the third to be the angle EAD.

*number of equal parts, such as 360, beginning from* I. *And from* A *let lines* AI, AH, AK, *and so on, be drawn through the points of division* I, H, K, *and so on, intersecting the circle about* B *at the points* D, C, L. *Then let it be that as* AI *is to* AD, *so is* AD *to* AG; *and as* AH *is to* AC, *so is* AC *to* AF; *and finally, as* AK *is to* AL, *so is* AL *to* AM; *and so on for all the rest. Let, I say, a geometer demonstrate that the sum of the 360 last constructed,* AG, AF, AM, *is equal to the sum of the first 360,* AI, AH, AK.

So, in this first method using the sums of the distances, I intended one thing (although erroneously and irrelevantly, namely, to add up the arcs CD or angles CBD, even though they were given at the beginning), but accomplished another, again erroneously. For I obtained, not the arcs, nor the angles, nor the path lengths, but the elapsed times on the unequal arcs of the planet's path, as if they were equal. And, following the rule of proportion, I said that as the sum of the means, AD, AE, AL, which is 35,924,252, is to the elapsed time of 360°, so is the sum of any number of distances to its elapsed time over the length of the path that includes these distances. *Let* A *be the sun,* B *the centre of the eccentric* CD, BC *the semidiameter. Let* B, A *be joined to* C. *Here the distances* CA *were found corresponding to integral degrees of the angle* CAD, *and thus to unequal arcs on the circle* CD, *something which escaped my notice. So let* CAD *be 45°. From* CB, BA, *the angle* CBD *is given as 48° 42′ 59″. Therefore, if there were no physical cause of the equations, and* CBD *were the measure of the time or mean anomaly, then there would correspond to it this value for* CAD, *truly equated. But the planet is slower on* CD, *owing to its greater distance from* A, *and the distances are the measure of the elapsed time. Therefore, at the anomaly* CAD *of 45° I collected 45 distances at the beginning of the arcs, which are longer. Their sum was 4,869,307. I also collected 45 shorter ones, the ones at the ends of the arcs, by subtracting the longest,* AD, *109,165, from the sum of the 46 distances, 4,975,577. The remainder was 4,866,412. The mean between the two sums, 4,867,852, I reduced to degrees, where 35,924,252 have the value 360°, or 99,790 have the value 1°. The result of this procedure was 48° 46′ 51″. And this ought to have been the time corresponding to the angle* CAD, *But the arc* CD *or the angle* CBD *was also found to be about that much, 48° 42′ 51″. This is absurd, and contrary to the hypothesis, which requires the planet to be slower at* CD. *The cause of this absurdity was immediately clear, namely, that in order to know the elapsed time for* CD *it was necessary to take the*

*distances corresponding to equal arcs of* CD, *while these distances just taken correspond to unequal arcs of* CD, *and are greater to the extent that the distances themselves are longer, by ch. 32. Therefore, these distances had too small a numerical value.* Nevertheless, in order that I not lose all this labour, I subtracted the excess of this number of the elapsed time over the number of the angle CAD, from CAD, so as to leave as a remainder EAD, 41° 13′ 9″, and so that AC, AE might be equal. Here it was supposed that in the time CBD the planet traverses an angle EBD about the centre of the eccentric B equal to CAD, and therefore, that as many distances from A were collected for the equal arcs ED of its eccentric as we found here on equal degrees of CAD. Thus, the same number of arcs which in our calculation were spread out over CD, which were unequal and, in this locality, too great, are now understood to be compacted within the confines of arc ED, now divided into equal parts. Therefore, the angle CBD is here the mean anomaly of distance, giving the angle CAD, for finding the distances CA, from which the angle CAE, the physical retardation and translation of CA to EA, is deduced.

I call it 'mean', not from its being an intermediate quantity among three, but from the uniform and mean motion of time which this

241

measures, insofar, that is, that it is the distances that are being sought.

Although it cannot show much of a discrepancy from the prior method of chapter 49, this procedure assumes without demonstration that CAD and EBD are equal, and consequently, that CA and EB are parallel, which was refuted above in the second diagram of ch. 46. But see how close this operation comes in its results. For

This differs slightly from the theory of ch. 49 and the two in ch. 47.

| At a mean anomaly of | Was found an equated | Which in the vicarious is | Difference |
|---|---|---|---|
| 48° 42′ 59″ | 41° 13′ 9″ | 41° 21′ 0″ | 8− |
| 95 15 31 | 84 44 18 | 84 39 18 | 5+ |
| 138 42 59 | 131 20 24 | 131 4 7 | 16+ |

The eccentricity was charged with being too small, and indeed, it really is greater, 9264 instead of 9165. Also, the planet was made to go too slowly near the apsides, and too fast near the middle longitudes. But, dismissing this first method, which we seized upon by chance in thinking over the error committed at the beginning, let us turn to the implementation of the second method, born from a consideration of the same error.

In the second attempt, the

The distances scattered over CAD have approximately the same numerical value as the sector CBD, and have led the argument to an

absurdity (for just as the area CAD, an approximate measure of the distances, is greater than the area of the sector CBD, the numerical value of the distances CD also had to have been greater than the sector CBD). The question therefore follows: do the lines AF, AG proportional to AC, AD correctly express the elapsed time over CD, thus allowing CAD to remain the true equated anomaly? The answer is, they do not. For if so, AC will remain in its place, which is the same at which its length was computed. Therefore, the orbit will be a perfect circle, which was refuted in ch. 44. Therefore, the distances at the middle longitudes, coming out longer than they should be, will make the planet slower than it should be there, and hence it will be faster at the apsides. But look at the result of the operation, which testifies to this itself. For

third anomaly is CAD, the second is CD or CBD, and the first, the sum of the few lines AG, AF, whose measure is, by supposition, the area CAD, approximately as in ch. 43.

| At an equated anomaly of | There followed a mean anomaly of | But in the vicarious | Difference |
|---|---|---|---|
| 45° | 52° 39′ 40″ | 52° 53′ | 13′ − |
| 90 | 100 29 12 | 100 34½ | 5 − |
| 135 | 142 10 47 | 142 9 | 2 + |

Nearly coincides with the physical hypothesis of a perfect circle, ch. 43.

First, the eccentricity is charged with being too small, since the maximum equation comes out to be 10° 29⅕′, which is 10° 34½′ in the vicarious hypothesis. Second, at the time 52° 39⅔′ the planet is found to have traversed as much of its path from the apsis as was traversed in the vicarious in the longer time of 52° 53′. If the eccentricity were adjusted, all the values of the equated anomaly would be increased, so that, in the lower quadrant also, in the time of 37° 44′ (the supplement of 142° 16′, which has been adjusted by increasing the eccentricity), the planet will traverse as much of the path as it traverses in the vicarious in the longer time of 37° 51′, which is the supplement of 142° 9′: that is, both will traverse 45°, which is the supplement of 135°.

By the way, this false hypothesis has come very close to giving us true results: the difference, after correction, is not more than 8′ or 7′ at either place. You thus see that results must not be trusted. And you
242 will note again what was observed in ch. 47, that the truth lies between these two methods (the latter of which describes a perfect

circle, and the former an oval, following the opinion of ch. 45). Therefore, you can at least conclude now, as well as before in ch. 47, that the lunules to be cut off from the perfect circle should have only half the breadth of the one that follows from the opinion of ch. 45.

### Third and fourth method

So, since this second method is not in accord with reason either, and in the first I learned that the distances are to be sought out corresponding to integral degrees of the angle CBD or to equal arcs of the eccentric CD, I also proceeded to the distances.

So, fifth (I am enumerating for you only those operations each of which was performed 180 times), I made use of interpolation to relate the distances found previously by dividing the mean anomalies CBD minutewise, or unequally, to the mean anomalies which are equal or are of an integral number of degrees. But now, CBD no longer remained the [mean] anomaly, as it formerly was in the first method, but was made the eccentric anomaly by this relating of the distances, as it also is in the second method.

I call an anomaly 'minutewise' when it is not expressed as a whole number of degrees, but has some minutes added on.

Sixth, using the same distances as before, I sought out their proportionals, that is, the lines that are to the distances as the distances are to the radius, 100,000. But this was unnecessary. Still, I wanted to be aware of all the possibilities.

Seventh and eighth, I again added the individual magnitudes, both of the distances AD, AC, and of their proportionals AG, AF. The sum of the distances came out to be 36,075,562. The reason for its coming out greater than 36,000,000 you have in ch. 40. The sum of the proportionals came out to 36,384,621.

Now, in the previous diagram, we shall proceed demonstratively, using the equated anomaly CAD to obtain the eccentric anomaly CBD, and through this eccentric anomaly CBD obtaining the sum of the distances found on the arc CD. And by this sum of the distances, we shall obtain the elapsed time over the arc CD, or the mean anomaly. Or, in reverse order, for convenience's sake, if an angle CBD of an integral number of degrees (such as 45°) is used to find CAB, and 45 correct distances are obtained, these things, I say, follow demonstratively. But again, as before in the second method, CAD becomes the true equated anomaly, and thus CA remains in place, and the orbit DC will be a perfect circle. Since this is false, as was shown in ch. 44, it necessarily follows that the distances at the

In the third attempt, as in the second, CAD is again the third anomaly, CBD or CD the second, and the more densely crowded lines AD, AC, or the area measuring their sum (the area CAD).

are the first anomaly, which is usually called the mean.

middle longitudes are taken to be too long here, and consequently that the times are made longer than they should be, and shorter at the apsides.

This method will in all respects be nearly equivalent to the previous method, using proportions. For the number of distances we have gathered is now greater than before, to about the same extent that the proportionals then were longer than the distances, when there were as many proportionals as there were distances. But, for safety's sake, witness the result of this calculation. For

| at a simple anomaly | there results an equated | Which in the vicarious | Difference | |
|---|---|---|---|---|
| 48° 38′ 31″ | 41° 31′ 0″ | 41° 17′ 6″ | 14′+ | nearly co- |
| 95 13 58 | 84 45 50 | 84 37 45 | 8 + | incides with |
| 138 45 41 | 131 1 52 | 131 7 13 | 5 − | the preceding. |

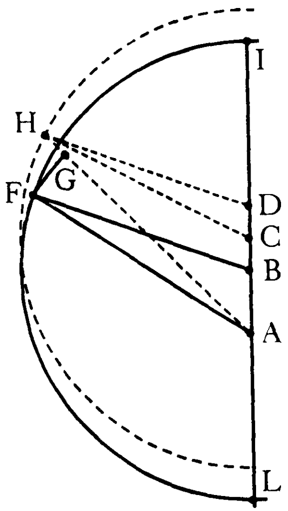
243 The eccentricity is again charged with being less than it should be. In other respects, the errors are the same as in the preceding one. The reason why the signs for excess have been turned into signs for defect, is that here the difference shows errors in the equated anomaly, while there they were in the mean anomaly. And this is the third method.

In the fourth try, if any remedy were attempted, a monster would be produced, with CBD the third anomaly, and the area CAD the second anomaly. But the sum of the more densely packed lines FA, GA, is the first anomaly.

In substituting the proportionals AG, AF for the distances AD, AC, which is the fourth method, we are going to make the two parts of the equation into three. For the area CAD measures the sum of the distances CA, DA. It is therefore much less than the sum of the lines FA, GA. And even if we attempt a remedy like the one used on the first method, we shall still have doubled our errors. For since the distances themselves cannot be admitted, owing to their excessive length at the middle longitudes, the proportionals will be even less tolerable, since they are longer. And if you want to test them with the results of a calculation, you will find that to a mean anomaly of 53° 23′ 56″ there corresponds an equated anomaly of 46° 0′, which in the vicarious hypothesis comes out to be about 45° 27′, a difference of 33′, clearly absurd.

### Fifth and sixth methods

So, since I accomplished nothing with these four methods, I then took the mean anomaly and the distances assigned to it (in the fifth

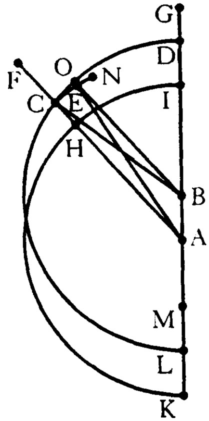



operation) and went over to the table of the vicarious hypothesis of chapter 16, and the true equated anomaly. Let the second diagram in chapter 46 be taken up again. Then, the distances AF belonging to integral degrees of mean anomaly IBF or IDH, also belong to the degrees and minutes of the equated anomaly IAH, which in the table mentioned corresponded to the mean anomaly IDH itself. Therefore,

Ninth, I related these distances obtained from the minutewise equated anomalies of the vicarious hypothesis of ch. 16, that is, from the unequal angles HAI, to the individual integral degrees of the equated anomaly HAI, that is, to equal parts.

Tenth, with the same distances, thus set up, I sought the [third] proportionals, as in the second and sixth operations.

Eleventh and twelfth, I added each, according to its kind, and the sum of the distances was 35,770,014, and the sum of the proportionals, 35,692,048. In this translation of distances we moved all the long ones upwards, and made them fewer, establishing large arcs IG of the oval path above, at aphelion, and thus attributing the individual distances to the individual degrees, not of the anomaly FAB, as in the first method, but of HAB, which is the true equated anomaly. There are no more of these degrees in the upper semicircle than in the lower. Therefore, there are now more of the short




distances than there are of the long ones, whence it not only happens that the sum of the 360 distances comes out smaller than the sum of the 360 diameters, but also the sum of the proportionals comes out smaller than was the sum of the distances themselves.

244 So, in the matter of the fifth method and the sums of the distances, reason again cries out against the method of basing equations upon it. Let the diagram belonging to this chapter be repeated, and let it be recalled to mind what was said about the first method. For in it the equated anomaly of distance CAD was divided into equal degrees. Hence it happened that CD was divided into unequal and [excessively] large parts, and had too few distances. From this, by a sort of accidental remedy derived from error, we concluded from the sum of the distances on CD that to those distances belonged a shorter arc ED, so as to transfer AC over to AE, and thus ED could be obtained divided into equal parts with a distance established at each of its degrees. However, it was not from the sum of the distances found on CD, but from a mingling of the vicarious hypothesis with the hypothesis of the distances framed in chapter 46, that the translation of AC to AE was now made and perfected, and the arc ED was attributed to the mean anomaly (which we have numbered on the arc CD for finding the distance CA or EA). And this was nonetheless done in such a way that BE and AC are not exactly parallel, as in the first method. And now that this has been done by a mingling of hypotheses, as I was saying, there is no need to go through the

Note the respect in which this is 'mean'. See the marginalia above.

operation again, as in the first method. Instead, only one thing needs to be found: do the few distances AC, AE gathered into one sum in this fifth method, produce the same physical equation that resulted artificially from the two mingled hypotheses?

In this fifth method the third anomaly is indeed EAD, and its mean anomaly (first in order) is CD or CBD, and also the same anomaly of distance for the distance CA or EA. But the area EAD measures some sum of distances EA, DA, which is alien to this equated anomaly EAD, since it belongs to the measure of time on the arc DN, and the equated anomaly DAN. Again, therefore, a monster.

Consider here how the distances are arranged in this last procedure. The angle EAD, whose terminus E is at distance AC from the sun, was divided into equal degrees in this last procedure, to each of which was given a distance. In this way the arc ED of the oval path standing upon the angle EAD ends up divided into unequal parts, and it receives too few distances. So the mean anomaly obtained in advance from the vicarious hypothesis cannot be had from the sum of the distances on EAD.

Now in the first method above, when CD received too few distances, with the angle CAD divided into equal degrees we substituted ED for CD as the arc suited to those distances. So similarly, in this fifth method, since ED has received too few distances, with the angle EAD divided into equal degrees, if it is permissible again to make use of a clumsy remedy, let us substitute ND for ED, as the arc to which those distances belong. *In order to find the distance* CA *let the mean anomaly* CBD *be 48° 44'. Given angle* B, *and* CB, BA, CA *is given as 105,784*[2], *and* CAB *as 45°. The vicarious hypothesis requires* AC *to be transferred to* AE. *And we now divide* ED, *which the vicarious hypothesis indicates to be 41° 22', into equal degrees, and through them we collect no more than 41 distances and part of a 42nd. And these, gathered into a single sum, constitute a mean anomaly which is by no means equal to the one first taken,* DC, *but is equal to another,* DO *which shows the distance* AO, *to be transferred to* AN. 'Work was begun on an amphora: why has a pitcher come forth from the whirling wheel?'[3] For the question was whether all the distances on the equal degrees of ED, gathered into a sum, would show the mean anomaly DC. But the operation gave me an answer concerning ND, and the anomaly DO.

**245**

This sixth method, after a very simple correction of those things in which the opinion of ch. 45 still errs, can also be of use

Finally, let us turn to the sixth method, and the proportionals that are adapted to the demonstration of ch. 32. For the true quantities of orbital arcs that appear equal from the centre of the sun are in the ratio of the distances: thus to the extent that AE is longer, so is ED.

[2] This is the number that would be obtained if AB were 8478, CBD meanwhile remaining 48° 44'. Where AB is 9165, as it has been throughout this chapter, CA is 106,268 at this angle. If CAB is to be 45°, CBD should be 48° 43', giving CA a value of 106,270.

[3] Horace, *Ars Amatoria*, 22.

in the truest physical hypothesis; and it is succinct and evident.

But the times of truly equal arcs, as measured on the orbit, are also in the ratio of the distances. For to the extent that ED is farther away from A, the planet also takes longer to traverse the arc ED.

Therefore, the times that the planet takes on those arcs that appear equal from the centre of the sun, are in the duplicate ratio of the distances.

But AF is to AH likewise in the duplicate ratio of the distance AC or AE to the mean AH. And so the measure of the times that the planet takes over equal degrees of the angle EAD is the [magnitude of the] proportional lines AG, AF, belonging to the integral degrees or equal parts of the angle of true equated anomaly EAD.

Therefore, let the proportionals to the distances at equal degrees of equated anomaly be tested, just as other distances were also tested above in this chapter. As, since 35,692,048, the sum of all 360 distances, at all 360 equal parts of the angle at the sun, is equivalent to an elapsed time of 360°, what is the value of a just and correct sum at any given degree of equated anomaly?

In this manner is found

| at equated anomalies of | Mean anomalies | Those yielded by the vicarious | Difference | |
|---|---|---|---|---|
| 41 | 48° 24′ 3″ | 48° 19′ 2″ | 5 + | coincides with |
| 81 | 91 30 39 | 91 34 8 | 3½− | those of |
| 91 | 101 28 10 | 101 34 7 | 6 − | chapter 49 |
| 131 | 138 28 5 | 138 39 28 | 11 − | |

In this sixth method, the third anomaly is EAD, the second ED, while the first is the sum of the lines AG, AF, where AF or AC is understood to be translated to AE. Nonetheless, in computing

The eccentricity is again charged with being less than it should be. With this corrected, the difference above, at 41°, will be about 8′+, and below, about 7½′−. Thus here, too, the planet is not made to go fast enough at the apsides, and so there are too many distances near the apsides, and consequently less than there should be at the middle longitudes. But it comes quite close to the truth, and clearly coincides with the method of chapter 49. For if you consider well, the same thing is done here as was done in ch. 49. There we computed the optical part of the equation by itself, and the physical part also by itself, while here we are computing them both together. There we had introduced fictitious rays of power in order to be able also to ascribe to the epicycle its own task of disengaging itself from those fictitious

the distance AE, that is, AC (from which AF is derived), DC or DBC is also the first anomaly. It is depicted twice here because two things are being investigated: time and distance.

rays (for in the truth of the matter no rays go around as slowly as the centre of the planetary epicycle goes, as was said in ch. 39). Nevertheless, as concerns the effects of all the physical force carrying the planet around, we left it to the sun, so that the epicycle would only serve to adjust the distances. Here, we made use of the same power of the sun for the physical translation, while we again computed the distances from the epicycle, and gave its equal parts to equal times, that is, to equal degrees of mean anomaly, as the opinion of ch. 45 would have it. And if we ended up taking as many distances in any given part of the time as there are degrees of equated anomaly, they are still derived from the distances of the mean anomaly, and in
246 length they are the same. And this form is so much easier that we can here put aside the other persuasion concerning the planet's epicyclic motion, and take one step closer to the truth of the physical cause, leaving to the epicyclic mode nothing but a reciprocation on the diameter – although this is still flawed, as has appeared from these equations, at least. For, as was noted just above in considering the second method, this preoccupation with epicyclic motion is excessive, showing distances at the middle longitudes which are too small, from which it happens that at that place the planet exceeds its measure of velocity, and at the apsides falls short of its measure. But it suffices us to express the opinion of ch. 45 in our calculation. Therefore, although one might here raise the objection from ch. 32 that this ratio of diurnal motions cannot be constant, since the parts of the eccentric near the apsides are presented directly to the sun, and the intermediate are presented obliquely, so that they thus appear differently from the way they would if they were presented directly, if anyone, I say, were to raise this objection, I shall answer as I did in ch. 49, that this obliquity at the intermediate parts is added by the planet in its own right, and occurs through its descent. Thus, it is not to be imputed to the motive cause arising from the sun, nor is it affected by that cause.

Therefore, studious reader, from such a great number of chapters and methods, you have only two methods of equating that conform to the opinion of ch. 45. One is by the physical hypothesis intermingled with an epicycle so arranged as to effect the longitude, described in ch. 49. The other is the sixth method of this chapter, in accordance with a more purely physical hypothesis, where the epicycle governs nothing but the descent towards the sun, or if anyone wishes to set it up to affect the latitude, perpendicular to the plane of the ecliptic.

And both of these two use different means to produce the same effect. You will thus be able to place confidence in them more safely when examining the opinion of chapter 45.

It is to be hoped that through a hitherto empty trust in the true physical causes that have been discovered, there will finally be a triumph over Mars. Now, some rumour, I know not what, calls me to new tumults and new labours.

# 51

## Distances of Mars from the sun are explored and compared, at an equal distance from aphelion on either semicircle; and at the same time the trustworthiness of the vicarious hypothesis is explored

While I am thus celebrating a triumph over the motions of Mars, and fetter him in the prison of tables and the leg-irons of eccentric equations, considering him utterly defeated, it is announced in various places that the victory is futile, and war is breaking out again with full force. For while the enemy was in the house as a captive, and hence lightly esteemed, he burst all the chains of the equations and broke out of the prison of the tables. That is, no method administered geometrically under the direction of the opinion of ch. 45 was able to emulate in numerical accuracy the vicarious hypothesis of chapter 16 (which has true equations derived from false causes). Outdoors, meanwhile, spies positioned throughout the whole circuit of the eccentric – I mean the true distances – have overthrown my entire
247 supply of physical causes called forth from ch. 45, and have shaken off their yoke, retaking their liberty. And now there is not much to prevent the fugitive enemy's joining forces with his fellow rebels and reducing me to desperation, unless I send new reinforcements of physical reasoning in a hurry to the scattered troops and old stragglers, and, informed with all diligence, stick to the trail without delay in the direction whither the captive has fled. In the following few chapters, I shall be telling of both these campaigns in the order in which they were waged.

And, to speak initially of the first of these, I shall begin by seeking out the distances of several places on the eccentric where the evidence was most trustworthy. Therefore, let it be our intention to explore the distances near the mean anomaly of 90° and 270°.

On 1589 May 6 at $11\frac{1}{3}^{h}$ Mars was observed at 27° $7\frac{1}{3}'$ Libra with

At a mean anomaly of 87.

latitude $0° 6\frac{2}{3}'$ north. The true position of the sun at this time was calculated as $25° 48\frac{2}{3}'$ Taurus, and its distance from earth 101,361. The mean longitude of Mars was $7^s 26° 0' 36''$, and therefore its eccentric position was $15° 32' 13''$ Scorpio. But our vicarious hypothesis of chapter 16 did not come nearer than $2\frac{1}{3}'$ to the true or observed position of Mars in an acronychal situation, and thus in so sensitive a procedure the computation of the equated anomaly cannot be trusted. Therefore, to the method of chapters 27, 28, and 42 I shall add another observation, which nevertheless uses a freer method. Indeed, as I remarked in ch. 12 above, Mars was not very often observed twice in this region. Therefore, we should be content with two observations. So, as a counterpart to the one just now presented, let there be taken another observation from 1594 December 28. At $7\frac{1}{4}^h$ on the morning of that day, Mars's mean longitude was calculated to be $7^s 26° 13' 39''$[1], a few minutes beyond the other. And at that time Mars, at an altitude of eight or nine degrees, was observed to be $50° 34'$ from Spica Virginis. *So, since it stood very close to the ecliptic, in the right triangle between Spica, its ecliptic position, and Mars, the base is given as 50° 34' and the side between Spica and the ecliptic is 1° 59', which is Spica's latitude. Therefore, the remaining side is 50° 32' 18" Thus, since Spica was at 18° 11' Libra, Mars fell at 8° 43' 18" Sagittarius. The declination of this position from the equator was 21° 50' 20".*

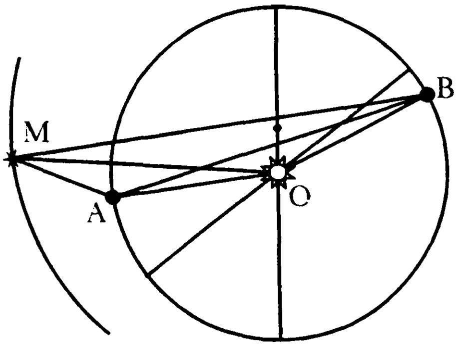
A short cut: given the distance, from a fixed star with a known latitude, of a planet with no latitude, to find the planet's longitude.

However, Mars was found to have a declination of $21° 41'$. Therefore, it displayed a small amount of north latitude, $9' 20''$. And on the following 1595 January 4 it still had $3'$ of north latitude. Our observation is hereby confirmed. But if you assume this to be the true latitude of Mars, its ecliptic position will not be changed perceptibly. So you may safely pronounce its position to be $8° 43'$ Sagittarius. And because Mars was near the sun, it was very far from earth, and thus had a much smaller parallax than the sun, which we shall ignore. But we cannot similarly ignore the refraction, which I shall now remove. *For the sun's position was 16° 47' 10" Capricorn, distance from earth 98,232, and its right ascension was 288° 12'. Therefore, 306° 57' on the equator was rising, and along with it 29° Sagittarius*[2]*, at which the*

A way of separating out the latitudinal and longitudinal refractions.

[1] This should be $7^s 26° 17' 40''$. The mean longitude given by Kepler corresponds to a time of 4.15 am; perhaps he used the wrong hour. In any event, this has little effect upon the positions, which he calculates with respect to the sun, by triangulation.

[2] Recomputation indicates this to be incorrect. The correct rising degree would be 9° Capricorn, and the angle between the ecliptic and the horizon would be 38°, complement 52°. This change has little effect, however.




*angle between the ecliptic and the horizon is 26°, its complement 64°. And because the altitudinal refraction as shown by the table of refractions of the fixed stars is 6′ 30″, and from the table of the sun, 11′, when the star is at an altitude of 8½°, 5′ 51″ or 9′ 53″ are to be subtracted from the latitude. The latitude from the former would be 3′ 29″ N., and from the latter 0′ 33″ S. And the longitudinal refraction is 2′ 39″ or 4′ 34″*[3].

Investigation of the refraction from the latitude.

Of these two methods of determining the refraction, I shall follow the one which is confirmed by the latitudes, as follows. *In the earlier*
248 *observation, the observed latitude was 6⅔ North. And because Mars was near the earth, and the angle at the sun was 10° 17′, while at the earth it was 28° 41′, this latitude requires an inclination of 2′ 30″. Therefore, in our later observation the inclination will also be 2′ 30″, or a little less, since we are 8′ closer to the node. But with the inclination assumed to be 2′ 30″, since here the angle at the sun is 61°, and at earth 38°, a latitude of about 1′ 50″ N. must follow, as indicated by our parallactic table*[4]. *But by using the refraction of the fixed stars, we are left with a latitude of 3′ 29″ N., while by using the solar refraction we are moved down to 0° 33′ S. Thus in the second our refraction was greater than would be correct, and in the first, less. So the correct refraction is between the two, namely, 3′ 36″.* This puts Mars at 8° 46⅓ Sagittarius. *Let* O *be the sun,* B *and* A *points on the earth's orbit,* A *the*

Use of the parallactic table in computing latitudes.

[3] The latitudinal refraction should be 5′ 7″ or 8′ 40″, making the corrected latitude 4′ 13″ N. or 0° 40″ N. The longitudinal refraction should be 3′ 28″ or 5′ 52″.

[4] The parallactic table appears in Kepler's *Astronomiae pars optica*. The table is bound into the 1604 edition at fol. 275, and in KGW 2 p. 240. Instructions for its use are to be found on fol. 320 of the 1604 edition, KGW 2 pp. 275–6, or in the translator's introduction to the present work.

*position of the earth in the earlier observation,* B *in the later,* M *Mars. Let the lines be connected. And although Mars does not return to exactly the same place, let it nonetheless be represented in both instances by the line* OM. *Thus* MAO *is 28° 41′ 14″, and* AO *is 101,365. Let* MO, *the distance of Mars from the sun (which is being sought here), be taken as if known, and let it be 154,200. Thus* OM *will fall at 15° 31′ 3″ Scorpio. And if* OM *is assumed to be 154,200 in the earlier observation, it ought to be taken as shorter in the later one. Now one degree at this place on the eccentric changes the distance by 240 units, whatever form you use for constructing the distances*[5]. *Therefore, since the mean longitudes here differ by 13 minutes, and when a subtraction is made to account for precession, only eight, the proportional part of 240 is 32. Consequently, in the second observation, we have assumed* OM *to be 154,168. But* OBM *is also known, it being 38° 0′ 40″, and* OB *is 98,232. Therefore,* OMB *is given as 23° 6′ 11″. Consequently, on the second occasion* OM *was at 15° 40′ 9″ Scorpio, differing from the earlier eccentric position by 9 minutes. It should have differed by somewhat more. For the mean anomalies differed by 8′ 3″ to which corresponds 7′ 49″ in the equated anomaly of the eccentric at this place. Add to these the intervening precession of the equinoxes of 4′ 48″. Thus 12′ 37″ were accumulated. Mars therefore ought to fall at 15° 43′ 40″ Scorpio. Therefore, we should take a somewhat different value for the distance* OM, *and should change it so that the lines represented by* OM *move about another 2⅔′ apart from each other. For when the earth is at* A, OM *should move back in longitude, and forward when the earth is at* B. *But this occurs if you increase* OM. *So let it be 154,400 in the first instance, and in the second, 154,368. For then* OM *falls first at 15° 29′ 34″ Scorpio, and second at 15° 42′ 18″ Scorpio.*

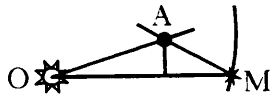
And for the first time the mean anomaly is 87° 9′ 24″ and 87° 16′ 30″[6] for the second. This will do for the mean longitude of the earlier one.

At a mean anomaly of the full-circle complement of 87.

For the other mean longitude, an observation in the month of December 1595 will serve, being well supported by the consensus of a number of consecutive days, and at a place where the vicarious hypothesis also exactly represented the acronychal position of Mars in the preceding October. For the sake of consensus, we shall also add

[5] At the middle longitudes, the change on the circle is nearly the same as the change on the oval, although the distances themselves differ appreciably. See note 2, p. 530.

[6] This should have been about four minutes greater, owing to the error in the mean longitude.




an observation of October 1597. In the other years it was not observed at this eccentric position. For the eccentric position falls at 10° Gemini. Thus Mars was observed last at this place in November
249 1580. In 1582 its arrival at the place fell in October, when [Tycho's] great interest in observing had not yet been aroused. In 1584 it came in September, in 1586 in July, in 1588 in June, in 1590 in April, and in 1592 in March, at which times, being near the sun owing to the short, bright nights in Denmark, it was neglected, while, whenever there was an opportunity, they were intent upon the fixed stars, the moon, and the other planets. But at the end of 1593 and the beginning of 1594, when it was at quadrature with the sun, the observation was not continued beyond this aspect because astronomers are usually chiefly interested in the quadrature. So, in 1595 Dec. 17 at $7^h\ 6^m$ in the evening the planet was observed at 11° 31′ 27″ Taurus, with latitude 1° 40′ 44″ N. The sun's position was 5° 39′ 3″ Capricorn. Its distance from earth was 98,200.

The mean longitude of Mars was concluded to be $2^s$ 2° 4′ 22″.

And since the aphelion is $4^s$ 28° 58′ 10″ the distance of the position from aphelion is 86° 53′ 48″ backwards. Previously, it was nearly the same as that, namely, 87° 9′ 24″. Therefore, these two positions are nearly the same distance from aphelion. Now in our vicarious hypothesis, there corresponds to this simple anomaly an equated anomaly of 76° 25′ 48″. Which, subtracted from the position of the aphelion, leaves Mars's eccentric position, 12° 32′ 22″ Gemini. *Let* A *be the earth,* O *the sun,* M *Mars.* AO *is given as 98,200. And because* OM *is at 12° 32′ 22″ Gemini, while* AM *is at 11° 31′ 27″ Taurus, therefore* AMO *is 31° 0′ 55″. And because* AO *is at 5° 39′ 3″ Capricorn, while* AM *is at 11° 31′ 27″ Taurus, therefore, the supplement of* OAM *is 54° 7′ 36″. Hence, since the sine of* OAM *is to* OM *as the sine of* AMO *is to* AO, OM *comes out to be 154,432.* And since this position is 15 minutes closer to the apogee than the one from 1589, and at this position on the eccentric 1° causes a change of 240 units, therefore 60 units are to be added for 15 minutes, since at positions farther removed from aphelion the distances are shorter. The distance thus comes out to be 154,372. On the other hand, since the node is at about

16° 20′ Taurus, and the eccentric position is 12° 32′ Gemini, the planet is 26° 12′ from the node. And the maximum inclination of the planes is 1° 50′. Therefore, the inclination at this place is 48′ 32″. The secant of this exceeds the radius by 10 units, or in our dimensions, $15\frac{1}{2}$. So the distance from the sun of that point on Mars's orbit is 154,387. And previously, at this distance from aphelion it was found to be approximately 154,400 from the sun. Therefore, the distance of these two points on the eccentric from the sun is equal to within a hair's breadth. For the 13 units that are wanting in the latter distance are of no importance. I shall rejoice if I am able to come within an uncertainty of 100 units everywhere.

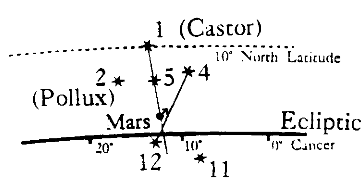
And now I shall add [the observation from] 1597, not so much to confirm the previous ones, which are certain in themselves, as to give the reader an opportunity to compare the observations of Tycho with the observations of others, by which means he will at length understand how much that man has benefited us. There do indeed exist observations of that author from the last days of October of 1597, but they were taken with a radius while travelling, and not brought to a calculation by the author himself, who knew how to correct distances taken with a radius, by applying a kind of parallactic table for the eye, as he informed us in the *Progymnasmata*. And so, since very different distances are ascribed to the same moment (possibly because they were supposed to be corrected immediately after the observations), they are to be dismissed. But at the same moment, I, though absent in Styria, made an observation, and (strange to say) did so with the eyes of Tycho Brahe, standing by the shore of the Baltic Sea. Here is the series of observations – hold your laughter, friends!

250 On 1597 November 8, Saturday, or October 29, in the morning, Mars was not yet on the line from the twelfth of Gemini to the fourth[7]. On the following day, it had already left it: it was nearer the ninth than the twelfth, and precisely on the line from 11 to 9, and also

[7] The numbers denote the order of the stars in Ptolemy's catalogue, under the constellation of Gemini. Tycho also followed the same order for the most part, except that some of his entries seem to denote different stars, as Kepler remarks below. Modern designations of the stars mentioned by Kepler are:

| | |
|---|---|
| First | Castor |
| Fourth | Tau Geminorum |
| Fifth | Iota Geminorum |
| Ninth | (uncertain) |
| Eleventh | Zeta Geminorum |
| Twelfth | Delta Geminorum. |

At the right is a diagram of the configuration.



on the line from 1 to 5, or a little farther east. And the fifth was halfway between the first and Mars.

From these the position of Mars can be elicited, when certain stellar positions are assumed from Tycho Brahe's catalogue, which I now profess to be my eyes. But because the ninth is not described in Brahe's catalogue (since in its stead in the ninth place is another, distant by more than three degrees from the Ptolemaic, and less than all of them), we shall call upon the latitude of Mars as our counsel. For an approximate knowledge of it will suffice. *Now the mean longitude of Mars on the morning of October 29 at $5^h$ (which is an estimate, since I did not write down the time) is found to be $1^s\ 29° 10' 43''$. Therefore, its eccentric position was 9° 43′ Gemini, 23° 20′ from the node. Therefore, the inclination was 43′ 52″. But the sun was at 15° 40′ Scorpio, and the apparent position of Mars, by anticipation, was about 12½° Cancer. Therefore, the latitude was 1° 36′ 24″*[8]. Let a computation be made of the longitude of a point on the line from the twelfth to the fourth, having a latitude of 1° 30½ North. *Since the fourth is at 9° 54′ Cancer, latitude 7° 43′ N., and the twelfth is at 12° 56′ Cancer, lat. 0° 13½′ S., the longitude of our point, by interpolation, will be 12° 16′ 17″ Cancer. But on 29 October, Mars was not yet here, and on the 30th it had already passed it. The diurnal motion was no more than 5 minutes, half of which is 2½′, so that on the morning of the 30th it was at 12° 18½ Cancer. And so indeed it was at the end of 1600, but in 1597 it was at 12° 16′ Cancer.* Three minutes of error in latitude barely produce one in longitude. So the position is certain enough. *If you also explore it using the first and the fifth, and use the point on that line whose latitude is 1° 30½′, it falls at 12° 9′ Cancer. And Mars was farther east, that is, forward in longitude, at about 12° 16′ or a little before, also intermediate. Therefore, the latitude computed by us is confirmed. For it should be approximately intermediate, and indeed it is. That is, between Mars's 1° 30½ and the 5° 42½′ of the fifth there is 4° 12′, and between this and the 10° 2′ of the first there is 4° 20′.*

Therefore, let Mars be at 12° 16′ Cancer. On 1597 October 30 at $5^h$ am the position of the sun is found to be 16° 38′ 8″ Scorpio, distance 98,820. [Mars's] mean longitude was $1^s$ 29° 42′ 10″, aphelion $4^s$ 28° 57′ 10″, supplement of the mean anomaly 89° 15′, of the equated anomaly 78° 43′ 23″, eccentric position 10° 13′ 47″ Gemini. Therefore, the

[8] This should be about 1° 30′, and indeed, immediately below Kepler restates this as if it had been 1° 30′ 24″. Most likely Kepler's printer made the easy error of mistaking a '0' for a '6'. Subsequent editions and translations have left this error uncorrected.

distance is inferred to be 153,753. And since we are 2° 6′ farther from aphelion than before, we will add twice 240, the sum of the units corresponding to one degree:

| | |
|---|---|
| | 240 |
| | 240 |
| And a tenth part: | 24 |
| Likewise another 15 parts, for the substitution of a line in the plane of Mars's orbit for the line in the plane of the ecliptic: | 15 |
| | 153753 |
| Produces | 154272 |
| Previous value | 154400 |
| Difference | 128 |

251 If you subtract three minutes from the position of Mars, so that it would be at 12° 13′ Cancer, which could be done in our observation, especially if the time were different, this difference would be reconciled.

Second, I shall prove the same thing at parts closer to aphelion. On 1589 April 5 at $11^h$ $33^m$ Mars was observed at 7° 31′ 10″ Scorpio with latitude 1° 28′ 13″ N. It was near the meridian, and consequently there were no horizontal variations. The mean longitude was concluded to be $7^s$ 9° 46′ 8″. And the aphelion was at $4^s$ 28° 51′ 8″. Therefore, the mean anomaly was 70° 55′ 0″ to which corresponds an equated anomaly of 61° 17′ 35″, by the vicarious hypothesis. And so the eccentric position was 0° 8′ 43″ Scorpio. The sun's position was 25° 52′ 43″ Aries, its distance from earth 100,560, the angle at the earth 11° 38′ 27″, at the planet 7° 22′ 27″. Therefore, the distance of Mars from the sun was 158,090[9]. But again, so as not to trust the eccentric position, on account of the error of about two or three minutes which the vicarious hypothesis commits at this position on the eccentric, we shall appropriate a counterpart from 1591 Feb. 19, when, at $5\frac{1}{2}^h$ am Mars was observed to be 28° 11′ from the southern pan of Libra[10] (which in that year was at 9° 23½ Scorpio), with a latitude of 0° 26′

At a mean anomaly of 71°.

[9] The translator's computations result in an eccentric position 20″ less than that given by Kepler – an insignificant difference, surely, yet this increases the distance to 158,208, owing to the small angles at the sun and Mars. Hence, this distance must be regarded as having a rather large uncertainty.

[10] This is $\alpha$ Librae.

North. So Mars fell at about 7° 34½′ Sagittarius, approximately[11]. But since that eccentric position has a declination from the equator of 21° 39′ 10″, [while] the observed declination of Mars was 20° 50′ 30″, its latitude was therefore 48′ 40″. From this, the longitude is corrected, which becomes 7° 34⅓′ Sagittarius. But the mean longitude was $7^s$ 8° 21′ 47″, to which corresponds an equated 59° 57′ 38″, and an eccentric position of 28° 51′ Libra. Therefore, the angle at the planet was 38° 43′ 20″. The sun's position was 10° 14′ 25″ Pisces. Therefore, the angle at the earth was 87° 20′ 0″. And the distance of the sun from the earth was 99,210. Thus the distance of Mars from the sun comes out here to be 158,428, longer than before, because here we are also nearer to aphelion by 1° 26′ 30″[12]. But at this place on the eccentric for one degree about 220 units are subtracted, or for the entire angular difference, 317, so that this place, if we carry it back to an anomaly similar to the preceding, would have a distance of 158,111 rather precisely. Whence it is argued that these two eccentric positions, treated by the method given above, will show exactly the same eccentric position as our vicarious hypothesis, except that on account of our nearness to 17° Scorpio we run the risk of being in error by one or two minutes. Moreover, in the latter of these, the distance from Aquila came out to 54° 12′, which is not within 12′ of agreeing with the other observed data, and consequently this observation is not perfectly certain. Also, some small quantity should be added, owing to the latitude.

A suitable observation at a similar longitude on the other semicircle occurred on 1582 November 12 at $6\frac{3}{4}^h$ am, when the sun's position was 29° 35′ 17″ Scorpio. Its distance was 98,503, mean longitude of Mars $2^s$ 15° 10′ 20″, aphelion $4^s$ 28° 44′ 20″. Hence, the full-circle complement of the mean anomaly was 73° 34′, and of the equated anomaly, 63° 45′ 18″. Hence, the eccentric position was 24° 59′ 2″ Gemini. At that time, as I was saying, the planet was observed at 26° 35′ 30″ Cancer, making the angle of vision, the one at the earth, 57° 0′ 13″, and at the planet, 31° 36′ 28″. By these data it is determined that the planet's distance from the sun was 157,631. And because the anomaly was previously 70° 55′, and is now 73° 34′,

At a mean anomaly of the full circle complement of 71°.

[11] Substituting 7° 34½′ for an obviously incorrect 7° 24½′.

[12] As can easily be seen from the two equated anomalies, the second position is actually only 1° 20′ nearer aphelion than the first, which would result in a smaller adjustment and consequently a larger distance. However, since the distance for the first position was probably greater than that stated by Kepler, the two are very nearly in agreement (158,200 for the first and 158,150 for the second).

we are therefore lower by 2° 39′. For this, in the previously men-
252 tioned ratio, 586 units are wanting. So the comparison afforded by this observation assigns a distance of 158,217 to an anomaly similar to the above, where again about the same amount as before, or a little more, is to be added for the latitude. The difference is about 127, which is excused owing to the uncertainty of the prior observation. For it is very small, and may be neglected in our present undertaking, where we are considering magnitudes of 1800 or 3600 or still more.

At a mean anomaly of 43 in the other semicircle.

But let us move yet higher towards aphelion, and explore those places where, by what was shown in ch. 6, the dislocation of the eccentric occasioned by exchanging the sun's mean motion for its true motion becomes most evident, namely, at the sun's apogee and the sign of Cancer.

On 1596 March 9 at $7^h$ $40^m$ pm when the sun's position was 29° 31′ 24″ Pisces, the distance from earth 99,764, Mars's mean longitude $3^s$ 15° 35′ 0″ aphelion $4^s$ 28° 58′ 31″, the difference between the mean anomaly and the full circle 43° 23′ 31″, equated anomaly 36° 40′ 2″, eccentric position from the vicarious hypothesis 22° 18′ 29″ Cancer – the planet was observed at 15° 49′ 12″ Gemini, latitude 1° 47′ 40″ N. Therefore, the angle at the earth was 76° 17′ 48″, at the Planet 36° 29′ 17″. Therefore, the distance of Mars from the sun was 162,994, or more correctly, this was the distance of the point in the plane of the ecliptic perpendicularly beneath the body of Mars.

But, to be sure, let an additional observation be taken. And Mars was at precisely the same sidereal position on 1584 Nov. 25 at $10^h$ $20^m$, when the sun was at 14° 0′ 3″ Sagittarius, distance from earth 98,318. The mean anomaly was not perceptibly different from the previous one, because the motion of the aphelion is only very slightly faster than the motion of the fixed stars. Therefore, the eccentric position is the same, 22° 8′ 44″ Cancer, if you subtract the precession of 9′ 45″. But the planet was observed on Nov. 12 at $13^h$ $26^m$ at 23° 14′ 5″ Leo, with latitude 2° 12′ 24″ N. On the 20th of Nov. following, at $18^h$ $30^m$, it appeared at 26° 0′ 30″ Leo. Thus in 8 days 5 hours it was moved forward 2° 46′ 25″. In Magini, this is 2° 48′. Therefore, since our time follows by 4 days $15^h$ $49^m$, to which corresponds 1° 28′ of motion from Magini[13], we shall add 1° 27′ according to the above

[13] According to the figures from Magini, the diurnal motion was 20′ 28″, and the total for four days 15 hours 49 minutes would be 1° 35¼′. This results in a distance of 162,658, considerably less than that computed by Kepler.

ratio. So Mars could have been observed at 27° 27′ 30″ Leo, approximately. Therefore, the angle at the earth was 73° 27′ 27″, at the planet 35° 18′ 46″. Hence, the distance of Mars from the sun here was 163,051, exceeding the previous one by 57 units. This can be absorbed by a very slight change in the eccentric position, as, indeed, the vicarious hypothesis is not trustworthy to within one minute. Furthermore, I could easily have made some slight error in the application of the observation.

At a mean anomaly of 43°.

For a similar longitude in the other semicircle, we shall again take up the observations of chapter 27. There I derived a distance somewhat less than 163,100 using the equation of the observations, but from the bare observations themselves I obtained 162,818, in the plane of the ecliptic as before. Now for one of the times introduced in that chapter, 1589 Feb. 12[14] at $5^h$ $13^m$ am, the mean longitude was $6^s$ 12° 38′ 44″[15], aphelion $4^s$ 28° 50′ 57″; and consequently, the mean anomaly was 43° 47′ 48″, lower than our previous one by 24 minutes. To this there corresponds [an adjustment of] about 64 units at this position on the eccentric. So the distance which was less than
253 163,137[16] at an anomaly of 43° 48′ will again be increased at anomaly 43° 24′ according to this ratio, so as to make it about 163,100 [*sic*] in this semicircle. In the previous one it was 163,051 and 162,996. Again, not an outstanding fit.

It should be noted, however, that in chapter 27, to which I am referring here, the observations led us to subtract 1′ 30″ from the eccentric position computed from our vicarious hypothesis at 5½° Libra, and this was through the observations of 1585, 1587, 1589, and 1590. Second, in chapter 18 above, the acronychal observation of 1589, at 5° Scorpio, gave the same testimony, namely, that our vicarious hypothesis needs to be diminished by $2\frac{1}{5}$′. And in 1591, at 26° Sagittarius, there still was one minute to be subtracted. Third, in this very chapter, about 16° Scorpio, the observations of 1589 and

[14] The text has February 11, probably because the observation was on the morning following February 11. In chapter 27 it is given as February 12, and this agrees with the eccentric position given there.

[15] This is the mean longitude for the same time one day later. The correct mean longitude for this time would be $6^s$ 12° 7′. The corresponding mean anomaly is 43° 16′, 7½′ *higher* (that is, nearer aphelion) than before. So instead of adding 64 parts, Kepler should have subtracted 20, which would have resulted in a distance of about 163,117, which is close to the distances in the other semicircle, as he stated them (but note that 163,051 should have been 162,658). Oddly enough, it is also close to what Kepler himself found: he would have noted more of a discrepancy, but in adding 64 to 163,137 he found the sum to be 163,100!

[16] That is, 163,100 increased by 37 to correct for the transfer from the plane of the ecliptic to the plane of Mars's orbit. See the end of chapter 27.

1594 required $3\frac{1}{2}$ minutes to be subtracted from the eccentric position computed from our vicarious hypothesis. So therefore this is constant about the middle longitude of this semicircle.

At a mean anomaly of 12°.

At a mean anomaly of the full-circle complement of 12°.

And likewise, near aphelion, we shall again take up the observations of chapter 28, where at a mean anomaly of 11° 37′ a distance of 166,180 or 166,208 was found (without correction for latitude). This was in the descending semicircle. But it was at a similar anomaly on the ascending semicircle at the following times.

On 1585 January 24 at $9^h$, when the position of the sun was 15° 9′ 5″ Aquarius, its distance from earth 98,590, the mean longitude of Mars $4^s$ 16° 50′ 10″, the aphelion $4^s$ 28° 46′ 41″, the difference between the mean anomaly and the full circle 11° 56′ 31″, and the consequent eccentric position from the vicarious hypothesis 18° 49′ 0″ Leo: the planet was observed at 24° 9′ 30″ Leo, latitude 4° 31′ 0″ N. The angle at earth was therefore 9° 0′ 25″, and at the planet 5° 20′ 30″. Therefore, the distance of Mars from the sun was 165,792. But if you subtract 1′ 30″ from the vicarious hypothesis here, as appeared necessary above in ch. 18 in the computation of the acronychal opposition, the angle at the planet becomes 5° 19′, and the distance of Mars from the sun 166,580. And the distance here is easily changed to this extent, because earth and Mars are close to one another. Therefore, for insurance, we shall bring in other positions.

On 1586 December 16 at $6\frac{1}{2}^h$ am, when the sun was at 4° 16′ 51″ Capricorn, 98,200 distant from earth, mean longitude of Mars $4^s$ 18° 39′ 9″, remainder of mean anomaly 10° 9′ 41″, eccentric position from the vicarious hypothesis 20° 20′ 30″ Leo: the declination of Mars was found to be 3° 54′, right ascension from Arcturus and Spica 177° 27′. Thus its longitude was 26° 6′ 24″ Virgo, latitude 2° 35′. Hence, the angle at earth was 81° 49′ 33″, at the planet 35° 45′ 54″. And the distance was 166,311, but by subtraction of 1′ 30″ from the eccentric position, 166,208. And at the previous distance from aphelion, 11° 37′, it would be about 70 units less. So it would be either 166,241 or 166,138.

On 1588 Nov. 6 at $6^h 50^m$ am, when the sun's position was 24° 3′ 43″ Scorpio, 98,630 distant from earth, mean longitude of Mars $4^s$ 20° 47′ 35″, remainder of the anomaly 8° 2′ 51″, eccentric position from the vicarious hypothesis 22° 7′ 48″ Leo: Mars was observed at 23° 16′
254 Virgo, lat. 1° 37′. Hence, the angle at earth was 60° 47′ 43″, at the planet 31° 8′ 12″. And thus the planet's distance from the sun was 166,511. But by subtraction of 1′ 30″ from the position of the vicarious

hypothesis, the distance becomes 166,396. And by this analogy, at the greater distance from aphelion of 11° 37′, where it is less by about 110, it is either 166,401 or 166,286. There is a discrepancy of 150 between this and the previous one. And if, keeping the correction of the eccentric position, we take a mean between the two, 166,230, as if saying that in the two observations of 1586 and 1588 there were some small observational errors in opposite senses in the determination of the distance, we hardly differ at all from the distance in the descending semicircle. Even this small difference will be able to be abolished by a slight retraction of the aphelion, of which more later. Thus, near aphelion also, as far as the senses can judge, we find the same distances from the sun at the same angle from aphelion in the two semicircles.

All three observations were made when Mars was in the east, and none with Mars in the west. For the rest of the observations [sc. of Mars at these mean longitudes] are lacking. Therefore, we will probably be safer to stay with the distance in the descending semicircle.

Third, let the same things we have explored above the middle longitudes now be explored below, near perihelion.

At a mean anomaly of 113 degrees.

In 1591 on the night following May 13, at 1 hour $40^m$ past midnight, when the sun was at 2° 8′ 43″ Gemini, distant from earth 101,487, while the mean longitude of Mars was $8^s$ 22° 18′ 4″, anomaly 113° 24′ 4″, equated 103° 15′ 48″, consequent eccentric position from the vicarious hypothesis 12° 9′ 48″ Sagittarius (or, by analogy with 26 Sagittarius nearby, just now brought to our attention, 12° 8¾′ Sagittarius): Mars was observed at 2° 24½′ Capricorn, latitude 2° 15′ S. Therefore, the angle at the earth was 30° 15′ 44″, and at the planet either 20° 14′ 39″ or 20° 15′ 42″. Hence, the distance of Mars (or of the point on the ecliptic) from the sun was 147,802, or more correctly, 147,683. Here, you see that an error of one minute in eccentric position causes the loss of 120 of our units at this distance of Mars from earth, and at this distance of the sun in the opposite position. So these slight discrepancies need no further attention. Besides, this observation is well supported by others on many of the days nearby, right up to the day of opposition with the sun. But since 12° 10′ Sagittarius is about 26½° from the node, the secant of the inclination at this place exceeds the radius by about 11 units, or 15 or 16 in our

present dimensions, so as to make the distance of Mars from the sun approximately 147,820 or 147,700.

At a mean anomaly of the full-circle complement of 113°.

For a similar distance from aphelion in the other semicircle, we shall take up again the observations of chapter 26, where I derived a distance of Mars from the sun of about 147,443 or 147,700 or 147,750. And at one of the times noted therein, namely, 1590 March 4 at $7\frac{1}{5}^{h}$, the [mean] longitude of Mars was $1^{s}$ 4° 11′ 20″. Hence, the full-circle complement of the mean anomaly was 114° 41′. We are thus lower down from aphelion than we were before by one degree and 17 minutes. And to one degree correspond 230 units at this position on the eccentric. Therefore, the distance of 113° 24′ on the ascending
255 semicircle would be 147,743 or 148,000 or 148,050 (extrapolating from the observations of ch. 26). But on the descending semicircle, 147,820 or 147,700 was found here. The difference is about 350 or 180 units, or none; it is rather uncertain. For the observations with Mars at perigee are rather poorly obtained, on account of the low elevation of the zodiac and many other causes. And you see in chapter 26 that the true distance, hesitantly accepted, fluctuates between 147,443 and 147,750, a difference of 300 units which, in our present undertaking, are of no great importance, since Mars is so low and close to the sun or the centre of the world.

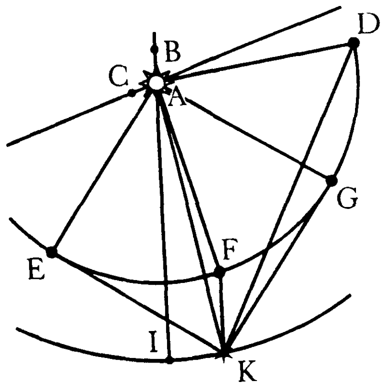
But let us descend here even farther towards perihelion, and explore the same thing about 22 degrees before and after perihelion.

At a mean anomaly of the full-circle complement of 162°.

On 1589 Dec. 3 at $5^{h}$ $39^{m}$, when the sun's position was 21° 44′ 56″ Sagittarius, and its distance from earth was 98,248, and the mean longitude of Mars was $11^{s}$ 16° 27′ 53″, full-circle complement of the anomaly 162° 24′ 11″, and the equated eccentric position 20° 4′ 32″ Pisces: Mars was observed at 15° 25′ 33″ Aquarius, lat. 1° 11′ 47″ S. But because it was found above in ch. 42 that our vicarious hypothesis errs somewhat near perihelion, we shall admit other positions, as many as we can obtain, and inquire of them, using the method of chapter 42, the distance of Mars from the sun, and at the same time a more correct eccentric position as well.

So, on 1591 Oct. 16 at $6^{h}$ $28^{m}$, when the sun was at 2° 39′ 15″ Scorpio, 99,142 distant from earth, Mars's mean longitude $11^{s}$ 13° 53′ 57″, full-circle complement of the anomaly 165° 0′ 9″, eccentric position from the vicarious hypothesis 16° 59′ 14″ Pisces: Mars was observed at 1° 27′ 18″ Aquarius, lat. 2° 10′ 52″ S.

Also, on 1593 Sept. 8 at $10^{h}$ $38^{m}$, when the sun was at 25° 41′ 0″




Virgo, 100,266 distant from earth, Mars's mean longitude $11^s\ 17°\ 10'\ 17''$, full-circle complement of the anomaly 161° 45′ 28″, and eccentric position from the vicarious hypothesis 20° 53′ 54″ Pisces: the planet was found at 8° 53′ 51″ Pisces with latitude 5° 14′ 30″ south.

Finally, on 1595 July 22 at $2^h\ 40^m$ am, when the sun was at 7° 59′ 52″ Leo, 101,487 distant from earth, Mars's mean longitude $11^s\ 14°\ 9'\ 5''$, and anomaly 164° 48′ 55″, and consequent eccentric position from the vicarious hypothesis 17° 16′ 36″ Pisces: the apparent position of Mars, from the most select observations, was 4° 11′ 10″ Taurus, lat. 2° 30′ S. Thus we twice have Mars in the most opportune position, in quadrature with the sun, while the positions of earth and Mars are also distant by a quarter circle.

And so, following the method of chapter 42, I shall make a test of the eccentric positions of the star, and to begin with I shall suppose that the distance of Mars at the first time was 139,212. Hence, the following ones were 139,033, 139,258, 139,045. For in such close proximity, the connection of the anomalies is easily known, as before. Let A be the sun, D, G, F, E, positions of the earth in 1589, 1591, 1593, 1595. Let K be the position of Mars, the same all four times (even though it is not quite the same in the observations). Let
256 the points be connected. AD, AG, AF, AE are given in position and length. And the length of AK is introduced four times. Moreover, the lines of observation DK, GK, FK, EK have known positions. Therefore, ADK, AGK, AFK, AEK are given. So, through the opposition of the sides to the angles, DKA, GKA, FKA, EKA are also given. Hence, so is the position of KA, four times.

| | | | | Anomaly | AK |
|---|---|---|---|---|---|
| DA | 21° 44′ 56″ Sag. | 98,248 | DK 15° 25′ 33″ Aqu. | 162° 24′ 11″ | 139.212 |
| GA | 2 39 15 Sco. | 99,142 | GK 1 27 18 Aqu. | 165 0 9 | 139.033 |
| FA | 25 41 0 Vir. | 100,266 | FK 8 53 51 Pis. | 161 45 28 | 139.258 |
| EA | 7 59 52 Leo | 101,487 | EK 4 11 10 Tau. | 164 48 55 | 139.045 |

| Compl. | Yields | Thus AK is | Vicarious |
|---|---|---|---|
| ADK 53° 40′ 37″ | DKA 34° 39′ 23″ | 20° 5′ 16″ Pisc. | 20° 4′ 32″ Pisc. |
| AGK 88 48 3 | GKA 45 28 27 | 16 55 45 Pisc. | 16 59 14 Pisc. |
| AFK 16 47 9 | FKA 12 0 4 | 20 53 55 Pisc. | 20 53 54 Pisc. |
| AEK 86 11 18 | EKA 46 44 30 | 17 26 40 Pisc. | 17 16 36 Pisc. |

So, since the first and third position here agree rather closely, some less thoughtful person will think that it should be established using these, the others being somehow reconciled. And I myself tried to do this for rather a long time. But since the second and fourth could not be reconciled, while the force of these observations was great, because in each the planet was observed in quadrature with the sun, and in the quadrilateral AEKG all the sides and angles are about equal, I therefore settled it as follows. From the vicarious hypothesis, you see that AK in the second observation ought to be distant from AK in the fourth by 17′ 22″. But by the assumption of this length, the two positions of AK are 30′ 55″ apart. So this is too much by 13′ 33″. And since all angles of the quadrilateral are about equal, I divided the excess in two, and added 6′ 46″ to the angles EKA, GKA. For in observation E, the line AK had moved forward too much, and not enough at G. So, with the two AK's moved back towards E and G, EK and GK staying fixed (for we are supposing the observations to be most certain), the angles at K will be uniformly increased. So now, given the angles GKA, 45° 35′ 13″, and EKA, 46° 51′ 16″, the other angles G, E, and the lines EA, GA remaining the same, AK comes out to be 138,765, and 138,787, differing from our assumed value by 258 units. So if we also subtract that much from the other two AK's, so as to make them 138,954 and 139,000, the resulting angles are DKA 34° 43′ 47″ and AK 20° 9′ 40″ [Pisces]; while FKA is 12° 1′ 24″ and AK 20° 55′ 15″ [Pisces]. But since I previously added 6′ 46″ at G and subtracted the same amount at E, I have therefore repositioned the eccentric positions at 17° 2′ 31″ Pisces at G, and 17° 19′ 54″ Pisces at E, increasing the position given by the vicarious hypothesis by 3′ 17″. Therefore, the same amount also ought to result at D, namely,

| | |
|---|---|
| | 20° 7′ 49″ Pisces |
| While I found | 20 9 40 |
| Difference | 1 51 more |
| At F | 20° 57′ 11″ Pisces |
| | 20 55 15 |
| | 1 56 less |

And so I have also brought the other two positions near enough together. For their errors lie both beyond and before the truth, which lends security. And to attribute an error of two minutes to observations at these positions, owing to the low elevation of the zodiac and horizontal variations, is not excessive.

257 At a similar anomaly on the descending semicircle, the observations at hand are no more than one, but it is certain enough. For on the night following June 29 in 1593, at $1^h\,30^m$ after midnight, when the sun was at 17° 25′ 42″ Cancer, 101,760 distant from earth, the longitude of Mars $10^s$ 10° 1′ 29″, anomaly 161° 5′ 29″, and the consequent position of Mars 6° 10′ 5″ Aquarius, it was observed at 13° 37′ 22″ Pisces, with latitude 4° 37′ S. Hence, the supplement of the angle at earth was 56° 11′ 46″, at the planet, or the parallax of the annual orb, 37° 27′ 23″. From which the distance of Mars from the sun comes out to be 139,036. But above, at an anomaly of 161° 45′ 28″, where Mars was 40 minutes farther from aphelion than here, the distance was found and established as 139,000. And at this position on the eccentric these 40 minutes effect a change of 52 units. So here too, by extrapolation from our anomaly, there results a distance of 138,984 at an anomaly of 161° 45½′, an admirable consensus much to be suspected. For it can hardly all be so certain and neat. Furthermore, both distances must be increased somewhat owing to the inclination of this position on the eccentric, which is at a maximum.

So, from this long induction, using a great many positions on the eccentric, it appears that those distances of Mars from the sun are equal whose points on the orbit are equally remote from aphelion, a question which we have investigated in ch. 16 and 42. This is an evident way of showing that the aphelion we have obtained is correct, by Euclid III. 7.

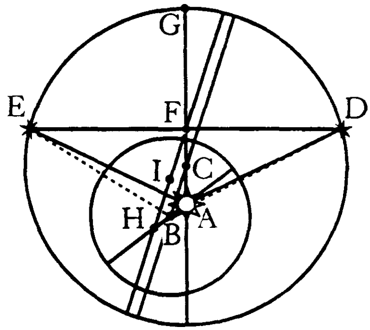
At the same time, the distances of the sun from earth are confirmed, which were derived in ch. 29 above and serve various purposes here. Nor is there any great discrepancy in the numbers that could testify to any flaw in them.

The implications of the observations presented in this chapter, and of the distances found through them, for the shaping of the planetary path, for which purpose we have produced them in this chapter, we shall postpone until chapter 55. First, there is something that must be proved in ch. 52 following, and in ch. 53 many more observations are going to be called upon to testify.

# 52

## Demonstration from the observations of chapter 51 that the planet's eccentric is set up, not about the centre of the sun's epicycle, or the point of the sun's mean position, but about the actual body of the sun; and that the line of apsides goes through the latter rather than the former

It is a happy accident that the distances found in chapter 51 also inform us about this, which, though promised in chapters 6, 26, and 33, I deliberately postponed until this point. For if I was correct in
258 constructing the eccentric of Mars about the body of the sun, it is necessary that the planet really be at its greatest distance from the sun in the region around 29° Leo, and that those positions which are at equal intervals from 29° Leo in either semicircle be at equal distances from the sun, and at unequal distances from the point that stands for the sun, which for Brahe is the centre of the sun's epicycle. More specifically, it should be less in the descending semicircle. When this is proved, it will follow in addition that the region around 24° Leo is neither the most distant from the sun's body, nor from the Copernican centre of the world, which for Brahe is the centre of the sun's epicycle, and also the centre to which the planetary circle is attached; and the regions at equal arcs' removal from 24° Leo in either semicircle are at unequal distances from the sun and from the point that stands for it. *For let there be set out the sun's centre* A, *Mars's line of apsides* AC, *eccentricity* AC, *and the eccentric* ED *with centre* C, *and let the point* F *above* AC *be the point of uniform motion,* G *the aphelion,* GFE *and* GFD *equal angles, and let* EA, DA *be connected, which will be equal, as has now been proven. And through* A *let the line* AB *be drawn towards Capricorn, and let* AB *be extended from* A *towards Capricorn until its length be 1800 of the units of which* AC *was 14,140 in chapter 42, and* AE, AD, *154,400; and let* B *be the centre of the earth's orb. Now because* BA *is directed towards* $5\frac{1}{2}$° *Cancer, and*




AE *towards 15½° Scorpio, the angle* EAB *is about 50° and acute, and* EBA *obtuse. Hence,* EA *is longer than* EB. *Likewise, since* BA *is directed towards 5½° Cancer, while* AD *is directed towards 12½° Gemini,* BAD *is therefore 157° and* ABD *is quite acute. Hence,* AD, *or* AE *which is equal to it, is shorter than* BD. *Therefore* BE *is much shorter than* BD, *and the difference is quite perceptible. For how can we neglect* AB, *which is 1800 or even more, when we could not tolerate observations with a mere 200 units' error? Hence, regions on opposite semicircles of the eccentric that are equidistant from* G, *such as* E *and* D, *are equidistant from no points off of the centre other than those on the line* CA *that goes through the body of the sun.*

*You may reply, however, that if* BC *is connected and extended, a new apsis is created where that line intersects the circle, and the point* D *is closer to that apsis than is the point* E. *So is it any wonder that* BD *is also longer? I answer that whatever lines are drawn,* AE *and* AD *always stay the same, since they are derived from the observations, in all three forms of hypotheses, and thus absolutely nothing in this derivation was assumed that could be subject to controversy. And so, with* AE *and* AD *remaining the same, let* BC *be drawn exactly as proposed in the objection. Nevertheless, that line* BC *will not give rise to a hypothesis that will fit the acronychal observations, as I proved in chapter 6. Instead, to save the acronychal positions, one needs to substitute for* BC *a line* FH *through* F *parallel to* CB, *passing through* F *and* H, *the centres of uniform motion for Mars and the sun. But when this is done, the centre of the eccentric is at the same time transferred from* C *to* I, *and there is more than a semicircle on the side of* E, *and less on the side of* D. *Nor are* AE *and* AD *left unchanged, but* AE *is*

*lengthened, and* AD *is shortened. And since these lines are altered, the observations at positions other than acronychal will never be saved, since they give evidence that* AE *and* AD *are equal.* I don't think there is any need for computation. Nevertheless, if there is anyone who
259 enjoys this labour (even though no astronomer should try anything with numbers whose foundations he has not previously seen in geometry, and geometry has just overturned the foundations of such an undertaking), he has an example of it in ch. 24 above. There, I computed the distances of earth from H, the point of uniformity of the earth's motion, and the distance of Mars from the same point H, simultaneously in a single operation, using the same observations by which I afterwards computed the distances of earth and Mars from the centre of the sun A, in ch. 26.

For the peculiar strength of the method I have used is this: that it shows that whatever point in the plane of the earth's circle is chosen that has a determinate position with respect to the sun's body, both in zodiacal longitude and in distance from the sun, described through a number of observations, also shows the distance of earth and Mars from that chosen point; and it does these things without any knowledge of the equated anomaly on the eccentric corresponding to that point. In fact, the only reason why I used that knowledge in chapter 26 was that it is a short cut.

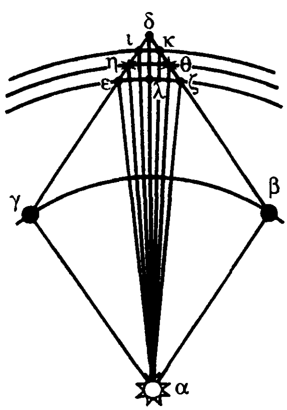
But in addition, there is another way to argue the point. It was proved in chapter 44 above that the planet's orbit is not a circle but an oval, such that the diameter on it which is called the [line] of apsides is the longest. Just now, in ch. 51, it was proved that regions that are equally removed from the point of the aphelion G also make an equal incursion at the sides. There is thus a real oval situated about the line AC, and therefore, it is not situated about the line FH. And one who would compute the various distances of Mars from the point H by the method just recommended will find a great irregularity in the distances, incapable of being included by any means in a circle or in any other possible figure set up about FH.

So again the faith that was pledged in chapter 6 and in many other places in this work, I have redeemed from all tincture of self-justification, and have shown that the eccentric of Mars cannot be referred to anything but the sun itself; and that, in addition, it is not only reason that stands with me, but the observations themselves, in my releasing the observations of Mars from the sun's mean motion and measuring them out by the apparent motion of the sun.

# 53

## Another method of exploring the distances of Mars from the sun, using several contiguous observations before and after acronychal position: wherein the eccentric positions are also explored at the same time

Since we are establishing new hypotheses here, in that we are enquiring into the natural cause of the eccentric equations, it is appropriate that we should explore everything as carefully as possible, lest in neglecting the foundations we build upon them a building doomed to ruin. And so it will further us to explore this same thing
260 (perfectly accurate distances of Mars from the sun) by many methods. *Let α be the sun, β the position of earth before opposition of Mars to the sun, and αβδ the angle of vision, or the arc of the elongation of δ from the sun. Likewise, let γ be the position of the earth after opposition, and αγδ the angle of vision. Thus, at the first time, let*



*the planet be on the line βδ, and at the second, on the line γδ, and let it actually traverse θη. So, when the time of the two observations is given, the angle θαη will be given precisely enough by the vicarious hypothesis, for whatever eccentric position*[1]*. If the pair of times are not far from each other, or if the planet is near the apsides or the middle longitudes, the difference in length of the lines αθ, αη will also be known approximately*[2]*. And we have included this much among our presuppositions only in order that there be no remaining difficulty here.*

*But if, in addition to the angles θβα, ηγα, given by observation, and βα, γα, which are known from Part III, we were to assume*[3] *[a value for] θα, and consequently also ηα, it is obvious that if this assumed value were longer than it should be, such as κα, ια, then the angle ιακ would come out less than it should be; and if, on the other hand, it were shorter than it should be, such as ζα, εα, the angle εαζ would come out greater than it should be.* So we must assume such distances as will make the angle of motion on the eccentric come out right.

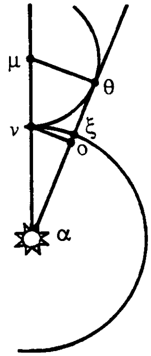
Any possible remaining error in the eccentric position will also occur in the same way. For let it be that θα, ηα are in their correct positions, and from that point let θα be carried forward, in error, through the angle θαδ. And let ηα likewise be carried forward, through the equal angle ηαε. You can see that αδ, substituted for αθ, is going to be very much too long, and αε, following αη, will be quite short, contrary to what, by hypothesis, is known at the start. Furthermore, the angle γαβ ought not to be as small as possible, so that no error of observation, or at least no minimal one, in opposite directions in the sky (as can happen) could have any great effect[4]. Now,

[1] What Kepler does not mention here is that he will use the foregoing data to find approximate values for the distances αθ, αη, which will then be adjusted by various means to produce the correct angle of motion on the eccentric.

[2] At aphelion and perihelion the rate of change in the distance vanishes; hence, near aphelion and perihelion the changes in the distance should be small, regardless of which theory is used to obtain the distances. At the middle longitudes, although the rate of change in the distance is considerable in any theory, it would be nearly the same no matter what theory is used, since the orbital curves are nearly parallel. At other places, however, it makes considerable difference which theory is used to obtain the difference in distances, a problem which Kepler never addresses directly.

[3] Kepler's use of this word here is revealing. The procedure he shows in his examples involves calculating, not assuming, a value for this distance. However, the distances obtained by this method are demonstrably incorrect, and there are grave difficulties in adjusting them to obtain correct distances, Although there are indications that Kepler did attempt to carry out this adjustment, the final table in this chapter is not (despite Kepler's claims) the result of such an attempt, as will be seen. There, the distances are simply 'assumed'; hence (presumably) Kepler's reluctance to represent them as having been calculated.

[4] The problem still remains that, although the distances will be wrong if the longitudes are wrong, and vice versa, one cannot tell where the error lies without some distance or




with the help of this method, we are going to go through the years 1582 in Cancer, 1585 in Leo, 1587 in Virgo, 1589 in Scorpio, 1591 in Sagittarius, 1593 in Pisces, and 1595 in Taurus. For in all these there are sufficient observations at hand.

If it seems a good idea to investigate demonstratively the elongation of the earth from the line through the sun and the planet at which any error in the distance of Mars from the sun would be most evidently perceived, let ch. 6 be consulted. For, following that chapter, we shall define it as that angle at the sun whose sine has to the radius about the same ratio as the excess of Mars's distance from the sun over the sine of the complement of the angle has to the distance itself. *For let* $\alpha$ *be the sun,* $\theta$ *the planet,* $\nu\xi$ *the earth's orb. From* $\theta$ *let the straight line* $\theta\mu$ *be drawn perpendicular to* $\theta\alpha$. *And on* $\theta\mu$ *let a number of centres be chosen, about which let circles through* $\theta$ *be described, until one of them be tangent to the earth's orb at* $\nu$. *The point* $\nu$ *will be where the defect of* $\alpha\theta$ *at* $\theta$ *appears most evidently, that is, where it subtends the greatest angle*[5]. *From* $\nu$ *let* $\nu o$ *be drawn parallel to* $\mu\theta$, *intersecting* $\alpha\theta$ *at* o. *I say that* $o\nu$ *is to* $\nu\alpha$ *as* $o\theta$ *is to* $\theta\alpha$. *For as* $\nu\mu$, *or (which is the same thing)* $\theta\mu$, *is to* $\mu\alpha$, *so is* $o\nu$ *to* $\nu\alpha$. *But* $\nu\mu$ *is to* $\mu\alpha$ *as* $o\theta$, *and (very nearly)* $\xi\theta$, *is to* $\theta\alpha$. *Therefore, etc.*

*Let* $\alpha\theta$ *be 161,000. Thus* $\xi\theta$ *will be nearly 61,000. And as 161 is to 61, so is 100,000 to 37,888. This, taken as a sine, shows the angle* $\nu\alpha\theta$ *to be 22° 15′, and greater, if instead of* $\xi\theta$ *you take* $o\theta$.

longitude that is known to be correct. Kepler's proposed approach to this problem is seen in the treatment of the data for 1593 (second table, below); however, his final solution is to abandon the process of adjustment and use the method of chapters 56 and 60 to obtain both the equations and the distances.

5 This is proved in ch. 6, pp. 163–167.

261 So, many days, nearly 45, must pass before the anomaly of relative motion is altered by 22¼ degrees. And before or after this time, αθ is much different. So at aphelion this angle of relative motion is about 28°, and at perihelion about 18⅓°.

And now, having found the termini at which any error that may arise will be most evident, owing to an incorrect distance of Mars from the sun, it is easy for us to choose suitable observations, since many are available.

We shall begin from the opposition of 1582, from which year we shall choose the following observations.

| | 1582 November 24 4ʰ am | December 26 8ʰ 30ᵐ | December 30 8ʰ 10ᵐ | 1583 January 26 6ʰ 15ᵐ |
|---|---|---|---|---|
| Observed at | 26° 38′ 30″ Cancer | 17° 40′ 30″ Cancer | 16° 0′ 30″ Cancer | 8° 20′ 30″ Cancer |
| Observed latitude | 2 49 10 N. | 4 7 0 N. | 4 8 0 N. | 2 52 12 N.[6] |
| Sun at | 11 40 40 Sagit. | 15 4 12 Capr. | 19 8 31 Capr. | 16 33 20 Aquarius |
| Sun–earth dist. | αβ 98,345 | αβ 98,226 | αγ 98,252 | αγ 98,624 |
| Mean anomaly[7] | 67 28 13 | 49 39 10 | 47 51 35 | 34 8 15 |
| Eccentric position | 0 43 34 Cancer | 16 7 10 Cancer | 17 57 32 Cancer | 0° 9′ 40″ Leo |
| On the ecliptic: αθ | 0 42 42 Cancer | 16 6 23 Cancer | αη 17 56 45 Cancer | αη 0 9 30 Leo |
| Resulting αθ | 158,920 | 163,082 | αη 158,842 | αη 164,116 |
| Because of the latitude[8] | 158,960 | 163,147 | 158,907 | 164,196 |

The two intermediate ones differ by 4240. And indeed, the later one αη, is shorter, although it should have been longer by 336[9]. So the sum of the two is 322,054. From this I subtract 336, and again add

[6] Comparison with the other data shows that this is much too low: the Mars–sun distance computed from this figure would be 222,573. Possibly the observed latitude was really 3° 52′ 12″, and the 3 was changed to a 2 in copying or typesetting: this would accord well with the distance.

[7] The mean anomalies given here are not consistent with those upon which the vicarious hypothesis was established (chapter 18), nor with the data given in the penultimate paragraph of this chapter, nor, indeed, with each other. However, if one considers the mean anomalies and longitudes of all eight of the dates in these first two tables, one finds that most of them are consistent with an aphelion of 29° 10′ 10″ Leo at noon on 1601 January 1/11 (9½′ farther forward than the position given at the end of this chapter). If the mean longitude for that time is decreased by 20″ to 10ˢ 7° 14′ 14″, computed mean anomalies and longitudes match Kepler's within a few seconds for five of the eight dates (the exceptions are those for December 1582 and March 1585). The different data may represent different stages in Kepler's attempts to 'tune' the vicarious hypothesis to fit all the observations more accurately. The 15′ Kepler adds to the mean anomalies in chapter 54 may be a similar vestige.

[8] Not 'By the latitude', as the Latin *Per latit.* would suggest. It is indeed possible to use the latitude to find the distance independently, but this procedure results in distances very different from Kepler's (the distance in footnote 6 above was obtained in this way). Furthermore, there would not be much point in performing such a computation, as the resulting distances would not be very accurate. The numbers Kepler presents here make it clear that he was simply adding a small correction for the transfer from the plane of the ecliptic to the plane of Mars's orbit.

[9] This difference is in accord with the distances on the ellipse, as can be seen by subtracting the distances in the final table in this chapter, which were obtained from the correct ellipse, as Kepler admits in chapter 56. To compare this difference with those which would result from the circle or the oval is harder than would at first appear, since one would have to decide

it. The halves of these are 160,859, which is $\alpha\theta$, and 161,363[10], which is $\alpha\eta$. And $\alpha\theta$ will be at 16° 5′ Cancer, and $\alpha\eta$ at 17° 55′ Cancer[11]. So here, the vicarious hypothesis would have lost $1\frac{1}{2}$ minutes.

But the distances themselves are not to be trusted, owing to the angle's being too small. For if the angle at $\delta$ be varied by one minute, through an error in observing, as easily happens, we shall be in error by a thousand units in either distance.

Therefore, let the two more remote ones be taken, which are found to differ by 5236. But we already know that they should differ by about 5570. So by an operation conducted as before, the more nearly correct values resulting are: $\alpha\theta$ 158,792, and $\alpha\eta$ 164,364, placing $\alpha\theta$ at 0° 41′ 0″ Cancer, and $\alpha\eta$ at 0° 8′ 30″ Leo. And it becomes certain, through observations on the four days at this position, that about $1\frac{1}{2}$ minutes must be subtracted from the eccentric positions derived from our vicarious hypothesis.

The distances found before are approximately confirmed as well, both before and beyond opposition, which turn out to have a magnitude between these. Unless, as the comparison indicates, they ought to be somewhat longer[12].

But at the same time it is clear that if the angle $\theta\delta\eta$ had been off by one minute, both distances would have been off by about 50 units, no more. So in these distances there can barely be an error of the hundredth part of the uncertainty that there was in the previous ones.

Now, if a longitude that was taken up expresses satisfactorily the observed values for the distances for these four days, it will also express the observed values for the intervening days, namely, November 25, 26, and 27, and December 3, 17, 27, 28, and 29 of 1582, and January 16, 17, 18, 19, 21, and 22 of 1583.

which of Kepler's data to take as primary (the mean anomaly, the eccentric anomaly, or the equated anomaly), and how they are to be determined from this table (since Kepler gives no position for the aphelion). However, if we follow Kepler's example and use the orbital parameters by which he computed the final table in the chapter, and use whichever anomaly is most convenient to compute the distances on the circle and oval, we find them to be about 321 and 366, respectively, for these dates and times. (It turns out not to matter much which anomaly is used: although the distances are different, the differences between them stay about the same.) Evidently, the difference given by Kepler is not consistent with those on either the circle or the oval, but is about midway between them, as one would expect if they came from the correct ellipse. Alternatively, it would have been possible to take the mean between the differences on the circle and the oval.

10 This number is in error, in that in the computation Kepler added 336 to the half of 322,054. However, the computed longitude corresponding to this distance does not reflect this error: hence, Kepler must have used the correct distance to find it.

11 These positions are computed using the adjusted earth-Mars distance, the earth-sun distance, and the angle at earth, to find the angle at Mars, and using this together with Mars's observed position to find Mars's heliocentric longitude.

12 This comparison is necessary because, as was noted above, Kepler's procedure will not show whether both distances are too short or too long.

262 Let us proceed to the opposition of 1585. For while the sun and Mars were at opposition on January 31 of that year, the planet was observed at many closely-spaced positions over the two months preceding and the same number following. From among them we shall take these four observations.

| | 1584 December 21 14ʰ | 1585 Jan. 24 9ʰ | February 4 6ʰ 40ᵐ | March 12 10ʰ 30ᵐ |
|---|---|---|---|---|
| Mars was observed at | 1° 13′ 30″ Virgo | 24° 7′ 30″ Leo | 19° 47′ 30″ Leo | 11° 46′ 0″ Leo |
| Latitude | 3° 31′ North | 4° 31′ North | 4° 28′ North | 3° 22′ North |
| Sun at | 10° 43′ 5″ Capr. | 15° 9′ 5″ Aquarius | 26° 10′ 31″ Aquarius | 2° 16′ 42″ Aries |
| Distance from earth | 98.210 | 98.595 | 98.840 | 99.850 |
| Mean anomaly of Mars | 29 46 53 | 12 4 21 | 6 21 31 | 12 47 15 |
| Eccentric position | 3° 54′ 34″ Leo | 18° 49′ 0″ Leo | 23° 34′ 47″ Leo | 9° 23′ 28″ Virgo |
| On the ecliptic | 3 53 56 Leo | 18 49 3 Leo | 23 35 0 Leo | 9 24 7 Virgo |
| Resulting distance αθ | 165.101 | 166.290 | and αη 166.182 | 166.131 |
| Because of the lat. | 165.184 | 166.378 | 166.260 | 166.206 |

The two intermediate ones differ by 118. They should have differed by 187 in the opposite sense, so that αθ would be 166,226 and αη 166,412. Therefore, αθ falls at 18° 48′ 47″ Leo, and αη at 23° 34′ 48″ Leo. And the contemptibly small alteration of the eccentric position confirms the vicarious hypothesis for this place. But we learn from this that an error of one minute in observation at this place would vitiate the two distances by about 100 units.

When the more remote ones are consulted, their difference is found to be 1022. From what is known already approximately from the hypothesis, the difference should have been greater, namely, 1275. And, in fact, the fourth [degree] of Leo is close to the eighteenth of Cancer, where previously something had to be subtracted from the eccentric position of the vicarious hypothesis. So, if you were to subtract one minute at the fourth of Leo, you would now make αθ a hundred units shorter, and if $2\frac{1}{2}'$, you would make it about 164,934, which is short enough that αη can also keep the length 166,206; and the last observation in the previous year 1583, which showed a length of 164,364, can be reconciled with it. For they should

have differed by 488, a certain enough value provided in advance by the hypothesis of the distances, while they actually differ by 570[13].

Furthermore, it is possible to transfer half of this $2\frac{1}{2}'$ change in the eccentric position to the observations. For if either of them has erred by one minute, that could have effected an error of 50 units in either distance.

It would be tedious to repeat the same method, using the same words, for all the years of the oppositions. And so, in the following table, I have placed the observations themselves which I have consulted, and added what resulted from the computations. The hypotheses underlying the calculations are these. The sun's position is taken from Brahe. The sun-earth distance is from ch. 30. The aphelion of Mars for the end of 1600[14] is 29° $0\frac{2}{3}'$ Leo[15]. The mean motion at the same time is $10^s$ 7° 14′ 34″[16]. The eccentricity and ratio of the orbs is as in ch. 54[17]. To this I have added the distances of Mars from the sun as if previously known[18]. So if, using these distances, we match the proposed observations, these distances will be the correct ones, which is what I proposed to show in this chapter.

[See table on p. 537.]

[13] It was ostensibly through this external reference that Kepler surmised that the distance for December was too long, and that the best values would be obtained by changing it and leaving the distance for March the same rather than adjusting both equally, as before. However, his subsequent use of the diametral distances (that is, distances as in the correct ellipse) leads one to suspect that the later theory played some part in his adjustments here.

[14] From the mean motion given, this would appear to have been noon on 1601 January 1 (old style), or January 11 in the Gregorian calendar.

[15] If one compares this with the earlier figure of 28° 48′ 55″ Leo on 1587 March 6 old style (see ch. 18), with 1′ 4″ per year added for the aphelial motion, one finds that Kepler has shifted the aphelion back 3′.

[16] Comparison with the mean motion established through Kepler's previous data shows that he has subtracted about one minute from the mean longitudes. The combined effect of this change and the change in the aphelion is to add 2′ to the mean anomalies.

[17] What Kepler does not say here is that he actually used the method of ch. 60 (that is, 'Kepler's First and Second Laws') to compute the equations. The translator's recomputation comes within a few seconds of most of Kepler's positions (notable exceptions being the positions for December 1582, off by 3′ 15″ and 2′ 26″, respectively). The vicarious hypothesis, in contrast, shows differences that gradually wax and wane, reaching a maximum of $2\frac{1}{2}$° around the beginning of Pisces.

[18] As Kepler states in chapter 56 below, these distances were obtained from the eccentric anomaly β for the ellipse, using (in effect) the equation:

$$d = r + e \cos \beta,$$

where r is the semimajor axis and e is the distance by which the sun is eccentric from the centre of the ellipse.

These, then, are the distances that will result from an investigation using the method of this chapter and the observations proposed. The apparent positions, on the other hand, when Mars's eccentric position is in Cancer, will come out about 4 minutes back from these, and in Sagittarius and Capricorn the same number of minutes forward. These small errors do not come from incorrect distances, for they would then be in opposite senses on opposite sides [of opposition], and not in the same sense[19]. I believe they can be reconciled by changing the sun's apogee by one degree, which is easily permitted by Brahe's observations. Nevertheless, I am not going to say anything definite at present. For the correction of both this apogee and the entire hypothesis is reserved for the book of Tables.

[19] That is, for two adjacent observations on either side of opposition (such as those of December 1582), incorrect distances should have opposite effects upon the computed apparent positions, while in fact the two computed positions appear to be in error in the same direction. It should be noted, however, that for these two dates, the heliocentric longitudes are incorrect; thus, Kepler is trying to find a natural explanation for data that are actually erroneous.

| Time | | Sun's position | Sun-earth distance | Sun–Mars distance[20] | Mars's eccentric pos. on ecliptic[21] | Computed position | Observed position | Difference | Latitude |
|---|---|---|---|---|---|---|---|---|---|
| | | | | | | | | | North |
| 1582 23 Nov. | $16^h$ 0 | 11°41′ Sagit. | 98,345 | 158,852 | 0°42′11″ Cancer | 26°40′ 0″ Cancer | 26°38′30″ Cancer | 1′30″ + | 2°49′ |
| 26 Dec. | $8^h$ 30 | 15 4 Capr. | 98,226 | 162,104 | 16 7 18 Cancer | 17 44 19 Cancer | 17 40 30 Cancer | 3 49 + | 4 7 |
| 30 Dec. | $8^h$ 10 | 19 9 Capr. | 98,252 | 162,443 | 17 56 32 Cancer | 16 6 20 Cancer | 16 0 30 Cancer | 5 50 + | 4 8 |
| 1583 26 Jan. | $6^h$ 15 | 16 33 Aquar. | 98,624 | 164,421 | 0 6 24 Leo | 8 17 57 Cancer | 8 20 30 Cancer | 2 33 − | 2 52 |
| 1584 21 Dec. | $14^h$ 0 | 10 16 Capr. | 98,207 | 164,907 | 3 51 45 Leo | 1 14 34 Virgo | 1 13 30 Virgo | 1 4 + | 3 31 |
| 1585 24 Jan. | $9^h$ 0 | 14 53 Aquar. | 98,595 | 166,210 | 18 47 8 Leo | 24 3 58 Leo | 24 7 30 Leo | 3 32 − | 4 31 |
| 4 Feb. | $6^h$ 40 | 26 10 Aquar. | 98,830 | 166,400 | 23 33 41 Leo | 19 43 52 Leo | 19 47 0 Leo | 3 8 − | 4 28 |
| 12 Mar. | $10^h$ 30 | 2 16 Aries | 99,858 | 166,170 | 9 23 14 Virgo | 11 43 31 Leo | 11 46 0 Leo | 2 29 − | 3 22 |
| 1587 25 Jan. | $17^h$ 0 | 16 1 Aquar. | 98,611 | 166,232 | 8 13 40 Virgo | 4 41 50 Libra | 4 42 0 Libra | 0 10 − | 3 26 |
| 4 Mar. | $13^h$ 24 | 24 0 Pisces | 99,595 | 164,737 | 24 56 50 Virgo | 26 24 41 Virgo | 26 25 40 Virgo | 0 59 − | 3 38 |
| 10 Mar. | $11^h$ 30 | 29 52 Pisces | 99,780 | 164,382 | 27 35 54 Virgo | 24 5 15 Virgo | 24 5 15 Virgo | 0 0 | 3 29 |
| 21 Apr. | $9^h$ 30 | 10 48 Taurus | 101,010 | 161,027 | 16 44 51 Libra | 15 49 50 Virgo | 15 48 20 Virgo | 1 30 + | 1 48 |
| 1589 8 Mar. | $16^h$ 24 | 28 36 Pisces | 99,736 | 161,000 | 16 55 14 Libra | 12 14 7 Scorp. | 12 16 50 Scorp. | 2 43 − | 2 4 |
| 13 Apr. | $11^h$ 15 | 3 38 Taurus | 100,810 | 157,141 | 4 1 50 Scorp. | 4 45 0 Scorp. | 4 43 20 Scorp. | 1 40 + | 1 10 |
| 15 Apr. | $12^h$ 5 | 5 36 Taurus | 100,866 | 156,900 | 5 1 41 Scorp. | 3 58 57 Scorp. | 3 58 20 Scorp. | 0 37 + | 1 4 |
| 6 May | $11^h$ 20 | 25 49 Taurus | 101,366 | 154,326 | 15 30 36 Scorp. | 27 8 17 Libra | 27 7 20 Libra | 0 57 + | 0 7 |
| | | | | | | | | | South |
| 1591 13 May | $14^h$ 0 | 2 10 Gemini | 101,467 | 147,891 | 12 7 38 Sagit. | 2 15 36 Capr. | 2 20 0 Capr. | 4 24 − | 2 25 |
| 6 Jun. | $12^h$ 20 | 24 59 Gemini | 101,769 | 144,981 | 25 38 48 Sagit. | 27 11 45 Sagit. | 27 15 0 Sagit. | 3 15 − | 3 55 |
| 10 Jun. | $11^h$ 50 | 28 47 Gemini | 101,789 | 144,526 | 27 56 49 Sagit. | 25 57 57 Sagit. | 26 2 36 Sagit. | 4 39 − | 4 8 |
| 28 Jun. | $10^h$ 24 | 15 51 Cancer | 101,770 | 142,608 | 8 29 32 Capr. | 21 4 21 Sagit. | 21 10 0 Sagit. | 5 39 − | 4 45 |
| 1593 21 Jul. | $14^h$ 0 | 8 26 Leo | 101,498 | 138,376 | 20 1 38 Aquar. | 17 43 14 Pisces | 17 45 45 Pisces | 2 31 − | 5 46 |
| 22 Aug. | $12^h$ 20 | 9 11 Virgo | 100,761 | 138,463 | 10 15 25 Pisces | 13 9 39 Pisces | 13 10 15 Pisces | 0 36 − | 6 7 |
| 29 Aug. | $10^h$ 20 | 15 54 Virgo[22] | 100,562 | 138,682 | 14 37 15 Pisces | 11 11 41 Pisces | 11 14 0 Pisces | 2 19 − | 5 52 |
| 3 Oct. | $8^h$ 0 | 20 15 Libra | 99,500 | 140,697 | 6 19 39 Aries | 7 49 54 Pisces | 7 50 10 Pisces | 0 16 − | 3 17 |
| 1595 17 Sept. | $16^h$ 45 | 4 18 Libra | 99,990 | 143,222 | 22 49 19 Aries | 26 5 45 Taurus | 26 7 12 Taurus | 1 27 − | 1 42 |
| 27 Oct. | $12^h$ 20 | 13 59 Scorpio | 98,851 | 147,890 | 15 35 38 Taurus | 18 50 46 Taurus | 18 51 15 Taurus | 0 29 − | 0 6 |
| | | | | | | | | | North |
| 3 Nov. | $12^h$ 0 | 21 2 Scorpio | 98,694 | 148,773 | 19 26 33 Taurus | 16 18 33 Taurus | 16 18 30 Taurus | 0 3 + | 0 17 |
| 18 Dec. | $8^h$ 0 | 6 43 Capr. | 98,200 | 154,539 | 13 2 29 Gemini | 11 39 1 Taurus | 11 40 0 Taurus | 0 59 − | 1 40 |

[20] Not the same as those found in the text: these are the 'diametral distances' as determined in ch. 56 (p. 546), as Kepler himself points out.

[21] Also not the same as those found in the text, nor are they the same as the adjusted positions given in the analysis of the data in each table. They are in fact positions computed from the ellipse and the area law. Hence, while Kepler presents this table as a test of empirically determined distances and positions, it is actually an empirical test of the theory of chapters 59 and 60.

[22] Corrected from 11°54′ Virgo on the basis of the other data.

264

# 54

## A more accurate examination of the ratio of the orbs

In chapter 42, we did actually establish the ratio of the orbs from observations at positions other than acronychal, but they were not ones that were in agreement with one another entirely and to our full satisfaction. Moreover, considered in itself, regardless of whether the most exact observations be available, the procedure is incapable of being brought to a certitude of 100 units. So it has to be done by polling and counting the votes. And in chapter 28, at a mean anomaly of 11° 37′, which, after the correction of chapter 53 preceding, becomes 11° 52′[1], the distance of the point on the ecliptic to which a perpendicular dropped from the body of Mars would descend, was found to be 166,180, or 166,208. And therefore, since this position is 23° from the northern limit, the inclination will be about 1° 43′, and excess of the secant will be 45 units, which will be about 70 in our dimensions. Therefore, the distance of Mars from the sun will be 166,250 or 166,278.

We shall now also compare the observations of chapter 51, so as to be supported by an approximate consensus. In 1586, with 10° 9′ 41″ of mean anomaly remaining, or 9° 54′ 41″ after correction, we found 166,311. But by subtracting $1\frac{1}{2}$′ from the position given by the vicarious hypothesis, we found 166,208. So for a subtraction of 3

[1] The reader will note that nowhere in chapter 53 is there any mention of adding a full 15′ to the mean anomaly. Furthermore, the mean anomaly computed from the orbital parameters of the vicarious hypothesis was 11° 19′ 42″ at that position in chapter 28. Again, apparently, something is going on here about which Kepler is not being entirely candid. It seems likely that this reference is to an earlier version of chapter 53, and Kepler neglected to change it when the chapter was rewritten.

minutes less than two degrees, about 95 should be subtracted, making it 166,113. For the latitude, 80 must again be added, making it 166,193. Thus in 1588, when the remaining [mean] anomaly was 8° 2′ 51″, or 7° 47′ 51″ corrected, by a subtraction of 1½′ from the position given by the vicarious hypothesis we found the distance to be 166,396. Thus, a position 4° 4′ lower will be shorter by about 102, making it 166,294. And, corrected for latitude, 166,284. This was previously found to be 166,193, from 1586. The mean is 166,238. In the descending semicircle, however, from 5 observations, we had found 166,250 or 166,278. So, although the difference is imperceptible, let us nevertheless take the mean, 166,260, giving more trust to the descending semicircle, as it is better confirmed by the observations.

Let it thus be [taken as] certain that at a mean anomaly of 11° 52′ the distance is 166,260. Hence, however great an hypothetical value you may conceive by some rough method, which is to be confirmed shortly thereafter, it follows that where the radius is 100,000, the distances at aphelion cannot increase more than 164 units, and even less if you use the hypothesis of a perfect circle. But those units accumulated through a preconceived ratio of the orbs, as it is set up in ch. 42, add about 250, and these added to 166,260 make 166,510. But above, in ch. 42, we found 166,780, using weaker observations. The difference is 270 units.

We shall also treat likewise the perihelial distance which in ch. 42 was found to be 138,500, from observations that were not solid enough.

265 Just now, in ch. 51, at a remaining [mean] anomaly of 161° 45½′, or 161° 30½′ after correction, we found the distance, before correction for latitude, to be 139,000 or 138,984. So let 139,000 be at 21 Pisces. Since this position is 35 degrees from the limit, the inclination is therefore 1° 31½′. The excess of the secant will be 35½, which is equivalent to 49 of our units. And so the true distance of Mars from the sun is 139,049. But if the radius is 100,000, the perihelial distance is 575[2] units shorter than that at an anomaly of 161½°, which becomes 876 of our units, or less, if you use a perfect circle. And when these

[2] Kepler's estimate of the change in distance on the oval here is too large, although above, near aphelion, it was nearly correct. The figure here should have been 506 (using a provisional eccentricity of 9265 where the radius is 100,000). This would result in an eccentricity of 9261, very close to Kepler's 'truest and best fitted' value of 9265.

are subtracted from 139,049, there remains the perihelial distance of 138,173. The difference from the value 138,500, found in chapter 42, is 327.

So, according to this method, these distances were found:

| | |
|---|---|
| Aphelial | 166,510 |
| Perihelial | 138,173 |
| Diameter | 304,683 |
| Semidiameter | 152,342 |
| Eccentricity | 14,169 |

And where 152,342 becomes 100,000, 14169 becomes 9301.

Nevertheless, because our observations, especially at perigee, do not bear out that great a difference, and since it can happen that the vicarious hypothesis, since it is false, also might introduce some falsity into the eccentricity, all the votes should be counted before the result is announced.

And so we shall adapt the aphelial distance found here, 166,510, to the eccentricity of ch. 42, which was 9265. And as 109,265 is to 90,735, so is 166,510 to 138,274, where the radius is about 152,400.

Also, manifest experience has shown that the eccentricity that is most true and best fitted to the physical equations is between 9230 and 9300; that is, the eccentricity of chapter 42, which is 9265.

Therefore, that we might not excessively abandon the perihelial distance found in this chapter, which is 138,173, nor put too much trust in the aphelial distance of 166,510, let us conclude that the truest aphelial is 166,465, and the perihelial, 138,234, where the radius is 152,350.

# 55

## From the observations of chapters 51 and 53, and the ratio of the orbs of chapter 54, it is demonstrated that the hypothesis seized upon in chapter 45 is in error, and makes the distances at the middle longitudes shorter than they should be

Indeed, I began to say this in chapter 51, But since more observations, and more suitable ones, were going to be provided to give evidence in chapter 53, from which at the same time something else was also inferred in chapter 52, the full demonstration was therefore postponed to this point.

266 There is no need for verbosity. At the mean anomalies of all the examples appearing in ch. 51 and 52 let the distances be computed according to the hypothesis of chapter 45 and the ratio of orbs of ch. 54, by the method I used from chapter 46 through chapter 50, and let them be compared to the distances of ch. 51 and 53, found using infallible observations. It will be apparent that the more we descend from the apsides, the more the computed distances are deficient with respect to the observed distances, a result quite the opposite of what we saw in ch. 44 above. For there, the distances computed according to the law of the circle were longer than the observed distances at the middle longitudes, while here, the distances resulting from the hypothesis that makes the planet's orbit oval are shorter. It is therefore obvious that the planet's path is neither a circle nor such as to make as great an incursion from the circle at the sides as does the oval that arose from the opinion of chapter 45 and was described in chapter 46; but takes a middle course. And if, in turn, using the distances of chapter 45, you compute the observed positions of Mars, especially those which, in ch. 53, stood at some distance on either side of opposition, the planet before opposition will fall too far forward, and after opposition, too far back. This is most evident in the

descending semicircle in 1589 and 1591, and in the ascending semicircle in 1582 and 1595. For in those places, the oval of chapter 45 is 660 units[1] too small, while the perfect circle is too large by the same amount, and this can have an effect upon the appearances of 20 minutes and more. Thus, David Fabricius was able to use his observations to charge my hypothesis of chapter 45, which I had communicated to him as true, with this error of having distances that are too short at the middle longitudes, writing at the very time when I was labouring to seek out the true hypothesis through repeated, careful operations. He was, in fact, quite close to arriving at the truth before me[2]. And since the perfect circle errs the same amount in the opposite direction, it is thus argued rightly that the truth is in the middle, between the two.

Moreover, the equations computed from physical causes in chapters 49 and 50 gave the same testimony, namely, that the lunule cut off from the perfect semicircle ought to have only half the breadth of the one which the opinion of chapter 45 cuts off. Therefore, nothing prevents our saying that the matter is most certainly demonstrated: that the opinion of chapter 45, in remedying the excess of the perfect circle, falls into the opposite defect.

So the physical causes of ch. 45 go up in smoke.

[1] Where the mean between the aphelial and perihelial distances is 152,350. This would be about 433 units where this mean distance is 100,000.

[2] See Fabricius's letter of 27 October 1604 (Letter 297, in KGW 15 pp. 58–62), and Kepler's reply of 18 December 1604 (Letter 308, in KGW 15 pp. 78–81). Despite Kepler's generous acknowledgement of Fabricius's ostensible near-precedence, the latter's attachment to circularity was such as to have eliminated any such possibility. See his letter of 20 January 1607 (Letter 408, in KGW 15 pp. 376–386).

# 56

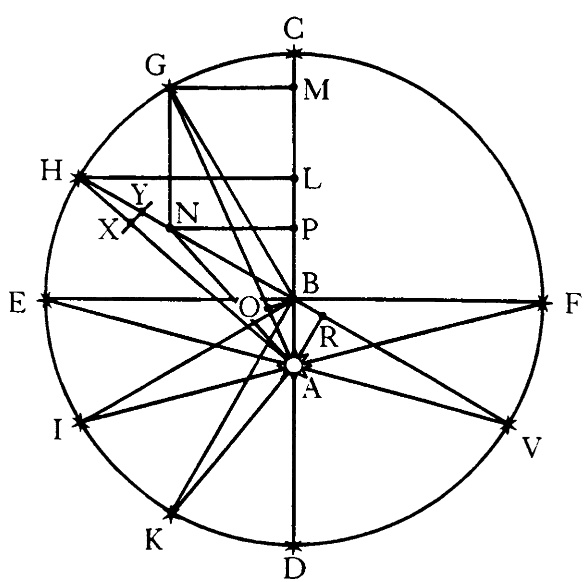
## Demonstration from the observations already introduced, that the distances of Mars from the sun are to be chosen as if from the diameter of the epicycle

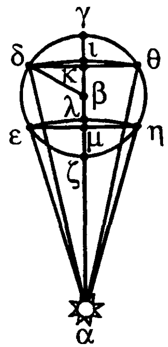
267 The breadth of the lunule of chapter 46 above, born to us out of the opinion of chapter 45 which instructed us to cut it off from the semicircle – this breadth, I say, was found to be 858 units, of which the semidiameter of the circle is 100,000. But then, by two arguments, by no means obscure, which I have already presented in chapters 49, 50, and 55, I concluded that the breadth of the lunule is to be taken as only half that, namely, 429, or more correctly, 432, and in units of which the semidiameter of Mars is 152,350, nearly 660. I therefore began to think of the causes and the manner by which a lunule of such a breadth might be cut off.

While I was anxiously turning this thought over in my mind, reflecting that absolutely nothing was accomplished by chapter 45, and consequently my triumph over Mars was futile, quite by chance I hit upon the secant of the angle 5° 18′, which is the measure of the greatest optical equation. And when I saw that this was 100,429, it was as if I were awakened from sleep to see a new light, and I began to reason thus. At the middle longitudes the lunule or shortening of the distances is greatest, and has the same magnitude as the excess of the secant of the greatest optical equation 100,429 over the radius 100,000. Therefore, if the radius is substituted for the secant at the middle longitude, this accomplishes what the observations suggest. And, in the diagram in chapter 40, I have concluded generally that if you use HR instead of HA, VR instead of VA, and substitute EB for EA, and so on for all of them, the effect on all the eccentric positions will be the same as what was done here at the middle longitudes. And

by equivalence, in the small diagram of chapter 39, ακ will be taken instead of the lines αδ or αι, and αμ for αε or αλ.

And so the reader should peruse chapter 39 again. He will find that what the observations testify here was already urged there, from natural causes, namely, that it appears reasonable that the planet perform some sort of reciprocation, as if moving on the diameter of the epicycle that is always directed towards the sun. He will also find that there is nothing more at odds with this notion than this: that when we proposed to represent a perfect circle, we were forced to make the highest parts γι of the reciprocation unequal to the lowest λζ, which parts correspond to equal arcs on the eccentric, the highest being short, and the lowest long. So, now that the planet's circular path is denied, and κα, μα are taken instead of δα, εα, that is, instead of ια, λα, as was said, it follows further that those parts of the reciprocation, such as γκ, μζ, are equal. And that which had tormented us for a long time in chapter 39 now surrenders to us in the statement of the truth we have perceived.






As for the middle parts κμ still being larger than the extremes γκ,
268 μζ, it will be said in chapter 57 following that this is in accord with nature, contrary to what we had been able to understand in chapter 39.

But in addition, the difficulty that arose in chapter 39 through supposing that the increase of the sun's [apparent] diameter serve the planet as an index for its approaching and receding, now vanishes entirely, as will appear in chapter 57.

Thus, concerning the eccentric anomaly of 90°, I easily was able to see in the manner just mentioned, that instead of the distance EA of the perfect circle, EB is to be taken, corresponding to an equated anomaly EAB.

And although I have drawn a general conclusion concerning all the anomalies using a single one as an example, this was not a consequence just of that one anomaly: there was need to strengthen it using closely spaced observations.

So now you understand the capacity in which the observations of chapters 51 and 53 are appointed to serve us, namely, to give this evidence.

Come, then: let the eccentric anomalies CBG, CBH be computed at the equated anomalies set out in those chapters, that is, at the angles CAG, CAH, and so on. Nor is there any need to strive after minute parts, nor be concerned about the imperfection of the eccentric equations that still remain in ch. 19, 29, 43, 47, 48, 49, and 50. Use any of these methods, particularly the one in ch. 43. You will not err in the equations by more than eight minutes.

When the angles are set up, seek out the lines, HR corresponding to the equated angle HAC, RV corresponding to the equated VAC, and so on for the others, and transpose them to the dimension of the orbs found in ch. 54. Your findings will be as in the following table.

| From the observations of ch. 51 | | |
|---|---|---|
| On the descending semicircle | On the ascending semicircle | Computed from the reciprocation |
| 166,180<br>166,208 | 166,401<br>166,296 | 166,228 |
| 162,994<br>163,051 | 163,100 | 163,160 |
| 158,091<br>158,111 | 158,217 | 158,074 |
| 154,400 | 154,278 | 154,338 |
| 147,820<br>147,700 | 147,743<br>148,000<br>148,050 | 147,918 |
| 139,000 | 138,984 | 139,093 |

In the observations of chapter 53, there is no need to do the same thing. For I previously used this same method of reciprocation to find out the distances of Mars from the sun which I presented in order to compute Mars's apparent positions. And since they are in agreement with the observations, they are therefore correct.

As you see, therefore, the distances measured on the diameter, found *a priori* in ch. 39, are confirmed by closely spaced and very reliable observations throughout the entire perimeter of the eccentric.

# 57

## By what natural principles the planet may be made to reciprocate as if on the diameter of an epicycle

269 It appears, then, from the most reliable observations, that the course of the planet through the ethereal air is not a circle, but an oval figure, and that it reciprocates on the diameter of a small circle in the following manner. Suppose that, after describing equal arcs on the eccentric, the planet comes to be at the diametral distances $\gamma\alpha$, $\kappa\alpha$, $\mu\alpha$, $\zeta\alpha$, instead of the circumferential distances $\gamma\alpha$, $\delta\alpha$, $\epsilon\alpha$, $\zeta\alpha$ (that is, $\gamma\alpha$, $\iota\alpha$, $\lambda\alpha$, $\zeta\alpha$), upon which the perfect circle lies. It is clear from inspection that a lunule is cut off from the perfect semicircle of the eccentric, whose breadth at any point is equal to the differences between the two diverse distances, such as $\iota\kappa$, $\lambda\mu$. This is proposed not on the basis of arguments *a priori*, but of observations, as I have just said; so now the physical theories will proceed more correctly than they had hitherto.*** For it is not by any ratiocinative or mental process that a planetary mind assigns the equal parts of the reciprocation $\gamma\kappa$, $\kappa\mu$, $\mu\zeta$, to equal arcs CD, DE, EF, of the as yet untraversed eccentric, for the former are not equal. Instead, the reciprocation is coordinated with the space traversed on the eccentric by natural means, which depend not upon the equality of the angles DBC, EBD, FBE, but upon the strength**[1] of the ever increasing angle DBC, EBC, FBC, which strength approximates the sine (so called by the geometers). The manner in which the ascent is thus

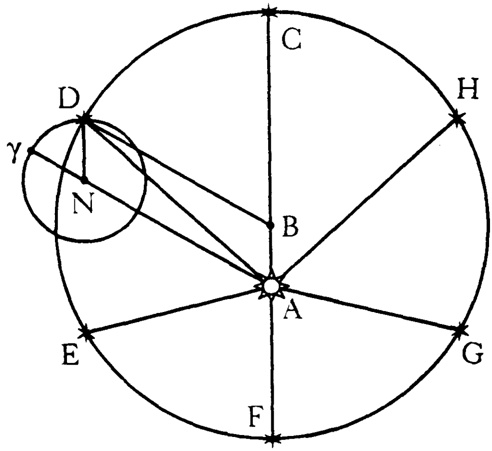
Terms: What the circumferential distance is, and what the diametral.

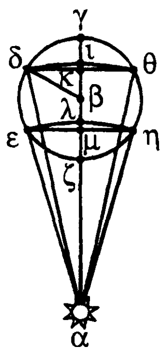
***The principle of this reciprocation is proved to be natural.

**This is the genuine measure of this reciprocation, supported by reason; in other words, it is the reason

[1] The phrase, 'supported by reason' (in the marginal note) is an attempt at rendering the Greek word 'απολογητος' which appears in Kepler's text. However, there seems to be no such word in classical Greek. The word 'απολογετικος' means 'suitable for defense, apologetic': perhaps this is what Kepler intended.

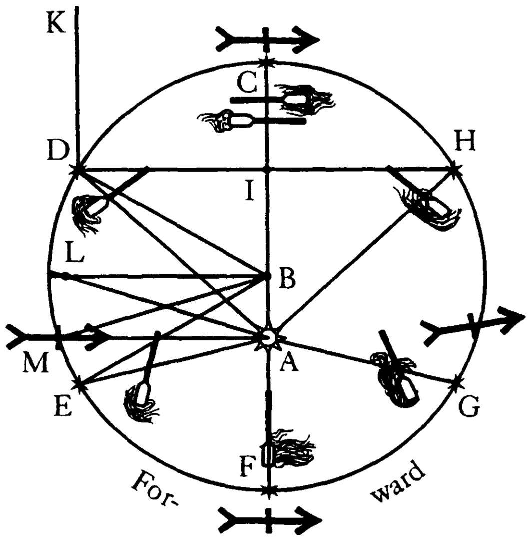
why the versed sine of the eccentric anomaly is the measure of this reciprocation.




imperceptibly changed into descent by continuous diminution is more probable than if the planet were said suddenly to turn its prow in the other direction – as we have indeed said in ch. 39, in clearly showing this to conflict with the experience [embodied in] the observations. And since the finger points to a natural way of measuring this reciprocation, its cause will also be natural; that is, it will be some natural – or better, corporeal – faculty, and not a planetary mind.

Also, in ch. 39, for the best reasons, one of our suppositions was




Natural examples of reciprocations of this kind.

Oars.

that a planet cannot make a transition from place to place by the bare effort of its inherent forces unless these be assisted or directed by an extrinsic force. If this conclusion still stands, we must as a consequence also ascribe this reciprocation in part to the solar power. In our exertions to this end, we shall be obliged once again to take up our oars which were introduced in ch. 39. Let there be a circular river
270 CDE, FGH, and in it a sailor who revolves his oar once in twice the periodic time of the planet, by an inherent and perfectly uniform force. Thus at C let the line of the oar be at right angles to the line from the sun, and at alternate returns let the bow and stern alternate in being forward. At F, however, let the line of the oar be part of the line from the sun, and at other positions let it have an intermediate inclination. Now the stream, flowing down upon the oar at DE, will push the ship[2] down towards A, while at C it will push very little, since the oar is also but slightly inclined. The same is true at F, because at this moment the stream strikes the oar directly. At D and E, however, it pushes down more strongly, because here the oar is

[2] Note that the 'ferryman' and the 'skiff' of chapter 38 (for which see the footnote to that chapter), which suggested ties to Charon and the classical underworld, have now become simply 'sailor' and 'ship'. The change in words primarily represents transformation of a concrete example to a more general and abstract model. Yet the language is also marvellously suggestive of the way Kepler's new universe is breaking out of the restrictions of the old 'cave world' and at the same time abandoning the old gods that made it live.

greatly disposed to such an approach by its inclination. The opposite happens in the ascending semicircle. For the river, coming beneath the oar at G and H, drives it away from the sun.

At the same time it will also happen, other things being equal, that the impulse will be less at C than at F, since our river is weak at C and strong at F. And this is is also in accordance with our wishes, since our reciprocation has been following equal spaces on the eccentric, and the planet spends longer in the upper ones than in the lower.

Defect of the example.

This example only shows the possibility of this arrangement. In itself it is inadequate, since the rotations of the oar and the river are accomplished, not in the same time, but a double time. Furthermore, to those looking at them from earth, the faces of the planets should appear to change, while the face of the moon, although it participates with the planets in that motion which we are discussing, does not change over the course of a month. Instead, it always is turned towards the earth, whence its eccentricity is reckoned. In addition, while the force of a river is material (for its water acts by its weight and material impetus), the force of the sun is immaterial. Therefore, the comparison with the planets ought to be different: they need no oar, no physical instrument, for catching hold of the force of some weighty thing (for that motive *species* of the sun has no weight). Nor do we deem it fitting that the stars have corporeal oars, seeing that we hold them to be round.

Example of the magnet.

William Gilbert's magnetical philosophy.

But from this very refutation, there comes another example, which will perhaps be more suitable. The river and the oar are of the same quality. The river is an immaterial *species* of magnetic power in the sun. So why not have the oar too borrow something from the magnet? What if all the bodies of the planets are enormous round magnets? Of the earth (one of the planets, for Copernicus) there is no doubt. William Gilbert has proved it.

Some magnetic arrangement in the body of the planet appears to be the cause of this reciprocation.

But to describe this power more plainly, the planet's globe has two poles, of which one seeks out the sun, and the other flees the sun. So let us imagine an axis of this sort, using a magnetic strip, and let its point seek the sun. But despite its sun-seeking magnetic nature, let it remain ever parallel to itself in the translational motion of the globe, except to the extent that over the ages it transfers the polar direction
**271** from one of the fixed stars to another, thus causing the progressive motion of the aphelion. I nevertheless admit the possibility that a mind may be needed for both of these, of such a nature as to be adequately instructed by the animate faculty for performing this

motion. For this is a motion, not of the entire body from place to place (which motion was rightly ascribed in ch. 39 above to a motive cause inherent in the planets), but of the parts about the centre of the whole, as if at rest.

Example of the earth.

Here again, in the globe of the earth there is an example of this directional property of the axis, from Copernicus. For as long as the axis of the earth, in the annual circulation of its centre, remains almost perfectly equidistant from itself in all its positions, summer and winter are brought about*. On the other hand, in that over a very long period it becomes inclined, the fixed stars are thought to move forward, and the equinoxes to retrogress.

*The precession of the equinoxes is like the progression of the aphelia.

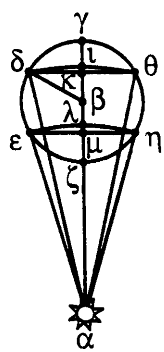
Why, then, should we have doubts about attributing to all the planets, in order to save the phenomena of eccentricity, something which is thought to be in one of them (that is, the earth) because of the phenomena of the precession of the equinoxes and the sun's annual cycle of rising and falling?

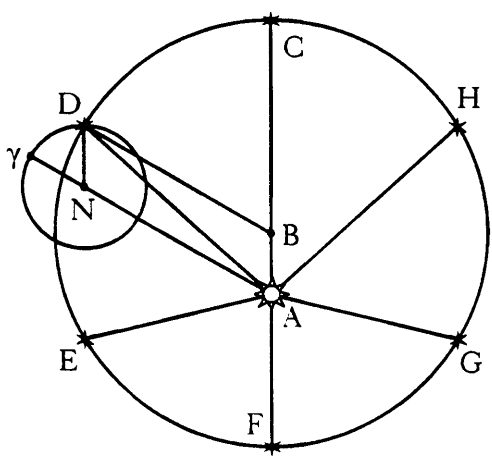
Copernicus was deceived here when he thought that he needed a special principle to cause the earth to reciprocate annually from north to south and back so as to produce summer and winter, and to bring about the equality of the tropical and sidereal years (to the extent that they are equal) by its efforts at producing equal periods. For all those effects are obtained by having the earth's axis, about which the diurnal motion is made, retain a single, constant direction: there is no need for extrinsic causes, except to account only for the extremely slow precession of the equinoxes. And so here, too, there is nothing to suggest that there will be a need for movers for the planet, which would carry its body about the sun in a parallel position, and at the same time perform the reciprocation. For the one will naturally depend upon the other. The only thing remaining to be considered is the extremely slow progression of the aphelia.

The reason why the reciprocation is swiftest in the middle.

To continue: when the strip is at C and F, there is no reason why the planet should approach or recede, since it holds its ends at equal distances from the sun, and would undoubtedly turn its point towards the sun if it were allowed to do so by the force that holds its axis straight and parallel. When the planet moves away from C, the point approaches the sun perceptibly, and the tail end recedes. Therefore, the globe begins perceptibly to navigate towards the sun. After F, the tail end perceptibly approaches, and the head end recedes from the sun. Therefore, by a natural aversion, the whole globe perceptibly flees the sun. And when it is across from A, where the length of the

axis is pointed directly at the sun, its approach in the former situation, or its flight in the latter, is strongest. Furthermore, our earlier presuppositions derived from the observations postulated this. For of the parts of the reciprocation γκ, κμ, μζ which correspond to equal arcs on the eccentric, the parts at the middle, such as κμ, were longest, and those near γ and ζ were short.




The reason why the reciprocation is slower at the top, and swifter at the bottom.

But it is also consistent that the observations would have γκ, μζ equal, although their arcs γδ, εζ, or better, CD, EF on the eccentric, though equal, are traversed in unequal times, longer for CD, so that the part of the reciprocation γκ is traversed in a longer time than μζ

which is equal to it. For similarly, magnets approach one another more slowly when they are at a greater interval, and more swiftly and more quickly at a shorter interval.

272 In fact, it is possible for us to transfer that force which keeps the magnetic axis in a parallel position, and does not allow it to remain pointed towards the sun, from the occupation of a mind, to which we had entrusted it a little earlier, to the function of nature. It would appear to be an objection to this, that nature always acts in one and the same way, while this retentive force appears to make its exertions differently at different times. This is seen, for example, in the tendency of the axis to incline towards the sun, for the impeding of which the retentive force is ordained, which tendency is evanescent at the middle longitudes but most strongly evident at aphelion and perihelion. Nevertheless, what is there to prevent this force of retention's being in many places stronger than the tendency to incline towards the sun, so that the force is either not at all or but little wearied by such a weak adversary? Let us again take an example from the magnet. In it are manifestly mingled two powers, one of directing it towards the pole, and the other of seeking iron. Thus if a strip or nautical needle be directed towards the pole, while some iron approach from the side, the needle gradually would decline from the pole and incline towards the iron, thus indulging somewhat in its intimacy with the iron, but in such a way that it gives most of it to the pole. Indeed, Gilbert thinks this to be the reason why a strip declines from the pole towards the continents of greatest magnitude, the cause thus lying in the tracts of land, being greater and having a more vigorous power on the right or left to the extent that they are higher in that vicinity.

The planet's axis of power is kept in a parallel position by natural force.

With, nevertheless, an exception.

A magnetic example.

The reason why a magnet declines somewhat from the pole.

Therefore, we can ascribe the same tasks, and a uniform action, to both natural faculties, and by the interplay of the two we can show a cause for the translation of the aphelia which will be neither obscure, nor, by Hercules, empty. For suppose that this force of directing the axis towards the sun does detract somewhat from the retentive power, commensurate with the ratio of the two. Accordingly, in the aphelial semicircle, as at C, the point will gradually incline towards H (that is, backward), and the tail end will turn away from the sun, gradually overcoming the retentive force. Thus the aphelion will become retrograde. But in the perihelial semicircle, as at F, the same point will incline towards G (that is, forward), again overcoming the contrary retentive force. Thus the aphelion will then be made to

The cause of the motion of the aphelia.

move forward, and to be fast. But because AF is shorter than AC, and the sun is closer to F than to C, the force tending to turn the magnetic axis towards the sun is therefore stronger at F than at C. Thus more will be detracted from the retentive [force] at F than at C. So the perihelial forward inclination not only compensates for the aphelial backward inclination, but even overcomes it. And so the reason is clear why the apsides progress, and do not retrogress. Thus the aphelion we have found will have that value at an equated anomaly of 90° and 270° when the axis of power is directed straight at the sun, which is its correct place. And the motion of the aphelion will be spiral, as will become clear below in chapter 68 for the motion of precession of the equinoxes also, which exists through another cause. So the direction of the magnetic axis in its parallel position, or the force which is its custodian, does not respect one or another of the fixed stars, but only the position of its body, as it is at any particular time. And, to think the matter through simply: because this direction is more like rest than motion, it is more appropriately sought in the material, and in the disposition of the body, than in some mind.

Why the aphelia do not retrogress.

273 But come: let us follow more closely the tracks of this similarity of the planetary reciprocation to the motion of a magnet, and that by a most beautiful geometrical demonstration, so that it might appear that a magnet has such a motion as that which we perceive in the planet. *Let* DFA *be either a round magnet or the body of Mars,* DA *the line along which the magnetic power is oriented,* D *the pole that seeks the sun,* A *the pole that flees the sun.* You will note, first, that in this theory it is all the same whether we consider the entire globe of the magnetic body, or one single physical line of its power, parallel to DA.

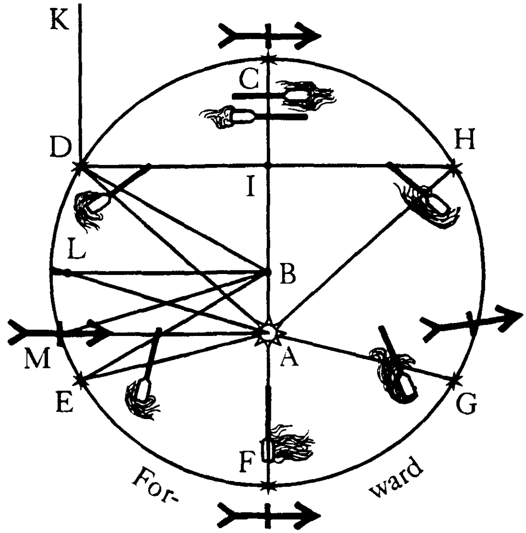
It is in accord with reason that any magnet correctly arranged would perform reciprocations of this sort.

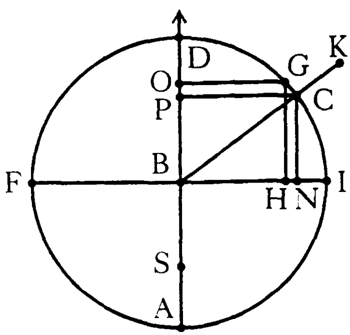
For this magnetic power is corporeal, and divisible with the body, as the Englishman Gilbert, B. Porta, and others, have proved[3], and its globe consists of an infinite number of physical lines, as it were, parallel to DA, whose power is extended in a straight line and in one direction in the world. Therefore, the judgements made about

Thou, O magnet, shewest sailors the hidden track: what wonder that the planets follow thy pleasure?

[3] The *Magia naturalis* of the Neapolitan Giovanni Battista della Porta was distributed in a very large number of editions in various languages in the second half of the sixteenth century. Although the first edition, divided into four books (Naples, 1558) hardly mentions magnetic phenomena, in the comprehensive edition of 20 books, which first appeared in 1589 in Naples, the entire book 7 is devoted to magnetism. With respect to the present passage, it is chapter 5 of that book that is chiefly of interest.

William Gilbert, whose work has already been mentioned above in footnote 5 to chapter 34, treated the phenomena consequent upon the division of a magnet in book I ch. 5. (Bibliographical information from KGW 3 p. 475.)




individual parts with respect to the quality of their motion will be the same as those concerning them all joined together. and vice versa. *So let the central axis* DA *be proposed for theorizing, in place of the whole body and all its filaments. Let* DA *be bisected at* B, *and let* FBI *be drawn perpendicular to* DA. *Thus, when the planet is so positioned*

*that* BI *points toward the centre of the sun, there will be no [tendency to] approach. For the angles* DBI, ABI *are equal, and thus have equal strength, the former for approaching, and the latter for receding. So this is like an equipoise in mechanics. Under these conditions, the centre of Mars will be on the apsides, at aphelion, say, at its greatest distance from the sun. Now let some arc* IC *be taken, measuring the angle of equated anomaly, and let* BC *be drawn and extended to* K. *And let the planet be so situated that* BC *points towards the sun, which is understood to be indicated by* K. The first thing to be sought is the measure of the strength of the planet's approach. *Now the approach occurs because the seeking pole* D *is inclined towards the sun* K *at the angle* DBK, *while the fleeing pole* A *is turned away at the angle* ABK. *And since the strength of this angle is natural, it will follow the same ratio as the balance. But when a line* CP *is drawn from* C *perpendicular to* DA, *between* DP *and* PA *there will be the ratio of the balance. For if a pair of scales is suspended from the balance support* KB, *and the arms come to rest at the angle* DBK, *the weight of the arm* BD *will be to the weight of the arm* BA *as* DP *is to* PA, *just as, if the balance arms were suspended from* CP *at* P, *and the weight of the arm* BA *were applied to* PD *while the weight of* BD *were applied to* PA, *then* DA *would be at right angles to the hanging balance support* CP. *See my*
274 Optics[4], *and do not be easily swayed by insufficiently careful experimentation. Therefore, as* DP *is to* PA, *so is the strength of angle* ABC *to the strength of angle* DBC. *Thus* DP *here measures the force of fleeing, and* PA *the seeking force. From* PA *subtract a magnitude equal to* DP, *and let this be* AS. *Therefore,* SP *is the measure of the seeking power alone, with the impediment of fleeing [power] subtracted, and it will be in the same proportion in which* AD *measures the single greatest force. But where the half* DB *measures the greatest force, the half of* PS, *which is* PB, *or the sine* CN *of the equated anomaly* CBI, *measures the net force of the planet's approach towards the sun at this location.* So the sine of the equated anomaly is the measure of the strength of the planet's approach towards the sun in this place. And this is the measure of the increments of power.

What is the measure of the speed of reciprocation at any point.

This reciprocation works according to the law of the balance: hence its name [lat. *libratio*].

The measure of the distance of the reciprocation traversed by these continuous increments of power is quite another thing. For the

What is the measure of the space traversed in the reciprocation up to a given moment.

[4] *Astronomiae pars optica* (Frankfurt 1604), ch. 1 prop. 20, pp. 17–20; KGW 2 pp. 28–30. Kepler's warning about 'insufficiently careful experimentation' may reflect the inability of this theory of the balance to describe the phenomena fully (although under certain circumstances it can give correct results).

observations show that if the eccentric anomaly GI corresponds to the equated anomaly IC, the versed sine IH of the arc GI is the measure of the reciprocation accomplished. If this can also be deduced from the previously indicated measure of the speed CN, then we shall have reconciled experience with the demonstration involving the balance. *Since the sine of any arc is the measure of the strength of that angle, the sum of the sines will be an approximate measure of the sum of the strengths or impressions over all the equal parts of the circle. And the completion of the entire reciprocation is the effect of all of these in common. Furthermore, letting* IC *and* IG, *though they are unequal elsewhere, be equal here to avoid confusion, the sum of the sines of the arc* IG *is to the sum of the sines over the quadrant, approximately as the versed sine* IH *of that arc* IG *is to the versed sine* IB *of the quadrant.* Approximately, I say. For at the beginning, when both the versed sine and its increments are small, it is less by half than the sum of the sines. For: *Let the quadrant be taken as 90 units. The sum of the 90 sines is 5,789,431*[5]. *In this instance I have added them all in order*[6]. *The sum of the sines at an arc of 1°, that is, the first sine, is 1745. And the former sum is to the latter as 100,000 is to 30. On the other hand, the versed sine of the quadrant is 100,000, and the versed sine of 1 degree is 15, which is half of 30.*

What is the ratio of the versed sine of some arc to the sum of the sines of all preceding degrees.

The ratio is approximately constant within the limits of sense perception.

The reader should not be at all deterred by this geometrical *faux pas* and fallacious principle. For before this becomes a perceptible portion of the reciprocation, the effects of the two procedures differ imperceptibly. *For the sum of 15 sines, which is 208,166, gives 3594*[7] *[as a fourth proportional]. And the versed sine of 15° gives 3407/100,000, which is only a little less than the other. Likewise, the sum of 30 sines, which is 792,598, shows, by the rule of proportions, a part of the reciprocation which is 13,691 out of 100,000. And the versed*

[5] Evaluated by modern methods, this sum is 5,779,433. Most likely, somewhere in the course of computation, an 8 was substituted for the second 7.

It has been remarked by A. E. L. Davis (*A Mathematical Elucidation of the Bases of Kepler's Laws*, unpublished doctoral dissertation, University of London, 1981, pp. 329–333) that in using this sum together with the radius to set out a proportion, Kepler in effect converted the angles to radian measure. However, it should be clear from Kepler's reasoning in setting up the proportion that it is only a mathematical accident that the sum of the sines happens to be the degree/radian conversion factor: Kepler had no deliberate intention of making such a conversion.

[6] This may mean that, instead of taking half the sum of the sines through 180°, which he already had computed in chapter 43 using the Cardan/Buergi rule (pp. 448–449), he actually added up the sines through 90° here.

[7] This value, as well as the corresponding figure at 30°, are somewhat low owing to the error in the sum of the sines to 90°.

*sine of 30° shows 13,397. Also, the sum of 60 sines, which is 2,908,017, shows a little more than 50,000, while the versed sine of 60° is 50,000*[8].

Application of the magnetic reciprocation, just demonstrated, to the observed reciprocation of the planet.

It has been demonstrated that if any magnet be set out as we have supposed the bodies of the planets to be set out in the heavens with respect to the sun, a reciprocation of the magnetic body will result whose displacements will be measured by the versed sine. And indeed, the observations testify that the planet's body reciprocates according to the measure of the versed sine of the eccentric anomaly. It is therefore perfectly consistent that the bodies of the planets be magnetic, and so disposed to the sun as we have described.

The ratio of the versed sines of the eccentric anomalies is the same as that of the sums of the sines of the equated anomalies corresponding to those eccentric anomalies, quite precisely.

**275** I must now show that it was not a great mistake to have taken the arcs IC and IG as equal. *When I say that the arc IC on the body of the planet is the measure of the equated anomaly, I am speaking properly, and CN is then the genuine measure of the strength possessed by the planet when it has the sun on the line BK.* However, when I say that IG is the measure of the eccentric anomaly which corresponds to the [equated] anomaly IC, I am speaking improperly, incorrectly using the circle of the planet's body to represent the eccentric. But on the eccentric's descending semicircle, since a greater arc of eccentric anomaly corresponds to a smaller arc of equated anomaly (namely, IG to IC), we will be adding up considerably more sines on IG than on IC, and rightly so. For since the sine measures the strength, and the strength acts in proportion to the time and to the closeness to the sun (magnets being stronger when closer) – that is, to put it briefly, in proportion to the arc IG – just as many sines should be set up on IC as are found on IG.

To the extent that the planet is slower on any arc, the parts of the equated anomaly are to be made smaller, so that the sum of their sines may be a true measure of the power sent forth over that equated anomaly.

Our only error is this, that we take those many sines to be longer than they should be, as GH is longer than CN.

But first of all this excess is in itself very small and imperceptible. For at the beginning of the quadrant the arcs IC and IG hardly differ, and the sines are small, and at the end of the quadrant, where the eccentric equation CG is greatest, the sines hardly differ.

8 The argument here may not be quite clear, and so a paraphrase may be useful. The strength of the attractive or repulsive force at any angle is proportional to the sine of that angle. The reciprocation, however, in its *geometrical* conception, is measured by the versed sine of the angle (that is, the difference between the cosine and the radius). Therefore, it had to be shown that the sum of the individual moments of force over any angle, which according to the *physical* theory should measure the reciprocation, is proportional to the versed sine of the *geometrical* conception. Lacking the means of proving this generally, Kepler resorted to the empirical test just presented.

In the *Epitome of Copernican Astronomy* Book V Part 1 Section 2 (p. 655 ff. of the original edition), Kepler attempted a general proof, not entirely successfully.

The defect in the ratio which we have established between the versed sine and the sum of the sines is compensated by our contrary error when in using the eccentric anomaly instead of the equated anomaly we take sines which are too long.

And then this error is in accord with our wishes. For the sums of the sines always come out a little greater than the versed sines, which, since they are commended to us by experience, we are using in our efforts to accommodate and reconcile the ratios characterizing the reciprocation with those of magnets. Therefore, this present error of ours, of accumulating long sines instead of short ones, is compensated if we use the versed sines instead of the sums of the sines themselves, for the sums of the sines are not exactly equal to the versed sines, but exceed them because of the effect of the reciprocation.

Therefore, by the best reasoning at our disposal, we have brought the calculation within the limits of observable error. Let us conclude that the body of the planet, like a magnet, approaches and recedes according to the law of the lever along an imaginary diameter of the epicycle tending towards the sun, and that the body's diameter of power, its true diameter DA, tends towards the middle longitudes, so that for this time BD tends toward 29° Taurus and BA towards 29° Scorpio. For the aphelion is at 29° Leo.

The magnetic force inhering in the bodies of the planets is excited and brought into action by a similar force of the solar body.

Thus this reciprocational approach is performed without the action of mind, by a magnetic force which, though it inheres in the planet and is independent, nevertheless depends for its definition upon the extrinsic body of the sun. For the force is defined as seeking the sun or fleeing it. And while the force between magnets tending to bring them together ought to be mutual, I have denied that the sun has the planets' attracting force, in chapter 39 above. Instead, that force was understood to be purely attractive only, as is clear from the argument presented there. The planets' force, on the other hand, is supposed to be simultaneously attractive on one side and repulsive on the other. Alternatively, one might suppose that the sun, like unmagnetized iron, is only sought after, and does not in turn seek other things. For in the above passage, its filaments were circular, while those of the planets are here supposed to be straight.

Difficulty and imperfection of this magnetic example.

I will be satisfied if this magnetic example demonstrates the general possibility of the proposed mechanism. Concerning its details, however, I have doubts. For when the earth is in question, it is certain that
276 its axis, whose constant and parallel direction brings about the year's seasons at the cardinal points, is not well suited to bringing about this reciprocation or this aphelion. The sun's apogee, or earth's aphelion, today closely coincides with the solstitial points, and not with the equinoctial, which would fit our theory; nor will it have remained at a

constant distance from the cardinal points. And if this axis is unsuitable, it seems that there is none suitable in the earth's entire body, since there is no part of it which rests in one position while the whole body of the globe revolves in a ceaseless daily whirl about that axis.

On the mental basis of this reciprocation. I am afraid to say 'rational' for fear that it would be understood to denote a discursive faculty.

So indeed, there may be absolutely no material, magnetic faculty that can accomplish the tasks entrusted to the planets individually, since there may be a lack of means, that is, no suitable diameter of the body which remains parallel to itself as the body is moved around. For this lack has just been made apparent in one of the planets, namely, the globe of the earth. Therefore, a mind must be summoned, which, as was said in chapter 39, arrives at a knowledge of the distances it assumes by contemplating the growth of the sun's diameter. This mind would need to govern a faculty, either animate or natural, that keeps its globe in a parallel position in a manner allowing it to be suitably impelled by the solar power and to reciprocate with respect to the sun. (For a mere mind, unassisted by faculties of a lower order, cannot by itself do anything in a body[9].) At the same time care should be taken that the periodic time of the reciprocation not be made exactly equal to the periodic return of the planet, so that the apsides will move. The plausibility of these things is argued in chapter 39 above.

Now that we have obtained from the observations the laws and quantitative characteristics of this reciprocation by which the sun's apparent diameter is varied, matters of which we were still ignorant in ch. 39, it now remains for us to see whether those laws might plausibly be known by the planets. The laws of the reciprocation were that the versed sine of the eccentric anomaly is the measure of the part of the reciprocation completed.

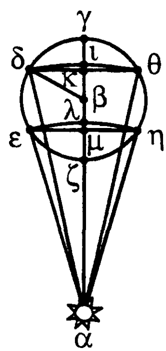
The increases of the sun's diameter are proportional to the versed sines of the equated anomaly.

To begin, therefore, I say that admitting as given the observational evidence, namely, that after equal arcs of the eccentric are traversed, the planet is found at $\gamma, \kappa, \mu, \zeta$ rather than at $\gamma, \iota, \lambda, \zeta$, the increment of the sun's diameter presents a legitimate measure of the versed sine

[9] This passage is a clear reference to the three faculties thought at the time to govern animal physiology. As Kepler is describing them, they are the natural faculty, controlling nutrition and growth; the animate faculty, controlling motion; and the faculty of mind, controlling purposive action.

of the equated anomaly[10], no less so than we know the versed sines of the eccentric anomaly to be a measure of the reciprocation.

Now, as was said in ch. 39, the planet's mind (if it has such an adjunct) perceives the spaces it traverses in the reciprocation none otherwise than by the evidence provided by the increase of the sun's diameter. It will therefore be fitting that it know the versed sine of the equated anomaly, in order that in its approach the sun's diameter might increase to the prescribed size.

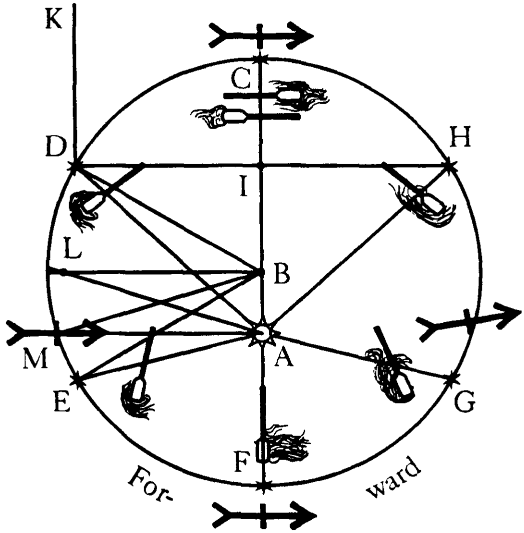



The proof is as follows. *Let the planet be at* γ, κ, μ, ζ *after traversing equal arcs of the imperfect eccentric* CD, DE, EF, *and let the points* D *and* H *be joined, the line intersecting the diameter* CF *at* I. *Therefore,*
277 *since the straight lines* δκθ, εμη *cut the epicycle into arcs similar to those on the eccentric, by construction, as* CF *is to* CI, *so will* γζ *be to* γκ, *one section being a measure of the other.*

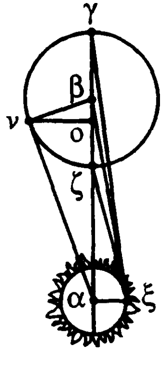
These things being so, I say it will also follow that the diameters of the sun at α, as observed from γ, κ, μ, ζ, will be augmented in the same ratio, namely, that by which the versed sine of the equated anomaly increases. It would be inconvenient to prove this solidly here. It will, however, easily be understood as holding solidly if we

[10] The versed sine of the $\left\{\begin{array}{l}\text{eccentric}\\ \text{equated}\end{array}\right\}$ anomaly measures the $\left\{\begin{array}{l}\text{planet's reciprocation}\\ \text{increase of the sun's diameter, as seen by a spectator supposed to be on the planet, and vice versa.}\end{array}\right.$

– Kepler's footnote.




prove it for both ends and the middle. *At* C *the equated anomaly is nothing, and the versed sine is nothing, and the sun, observed from* $\gamma$, *appears at its minimum, so that the amount of its increase is again nothing. At* F *the equated anomaly is 180°. The versed sine is equal to the whole diameter, 200,000. And the sun, observed from* $\zeta$, *appears at its maximum, so that it shall have acquired all of its increase.*

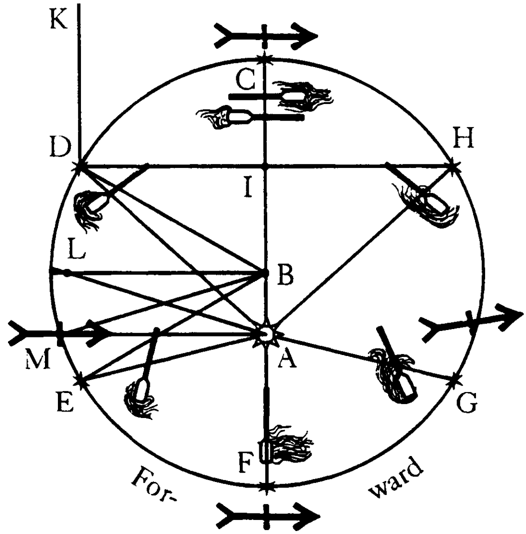


*Now for an equated anomaly of 90°, from* A *let* AM *be set up perpendicular to* CF, *and let* MB *be joined. Also, from* $\alpha$ *let a line be drawn tangent to the epicycle at* $\nu$, *and the tangent point* $\nu$ *be joined with the centre* $\beta$. *Now since* $\alpha\nu\beta$ *is right, by Euclid III. 18, and* MAB *is right by construction, and* $\beta\nu$, BA *are equal by construction, as well as* $\beta\alpha$, BM, *therefore, the triangles are equal and congruent. So* $\nu\beta\alpha$, ABM *are equal. From* $\nu$ *let* $\nu o$ *be drawn perpendicular to* $\gamma\zeta$. *Therefore, since* $\nu o \beta$ *is right, it is equal to* MAB, *and* $\nu\beta o$ *will be equal to* MBA. *So the triangles are similar, and as* $\nu\beta$ *is to* $\beta o$, *so is* MB *to* BA, *and vice versa. And since* $\nu\beta$, $\beta\gamma$, *and* $\beta\zeta$ *are equal, and also* MB, BC, *and* BF, *as* $\nu\beta$, $\beta o$ *together, or* $\gamma o$, *is to* $o\zeta$, *so are* MB, BA *together, or* CA, *to* AF. *Therefore, since* CA *is the versed sine of the eccentric anomaly* CBM, *and is supposed to be the measure of the corresponding part of the reciprocation,* $\gamma o$ *will be that part. Therefore, at this eccentric anomaly* CBM, *where the equated anomaly* CAM *is 90°, the planet will be at* o.

*But the versed sine of the equated anomaly of 90°, the angle* CAM, *is half the total diameter, or 100,000. I say also that the apparent magnitude of the diameter of the sun at* A, $\alpha$, *as seen from* o, *will be a mean between the magnitudes as seen from* $\gamma$ *and* $\zeta$, *so that it shall have acquired half of its increase when the planet is at* o *below* $\beta$.

*For let the diameter of the sun's body be* $\alpha\xi$, *and the apparent angles formed by joining* $\xi$ *with* $\zeta$, o, $\gamma$, *be* $\xi\zeta\alpha$, $\xi o\alpha$, $\xi\gamma\alpha$. *And because* AF, $\zeta\alpha$ *are equal, as well as* AC, $\alpha\gamma$, *and as* CA *is to* AF, *so is* $\gamma o$ *to* $o\zeta$, *therefore, as* $\gamma\alpha$ *is to* $\alpha\zeta$ *so is* $\gamma o$ *to* $o\zeta$. *But* $\gamma\xi$ *differs imperceptibly from* $\gamma\alpha$, *and* $\zeta\xi$ *from* $\zeta\alpha$. *Therefore, as* $\gamma\xi$ *is to* $\zeta\xi$, *so is* $\gamma o$ *to* $o\zeta$, *within the limits of perception. So in the triangle* $\gamma\xi\zeta$, *the angle* $\xi$ *is divided by the line* $\xi o$ *so that the base* $\gamma\zeta$ *is divided in the same ratio as the sides* $\gamma\xi$, $\zeta\xi$. *Therefore, by the converse of Euclid VI. 3, the angle* $\gamma\xi\zeta$ *is divided into two equal parts by the line* $\xi o$, *and* $\gamma\xi o$ *is half of* $\gamma\xi\zeta$, *the total increase of the sun's diameter. Q. E. D.* It is therefore certain at both ends and the middle that if the diameter of the reciprocation is divided by the planet in proportion to the versed sines of the eccentric anomaly, the
278 sun's diameter would increase in proportion to the versed sines of the equated anomaly.

To make it more evident, this is proved in part by the following. *Let the straight line* BL *be drawn from* B *perpendicular to* CF, *and about centre* A, *with radius equal to* BC, *let an arc be drawn intersecting* BL *at* L, *and let* AL *be joined. Since the eccentric anomaly* CBL *is 90°, the versed sine will be* CB, *100,000, half of the whole diameter, and*




*consequently the reciprocation will be* γβ, *half of the whole* γζ. *Also, the distance will be* βα. *But* AL *is equal to it by construction. Thus the planet will be at* L. *And because* AL *is equal to* BC *or* BM, *and* BA *is a common side, and* LBA *is right, as well as* MAB, *therefore, the triangles* BMA, ALB *are congruent. So* BL *is equal to* AM. *But* AM *is equal to* αν, *as above, and therefore* BL *is equal to it also. But* αν, *which lies opposite the right angle* αον, *is longer than* αο, *which subtends the acute angle* ανο. *Therefore,* BL *is also longer than* αο, *and* AL *is longer than* BL. *Thus* AL *is much longer than* αο. *Therefore, the sun appears smaller at the distance* AL *than at the distance* αο. *But the distance* αο *was just now seen to be the mean between the maximum and the minimum. Thus at distance* AL *the sun appears less than the mean.* So at L, even though half the semicircle of the eccentric has been traversed, the sun's diameter has attained less than half of its total increase. This is, of course, because the equated anomaly LAC is less than the mean value of 90°. And this was the problem that was tying us in knots in ch. 39, as was said in the preceding chapter (ch. 56). For if the planet's orbit had been a perfect circle, the increase of the sun's diameter would have been a measure of the increases of the versed sines of the eccentric anomaly, which cannot be observed as directly by the planet's mind as can the equated anomaly, as we have

just heard. You can see from this contrast just how conveniently this measure is attributed to the planet, and how plausibly.

The planet cannot obtain knowledge of the eccentric anomaly.

We might suppose that the measure of the reciprocation (the versed sine of the eccentric anomaly, as the observations show) is to be grasped directly by the mind. But then the planet's mind would be deprived of the assistance of the variable solar diameter, and thus does not adjust itself to the versed sines of this eccentric anomaly. For the planet's path is not a circle. And the planet's mind would have to intuit the parts of the reciprocation, or the distances to be traversed in them, without any indicators. This we long ago rejected as absurd. It would also have to intuit the eccentric anomaly, which is the angle between two straight lines projected from the centre of the eccentric, one through the aphelial point and the other through the centre of the planetary globe. In the diagram, it is DBC (or if the line DK be projected from D parallel to BC, KDB would then be the supplement of the same eccentric anomaly). Therefore, if the mind perceives the angle KDB, it must perceive the triad of points K, D, B. Concerning the point D there is no problem, since it is the centre of its globe. I am not much concerned about K, because, owing to the infinite distance of the fixed stars, BC and DK ultimately coincide at the same location among the fixed stars, and the fixed stars are real bodies. Therefore there is no absurdity in holding that the planet's mind uses some hidden sense to keep in view that fixed star which provides lodging for the aphelion at any particular time. It is only B the apprehension of which must be considered beyond the competence of the planet's mind, since B is not clothed in any body.

And besides, in the natural method proposed just above, there was no need for this idea.

279 Furthermore, when one removes the cause for keeping watch on B, the effect is also removed. But B needs to be watched if the circle CD is to be traversed. However, the planets' orbits are not perfectly circular, as was proven from the observations in chapter 42. Therefore, the planets do not take aim at B. And thus this putative centre B is actually secondary to the path CD. But if it were watched by the planet, it would have to be prior to the path.

For these reasons, therefore, I deny that the versed sine of the eccentric anomaly provides the planet with a measure of its reciprocation, not because this is not such a measure, but because even if it is, it cannot be discerned by the planet's mind.

But if we suppose it to be the increasing and decreasing of the sun's diameter that serves as the means or aid by which the planet arrives at the correct distances (imperceptible in themselves) in its recipro-

cation, and then for the variation of this diameter of the sun, from the demonstration just completed, we posit a rule or measure, to be perceived by the planet's mind, [namely,] the equated anomaly of the eccentric, DAC, or rather, KDA, in the diagram, we now stand closer to the truth. For both measures are perceptible: for the reciprocation, the increasing and decreasing magnitude of the sun's diameter, and for the measure, or angle, three points adorned with bodies. For at A there is the sun itself, at D the planet, and at K the fixed star that indicates the aphelia.

The planet can obtain knowledge of the equated anomaly.

It will perhaps be correct to say, as a consequence of this, that an ability to sense the light of the fixed stars and the sun should be attributed to the planet. We did in fact embrace this idea above, in ch. 39, when we considered the case in which the forces of nature might not suffice to administer the celestial motions. This would be needed so that the planet could estimate the angle of equated anomaly by the intersection of radiations at the centre of the planetary body.

There is but one difficulty to clear up. For what reason is it not the angle itself that is made to be the measure of the planetary operation (that is, to make the sun's diameter increase by approaching the sun), but the versed sine in place of the angle?*[11] And by what means might the planet perceive the sine of the equated anomaly? Does it proceed in the way humans do, by geometrical reasoning? Nevertheless, hitherto no faculty of administering the celestial motions has been attributed to the planet's mind that could not have been acquired by a divine inspiration imparted at the very beginning of the world and lasting even to this day, without any reasoning whatever.

The planet's mind, if it is indeed intent upon the angle of the equated anomaly, does not estimate its magnitude, but its sine.

*Just as a little earlier the sine of the eccentric anomaly (or of the corresponding equated anomaly) was the index of the strength of the reciprocation, while the versed sine of the eccentric anomaly was the index of the amount of the reciprocation traversed, so here the sine of the

Therefore, what was said just above should be repeated, namely, that the sine of the equated anomaly is the index of the strength of the angles KDA: on this point, see Aristotle's *Mechanics*, and what was said above in this chapter. For when the two balance arms are disposed at an obtuse angle, they are more easily directed than when they are at a right angle, the ease of direction being proportional to the sines. And, on the other hand, when the two arms are connected at an acute angle, they are more easily made to move together into a single line, head-to-head, than if they were connected at a right angle. Refer again to the demonstration contained in what was just presented.

[11] The resemblance of Kepler's thinking here (in the marginal note) to certain of Galileo's ideas is intriguing. See, for example, *Two New Sciences*, Third Day, 'On Naturally Accelerated Motion', Proposition I Theorem I, in Galileo, *Opere*, Vol. VIII pp. 208–9.

equated anomaly is the index of the speed by which the sun's diameter increases, while the versed sine of the equated anomaly is the index of the amount of in-
280
crease occasioned by all the antecedent [degrees of] speed.

A way in which the planet could acquire knowledge of the versed sine.

Thus, in a way, if it makes sense for the planet to have a sense of the strength of the angles, there will be no absurdity in our saying (using our human conception) that the sines of the angles are known to it. But why would it take note of the natural strength of the angles (for we are returning to natural principles)? As before, let there be certain regions of the planetary body in which there is a magnetic force of direction along a line tending towards the sun. However, contrary to the previous case, let it be an attribute, not of the nature of the body, but of an animate faculty of the sort that governs the body of the planet from within, that as it is swept along by the sun, it keeps that magnetic axis always directed at the same fixed stars, except to the extent that it turns slowly away over the ages. The result will be a battle between the animate faculty and the magnetic faculty, and the animate will win. It is no different from what we had said in ch. 34, that the bodies of the planets naturally seek rest, but are moved by the extrinsic force of the sun.

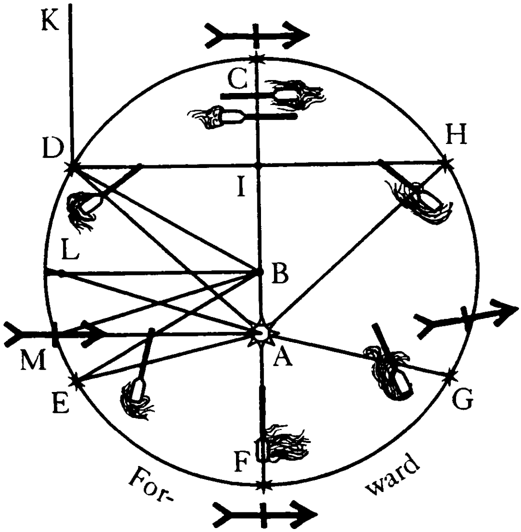
Or here is a more apt example. The weight of the human arm naturally tends towards the centre of the earth. However, in a flag bearer this tendency is subordinate to an animate faculty, by which he may extend that weight above his head and wave it in a circulatory motion. Here the animate faculty overcomes the natural weight, and would do so forever unless the body of the flag-bearer together with all its faculties were created mortal.

On the basis of these presuppositions, the planet's mind will be able to intuit and perceive the strength of the angle from the wrestling match between the animate faculty, which is designed to keep the magnetic axis in line, and the magnetic power of directing it towards the sun.

This arrangement seems also to be confirmed by the example of the moon, which is incontestibly more strongly propelled when it is on the diametral line of the sun and the earth, perhaps because of this strength of the angles.

The character of the celestial motions, if mind concurs in them.

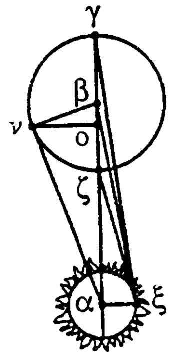
The final conclusion, then, will be this. A planet situated at aphelion makes no endeavour in the direction of the sun, but is carried along with a motion appropriate to the distance AC. The angle KDA results from its forward motion. In accord with the proportion of strength of this angle, the planet causes the sun's diameter to increase by approaching the sun. In its approach, it diminishes the distance, making it AD. Since the distance is diminished, the forward motion is increased. Therefore, the angle KDA is




changed more rapidly. Therefore, the planet causes the sun's diameter to increase more rapidly (other things being equal). Thus is established a perpetual circulation which does not occur by leaps such as we have supposed in our thinking and calculation, ignoring imperceptible errors, but is quite continuous.

A comparison of the mental principle with the magnetic.

What I have said so far holds in case the reciprocation supported by the observations could not be performed by a magnetic power, and it has become absolutely necessary for us to have recourse to a mind. Otherwise, if a comparison between the natural motion and the mental one is in order, the former stands on its own, requiring nothing external, while the latter, the mental motion, appears to give evidence of the magnetic one, and to require its assistance, no matter how you equip it with an animate faculty of moving the body. For in the first place, mind by itself can do nothing in a body. It is therefore necessary to provide the mind an adjunct faculty that performs its functions in making the planet's body reciprocate. This faculty will be either animate or natural and magnetic. In cannot be animate, for an animate faculty cannot transport its body from place to place (as the reciprocation requires) without the operation of another assisting body. Therefore, it will be a magnetic, that is, natural, faculty of sympathy between the bodies of the planet and the sun. Thus the mind calls upon nature and the magnets for assistance.




281 Second, at the halfway point in its pattern, which is the equated anomaly, it has traversed a greater part γo of its reciprocation above, and a smaller part oζ below, although it has completed half of its task, which consists of increasing or decreasing the sun's diameter. Nor do γo, oζ correspond to parts of the time. For more time is consumed on γo than its excess over oζ required. Nor do the parts increase continuously from ζ to γ, the ones about γ, κ being smaller, as well as those about μ, ζ. The operations of mind, however, are accustomed to constancy.

There was consequently a need for us to equip the mind with an animate faculty, as well as a magnetic one, and to contrive a battle between the two which would remind the mind of its duties, of which it could not have been reminded by the equality of either the times or the spaces traversed. So again we have asked nature to assist the mind.

On the other hand, all these modifications really appertain to the workings of the sun's extrinsic magnetic power, and of the magnetic [power] joined to it, which inheres in the planet, as was explained above. If, therefore, the magnetic powers can do the job on their own, what need have they of a directing mind?

Although we have remained uncertain about the magnetic force inherent in the planetary bodies, through our consideration of the earth's axis, which is different from the sun's line of apsides, this difficulty is common to both explanations. For even when we supposed a mind, we were compelled to admit the kind of axis that we wanted in the earth, through whose mediation the mind could apprehend the strength of the angle, or its versed sine. On the

contrary, probability strongly urges us to ascribe this reciprocation of the planets, which without doubt is in accord with the laws of nature, entirely to nature, whatever may be the means by which it occupies the planet's bodies.

Moreover, I do not know whether I have given sufficient proof to the philosophical reader of this perceptive cognition of the sun and the fixed stars, which I myself so easily accept, and bestow upon the planet's mind.

Furthermore, in those very modes of operation which we have prescribed to the mind, the soundest of all those which were deemed possible appear to involve some geometrical uncertainty. I am not sure whether this might not be repudiated by God Himself, as to this point He has always been seen to proceed by the path of demonstration. For suppose that a planet, in proportion as it has approached the sun partly through its inherent force, comes upon one or another degree of power acquired externally from the sun (as does in fact happen). And suppose that the various degrees in return intensify the planet's force of approaching, in increasing the angle that serves to govern the approach or the increase of the sun's diameter. Then the planet's own striving eventually will become in part a measure for itself, and in the intensification of the planet['s force], simultaneously prior and posterior. For in its parts it is unequal, and for this reason it requires a measure. Thus, the search for the forces tempering both powers will be concluded by a kind of iterative method rather than deductively, so that they may complete their cycles in the same time, and in the same revolution of the body.

**282** Someone might, however, want to think he had found the cause of the progression of the aphelia in the ungeometrical nature of this measure. But in ch. 35 we left it undecided whether this category of motion might not exist through another cause, namely, occultation. That is, just as a plate of iron intercepts the force of a magnet on a strip of iron, the bodies of the planets would have the effect upon one another of intercepting the magnetic powers proper to them, by which they incline towards the sun. And so that this might not happen to the solar power, so that, I mean, the solar power, common to all, could not be intercepted for one planet by the interposition of another, we have drawn a distinction between the essence of the solar body and that of the planets' bodies. So, since we have not drawn a distinction between the bodies of the planets themselves, this seems still to be a possible cause. It could, of course, not have been arranged

A cause for the progression of the aphelia that follows from the supposition of a mind.

On occultation, once again.

thus if the exact magnetic disposition of the planet's body, by which the reciprocation is administered, were not known.

Occultation does not transpose the aphelia, neither through natural means:

But to give an example of reasoning: let the planet have a magnetic disposition of the sort which, though we had introduced it somewhat earlier, we later denied the earth to possess. In this disposition, impediment through occultation does not have any place. For because it was the effect of the magnetic power to tend towards the sun and to recede from the sun, meanwhile keeping the fibres of its magnetic seat in line, if another planet, coming between the sun and the planet, impedes this travel towards the sun, or recession from it, while not impeding the common motion from the sun, the planet will approach or recede less than it should, and thus the size of the circuit will be altered along with the periodic time, over the ages, and will again be corrected by contrary eclipses. However, the aphelion will not change position through this occultation. So the cause for the motion of the aphelia previously proposed by us still reigns alone, without peer or rival.

Nor through the supposition of a mind.

Further, if a mind should preside over the reciprocation in the manner described, occultations will still do no harm. For as was said, the mind would use the angle of equated anomaly as its measure for increasing the sun's diameter, and it would be possible (the gods willing) to compensate for the loss of perception of it over the short time in which the sun is covered, so that it would be ignored when the sun reappeared and brought the equated anomaly back into view. For mind (where there is one) is master of the animate faculty, and uses it differently and unequally according to circumstances. So why should it not use the animate faculty in an unusual manner here too, in removing the discrepancy between the measure (the equated anomaly) and the measured quantity (the sun's diameter) which had insinuated itself through the means of the sun's eclipse? What about other slow motions of this kind, such as the precession of the equinoxes arising from the earth's axis being directed at one or another of the fixed stars, and not at the sun? For here, the removal of the sun's light can have no effect, since its presence has no effect either.

What the physicist could say to deny the effects of occultations.

We would like to avoid the inconvenient effects of magnetic occultations upon the reciprocations proper to the planets, just as we did in ch. 35 for the common revolving effect of the sun. It should therefore be said that the planets can indeed be similar in their magnetic dispositions, but either (1) they are so far from one another

that the planets' orbs of power would not overlap, or (2) the power coming from the sun is so strong (that which activates the planets' proper powers no less than that which makes them revolve in their orbs) that the interposition of a small, weak body could not impede it
283 at all, so that it would pass on through, just as light passes through a globe of water, or (3) the bodies of the planets are so meagre that they would have no effect, just as the sun is never substantially blocked by any planet from any of the other planets moved by the sun, as the sun is never substantially blocked from the earth by the moon. For although the whole sun can be covered for the moon for several hours, the moon performs its reciprocation with respect to the earth, not the sun, and it can never be deprived of its perception of the earth since there is no body between the earth and the moon.

Under what conditions, a mind being given, the motion of the aphelia can be ascribed to occultation.

Nevertheless, it might appear plausible to someone that the transposition of the apogees is instantaneous, and occurs through the cause of the sun's being eclipsed. Let him say, if he please, that to prevent the reciprocation's undergoing a sudden leap of speed when it is interrupted by an eclipse, during which the planet is moved by the sun to another angle and another degree of its strength, this angular leap is compensated by the planet itself, by having its axis incline towards the sun at the same angle after the eclipse that it had before the eclipse. For thus a transposition of the aphelia will be obtained, but one occurring by leaps, and remaining in the same sidereal position for many years, until there happens to be another occultation of the planet.

Another cause of the motion of the aphelia on the supposition of a mind.

On the other hand, the prior cause of the transposition of the aphelia, arising from the reciprocation's aberration from its sidereal circuit, produced by the ungeometrical interconnection of its components, favours the uniform transposition of the apogees.

A third.

Finally, if neither of these causes obtains, let the mind, through its animate faculty, which presides over the constant direction of the magnetic axis, have the additional task of inclining the axis over the ages. But if none of these causes stands, nor even the general idea of a mind, let us be satisfied with nature, which, as she has allowed everything else to be disentangled, has also shown a splendid occasion for the motion of the aphelia.

# 58

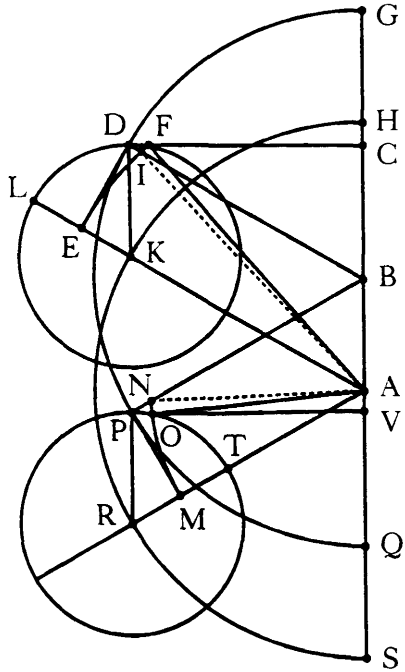
## In what manner the reciprocation discovered and demonstrated in chapter 56 may be accepted, and nevertheless an error may be introduced in a wrongheaded application of the reciprocation, whereby the path of the planet is made puff-cheeked

Galatea seeks me mischievously, the lusty wench:
She flees to the willows, but hopes I'll see her first.[1]

It is perfectly fitting that I borrow Vergil's voice to sing this about Nature. For the closer the approach to her, the more petulant her games become, and the more she again and again sneaks out of the seeker's grasp just when he is about to seize her through some circuitous route. Nevertheless, she never ceases to invite me to seize her, as though delighting in my mistakes.

Throughout this entire work, my aim has been to find a physical hypothesis that not only will produce distances in agreement with those observed, but also, and at the same time, sound equations,
284 which hitherto we have been driven to borrow from the vicarious hypothesis of chapter 16. So, while trying to use false method to do the same thing through this hypothesis, which is itself perfectly correct, I began once again to fear for the whole undertaking. *On the line of apsides, about centres* A *and* B, *let the equal circles* GD, HK *be described. And let* AB *be the eccentricity of the circle* GD. *Also, let the eccentric anomaly, or its number of degrees, be the arc* GD *or* HK, *by the equivalence established in chapter 3. Next, about centre* K, *with radius* KD *which shall be equal to* AB, *let the epicycle* LDF *be described, which will intersect the circle* GD *at* D, *through the*

[1] Vergil, *Eclogues*, 3. 64. There is a pun in the Latin: '*malum*' can mean either 'mischief' or 'apple'. Usually the latter translation is given, as being more direct and literal. However, the point here is that Nature is teasing Kepler: hence, the former has been chosen here (although the link between the apple and the puff-cheeked path is thereby lost).




*equivalence established in ch. 3. Let* AK *be drawn, and extended to intersect the epicycle at* L, *so that the arc* LD *is similar to the eccentric anomaly* GD *or* HK. *And let* BD *be joined. Now, from* D *let perpendiculars be drawn to* GA, LA, *and let these be* DC, DE. Therefore, by what has previously been demonstrated in ch. 56, AE will indubitably be the correct distance at this eccentric anomaly. The question remains how much time was taken to arrive at it. Now the versed sine of its arc, GC, which, after multiplication, becomes LE, when subtracted from GA, yielded the correct distance AE. These indications persuaded me that the other end of AE should be sought, not on the line DC (which was actually perfectly correct), but at the point I of the line DB, such that if I were to draw the arc EIF about centre A with radius AE, it would intersect DB at I. Thus, according to this persuasion, AI would be the correct distance, both in position and length, and IAG would be the true equated anomaly. But it is manifest that the arc EIF would intersect the line DC at a higher place, namely, at F. Thus the angles IAG and FAG differ by the quantity IAF.

I therefore erred in taking the line AI instead of AF[2]. I first discovered the error empirically. For when I explored the quantity of the area DAG, either using all the distances or using the small area DAB, and then fitted the angle IAG, rather than FAG, to this area DAG, now converted into time, in the upper part of the semicircle I found $5\frac{1}{2}'$ more, and in the lower half $4'$ less, than the vicarious hypothesis gave with sufficient certainty. And so, since the equations disagreed with the truth, I began once more to accuse these perfectly correct distances AE, and the planet's reciprocation LE, of the crime for which my false method, which took I in place of F, was to be blamed. What need is there for many words? The very truth, and the nature of things, though repudiated and ordered into exile, sneaked in again through the back door, to be received by me under an unwonted guise. That is, I rejected the reciprocation on the diameter LE, and began by recalling the ellipses, quite convinced that I was thus following an hypothesis far, far different from the reciprocation hypothesis, although they coincide exactly, as will be demonstrated in the following chapter. The only difference was that where I had erred before in my method, I proceeded correctly this time, using F instead of I, as it should have been.

My line of reasoning was like that presented in ch. 49, 50, and 56. The circle of ch. 43 errs in excess, while the ellipse of chapter 45 errs in defect. And the excess of the former and the defect of the latter are
285 equal. But the only figure occupying the middle between a circle and an ellipse is another ellipse. Therefore, the ellipse is the path of the planet, and the lunule cut off from the semicircle has half the breadth of the previous one, namely, 429.

Moreover, if the planet's path had been an ellipse, it would have been clear enough that I could not be taken in place of F, for if this is done, the planet's path is made to be puff-cheeked. *For let the angles* QBP, SAR *in the lower part be equal to* GBD, HAK, *and about centre* R *let the epicycle* PT *again be described, equal to the previous one, and from* P, *the intersection of the epicycle with the eccentric, let perpendiculars* PV, PM, *be dropped to* BQ, AR *[respectively], and let* PB *be joined. And about centre* A, *with radius* AM, *let the arc* MN *be described, intersecting* PV *at* O, *and* PB *at* N. *So, by analogy with the above, just as we took* I *in place of* F, *let us now take* N *for* O, *and let us*

[2] Max Caspar's analysis of this hypothesis (cited in KGW 3 p. 480) shows that it reaches a maximum error of 7′ at a mean anomaly of 45°.

*consider that just as* AN *is the correct distance in length, it is also correct in position. Now the points* I, N, *and the like do indeed make the planet's path puff-cheeked. For the arcs* GD *and* QP *are equal. And* BD, BP, *projected from a common centre, intersect the lunule cut off. But* DI *and* PN, *the breadths of the lunule as measured through the centre, are unequal. And* DI *is smaller, and* PN *greater. For since* ED *and* MP *are equal, and* EDI, MPN *are right, while* EI *is a greater circle, since its radius* AE *is greater, and* MN *is a smaller circle, since its radius* AM *is smaller, therefore,* PN *will definitely be greater, and* DI *smaller. Therefore, the lunule cut off is narrower above, at* D, *and broader below, at* P. *In the ellipse, in contrast, this lunule is of equal breadth at points equally removed from the apsides* G *and* Q. So it is clear that the path is puff-cheeked, so it is not an ellipse. And since the ellipse gives the correct equations, this puff-cheeked path should consequently give incorrect ones.

Nor was there any need to compute the equations anew from the ellipse. I knew they were going to perform their function without further prompting. I was only concerned about the distances, that if they were taken from the ellipse they might cause me trouble. But if this were to have happened, I had already prepared a refuge, namely, the uncertainty of 200 units in the distances. Consequently, I did not hesitate much here, either. The greatest scruple by far, however, was that despite my considering and searching about almost to the point of insanity, I could not discover why the planet, to which a reciprocation LE on the diameter LK was attributed with such probability, and by so perfect an agreement with the observed distances, would rather follow an elliptical path, as shown by the equations. O ridiculous me! To think that the reciprocation on the diameter could not be the way to the ellipse! So it came to me as no small revelation that through the reciprocation an ellipse was generated. This will be made clear in the following chapter, where it will be demonstrated at the same time, through the agreement of arguments from physical principles with the body of experience, mentioned in this chapter, that is contained in the observations and in the vicarious hypothesis, that no figure is left for the planet to follow other than a perfectly elliptical one.

# 59

## Demonstration that when Mars reciprocates on the diameter of an epicycle, its orbit becomes a perfect ellipse; and that the area of the circle measures the sum of the distances of points on the circumference of the ellipse.

286 **Protheorems**[1]

I

If an ellipse be inscribed within a circle, touching it at its vertices at opposite points, and a diameter be described through the centre and the points of contact, and further, if perpendiculars be drawn to the diameter from other points on the circumference of the circle, all these lines will be cut in the same ratio by the circumference of the ellipse.

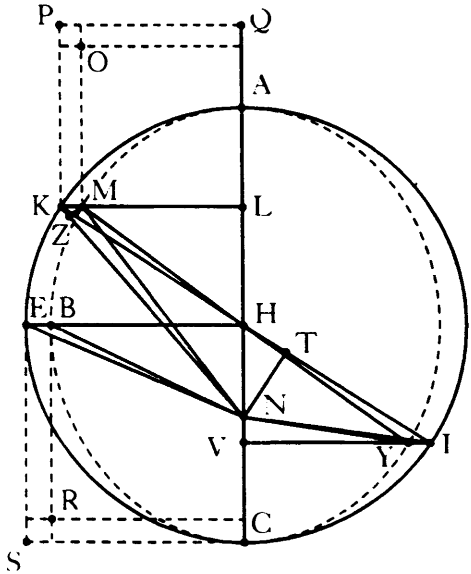
*Using Book I page*[2] 21 *of the* Conics *of Apollonius, Commandino*[3] *proves this in his commentary on Proposition 5 of Archimedes'* On Spheroids.

*Let there be the circle* AEC *with ellipse* ABC *inscribed in it, touching the circle at* A *and* C, *and let the diameter be drawn through the points of contact* A *and* C, *passing through the centre* H. *Then from the points* K *and* E *on the circumference let the perpendiculars* KL, EH *be dropped, cut by the circumference of the ellipse at* M, B. BH *will be to* HE *as* ML *is to* LK, *and so on for all other perpendiculars.*

[1] *Protheoremata*, in Latin. The word was used by Martianus Capella (5th century AD), presumably as a direct transliteration from the Greek, where it meant 'preliminary discussion'. It is clear from the context that this is not what Kepler meant; however, it is not clear what he did mean by this word, or why he used it instead of calling these simply '*theoremata*'. It therefore seems best to call them 'protheorems', rendering Kepler's Latin neologism with an English one.

[2] Although Kepler writes 'page 21', the correct reference is to proposition 21.

[3] F. Commandinus, *Commentaris in opera nonnulla Archemedis* (Venice, 1558), pp. 31–33. (Citation from KGW 3 p. 480.)




II

The area of an ellipse thus described in a circle is to the area of the circle in the same ratio as the lines just mentioned.

*For as* BH *is to* HE, *so is the area of the ellipse* ABC *to the area of the circle* AEC. *This is proposition 5 of Archimedes'* On Spheroids.

III

If from a given point on the diameter lines be drawn to the points on the perpendiculars where the circumferences of the circle and the ellipse intersect them, the areas cut off by these lines will also be as the segments of the perpendiculars.

*Let* N *be the point on the diameter and* KML *the perpendicular, and let* K *and* M *be connected with* N. *I say that as* ML *is to* LK, *or (by protheorem I) as the shorter semidiameter* BH *is to the longer* HE, *so is the area* AMN *to* AKN. *For the area* AML *is to the area* AKL *as* ML *is to* LK, *by Archimedes' assumptions in* On Spheroids, *Prop. 5, which Commandino demonstrates under letters* C *and* D *in his commentary on this proposition. But the altitude* NL *of the right triangles* NLM,

287 NLK *is the same, and the bases are* LM, LK; *and consequently* MLN *is to* KLN *as* ML *is to* LK. *Therefore, by composition, the whole area* AMN *is to the whole area* AKN *as* ML *is to* LK. *Q. E. D.*

IV

If the circle be divided into any number of equal arcs by perpendiculars such as these, the ellipse is divided into unequal arcs, whose ratio [to the arcs of the circle] is greatest near the vertices and least in the middle positions.[4]

*For about the vertices the ratio of the arcs is close to the ratio of the perpendiculars cut off [by them], to which they closely approximate in length, although they are less. About the middle positions they are nearly equal, but the elliptical arcs are smaller because they are less curved than the circular ones. This is self-evident.*

V

The entire elliptical circumference is approximately the arithmetic mean between the circle on the greater diameter and the circle on the smaller diameter.

*For it was proved in chapter 48 above that that circumference is longer whose diameter is the mean proportional between the diameters of the ellipse, the area of which circle, by Archimedes,* On Spheroids *prop. 7, being equal to the area of the ellipse. But, too, the arithmetic mean is longer than the mean proportional. Therefore, they are approximately equal.*

VI

The gnomons[5] of squares divided proportionally are to one another as the squares.

[4] Kepler's use of the terms 'greater ratio' and 'smaller ratio' differs from Euclid's (*Elements* V definition 7). Kepler's greater ratios are those which are farther from equality, regardless of which magnitude is greater. See, for example, *Stereometria doliorum vinariorum* (Linz, 1615), Proposition V, in KGW 9 pp. 82 ff.

[5] According to Euclid (Book II, definition 2), 'In any parallelogrammic area let any one whatever of the parallelograms about its diameter with the two complements be called a **gnomon**.' Hence, the **gnomon** is the figure remaining when in a parallelogram one draws lines through a point on a diagonal parallel to the sides and removes from the whole parallelogram one of the two small parallelograms through which the diagonal passes.

*Let there be two squares,* PL *and* SH. *Let their sides* KL, EH *be divided proportionally at the points* M, B. *Let the gnomons* KOQ *and* CRE *be described. Therefore, because* ML *is to* LK *as* BH *is to* HE, OL *will also be to* LP *as* RH *is to* HS. *But the gnomons are the differences of the squares. Therefore also, as* LP *is to its gnomon, so is* HS *to its, and permuted, as* PL *is to* HS, *so is the gnomon* KOQ *to the gnomon* CRE.

VII

If from the end of the shorter semidiameter on the circumference of an ellipse, a line equal to the longer semidiameter be extended, ending at the longer semidiameter, the distance between that point of intersection and the centre is the side of a square equal to the gnomon that the square of the longer semidiameter places about the square of the shorter semidiameter.[6]

*From the end* B *of the shorter semidiameter* HB, *let the straight line* BN
288 *be extended, equal to the longer semidiameter* AH. *I say that* HN *is equal in square to the gnomon* ERC, *that is, that it is the mean proportional between* EB *and the remainder of the circle's diameter. This was proved in chapter 46 above. But it is demonstrated more easily and briefly in the pure case here. For the gnomon is the difference of the squares* BH *and* HE *or* HA, *by the sixth of these protheorems. But the square on* HN *is also the difference of the squares* BH *and* BN, *that is,* HE *or* AH, *by Euclid I. 46.*[7] *Therefore, the square on HN is equal to the gnomon* ERC. *Q. E. D.*

VIII

If a circle be divided into any number (or an infinity) of parts, and the points of division be connected with some point within the circumference of the circle other than the centre, and also be connected with the centre, the sum of the lines drawn from the centre will be less than the sum of those from the other point.

[6] Here Kepler proves that in the ellipse, the square of the eccentricity (in the astronomical sense) is equal to the difference of the squares of the two semiaxes. This use of the semimajor axis to determine the eccentricity is the nearest Kepler comes to introducing the focus of the ellipse. Although he himself had originated the term 'focus' in his *Astronomiae pars optica* (Frankfurt, 1604), ch. 4 sect. 4 p. 93 (KGW 2 p. 91), he does not use it in the *New Astronomy* (except in the 'Epigrams'), and it is not clear whether he even recognized at this time that the eccentric point and the focus are the same. See footnote 9 to the 'Epigrams', above.

[7] The propositions meant is obviously the 'Pythagorean theorem', I. 47, not I. 46 as stated.

Also, a pair of lines close to the line of apsides drawn to opposite points[8] from a point other than the centre will be approximately equal to two drawn to opposite points from the centre, while a pair so drawn at intermediate locations will be much greater than those drawn from the same centre.

*This was proved in chapter 40.*[9] So[10] that excess does not increase uniformly with the number of lines, much less with the sines. For their differences vanish at the end, while the differences of the said excesses are greatest at the end. And since the area of the circle KNA increases uniformly, its part KHA increasing with the number of lines, by construction, and its part KNH with the sines of the arcs to which the lines are drawn, multiplied by HN, by chapter 40, the area of the circle is therefore not adapted to the measure of the sum of the distances to its circumference.[11]

IX

If, on the other hand, instead of the lines from the point other than the centre, those lines be taken which are bounded by perpendiculars drawn from that point to the lines which are
289 drawn to the centre – that is, if, in the terms of ch. 39 and 57, the diametral distances are taken in place of the circumferential ones – then their sum will equal the sum of those drawn from the centre.[12]

*For let any point whatever on the circumference of the circle be chosen,* K *in the present instance, and from* K *let a straight line be drawn*

[8] That is, opposite through the centre of the eccentric.
[9] The demonstration is found on pp. 422–423.
[10] The following is in effect a corollary to the protheorem.
[11] Again, an elucidation may be in order. Kepler knew from ch. 40 that the area KNA is proportional to the sum of the areas of the sector KHA and the triangle KHN, which is proportional to the sine of angle KHA. Could this area also be represented by lines from N to points on the equally divided arc KA? Kepler thought not, reasoning thus. The difference between KH (whose sum manifestly measures the area of the circle) and KN, if it were the required sine function, would increase maximally at the apsis and minimally at the quadrant. However, its rate of increase is actually the opposite. Therefore, the distances KN cannot represent the area.

Kepler's qualitative evaluations are correct: the area is a function of the sine of angle KHA, while the line KN is a function of its cosine. However, Kepler needed to compare changes in the area with changes in the *sum* of the distances KN. This comparison is not so easy, and it seems clear that Kepler never made it. Already aware that he would need to use the diametral distances instead of the circumferential ones, he apparently concocted this demonstration in an attempt to show more clearly why the circumferential ones could not be used.
[12] This clearly can only be true for the whole semicircle, for elsewhere the sum of the distances AN, KT, and those in between, cannot equal the sum of AH, KH, and those in between.

*through* H *to the opposite part of the circumference* I. *Now from* N *let a perpendicular be dropped to* KI, *and let this be* NT. *Then* KH, HI *together, are equal to* KT, TI *together. And any sum of the pairs* KH, HI, *is equal to an equal sum of the pairs* KT, TI.[13] And since, when AK *is divided into any number of equal parts, the sum of the lines* AN, KT *to those parts increases partly with the number of lines* HA, HK *and partly with the sines multiplied by* HN, *the sum therefore increases uniformly with the area* KNA, *by the foregoing. Thus the area of the circle, and the parts* KNA, *are a measure of the sums of the diametral distances.*

## X

The ratio of distances from a point not at the centre of an ellipse to equal arcs of the ellipse, no less than those on the circle in protheorem 8, is contrary to the ratio of arcs of the circle and the ellipse to one another, explained in protheorem 4. For the pair drawn from the point not at the centre exceeds the pair drawn from the centre in opposite directions, in the least ratio (that is, not at all) at the apsides; but at the middle longitudes they exceed the latter in the greatest ratio.

*This appears in chapter 40.* So, again, as in protheorem 8, the area of the ellipse is not suited to measuring the sums of the distances of equal arcs of its elliptical circumference.[14]

[13] At this point, Kepler has proved the theorem stated above (for the full circle only, although the proof could easily be revised so as to apply to the complete semicircle as well). However, he continues with a purported proof that the sum of the lines KT is equivalent to the area KNA. His reasoning is as follows. The lines KT are made up of KH and HT, the former of which is the radius, and the latter of which can be defined in terms of the angle KHA and the eccentricity HN. Here he apparently made a hasty mistake: he though that HT was equal to the product of HN and the sine of KHA, although it is actually the product of HN and the sine of the complement of KHA (this is the formulation Kepler would have used: he had no separately defined cosine function). His attention was no doubt distracted from the error by his excitement in recalling that he had already proved in ch. 40 that the area KNA is equal to the sum of the sector KHA (that is, the sum of the distances) and the area KNH which is proportional to the product of HN and the sine of KNA. Thus the area is equal to the sum of the distances KT.

Commenting on this passage, Max Caspar points out (KGW 3 p. 480) that in summing the diametral distances KT over equal small arcs on KA, Kepler found the sum to be equivalent to the sum of angle KHA and the product of HN and the sine of KHA (that is, the sum of sector KHA and area KHN).

[14] This last sentence might well have been labelled as a corollary or perhaps a porism, as it shows a purported consequence of the theorem.

XI

With these preliminaries completed, I shall now proceed to the demonstration.

If in an ellipse divided by perpendiculars dropped from equal arcs of the circle, as in protheorem 4 above, the points of division of the circle and the ellipse be connected to the point that was found in protheorem 7, I say that those that are drawn to the circumference of the circle are the circumferential [distances], while those that are drawn to the circumference of the ellipse are the diametral, which are established at an equal number of degrees from the apsides of the epicycle.

**290** *From the point* I, *opposite* K *from the centre* H, *let* IV *be dropped perpendicular to* AC, *intersecting the elliptical circumference at* Y. *And from the point* N *found in protheorem 7 let the lines* NK, NM, *and also* NI, NY *be drawn to the points of intersection* K, M, *and also* I, Y *made by the two perpendiculars, respectively. Further, let the diagram of ch. 39 and 57 be brought back, and let the semidiameter of the epicycle* $\beta\gamma$ *be equal to the eccentricity* HN, *and the arc* $\gamma\delta$ *beginning from the apsis* $\alpha\gamma$ *be similar to* AK *beginning from the apsis, and let* $\alpha\beta$ *equal the semidiameter* HA. *I say that* NK *is the circumferential distance* $\alpha\delta$ *(this was proven in ch. 2) and* NM *is the diametral distance* $\alpha\kappa$.

*First,* KN *is equal in square to [the sum of the squares on]* KL *and* LN. *Likewise,* MN *in square is equal to [the sum of the squares on]* ML *and* LN. *Let* LP *be the square on* LK, *and* LO *the square on* LM. *Thus when the square on* LN *and the square on* LM *(that is, the square* LO*), common to both, are subtracted, there will remain the gnomon* KOQ, *by which the square on* KN *exceeds the square on* MN.[15] *Now as* KL *is to* EH, *so is* KM *to* EB, *by the first protheorem. Therefore also, as* KQ, *the square on* KL, *is to* EC, *the square on* EH, *so is the gnomon* KOQ

[15] That is, the quantity ($LN^2$ + LO) is subtracted from both sides of each equation. Thus the first equation becomes:

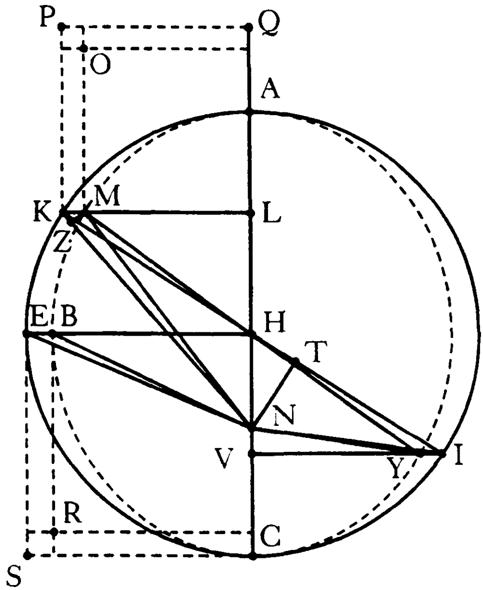
$$KN^2 - LN^2 - LO = \text{gnomon KOQ};$$

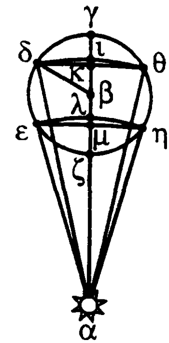
and the second:

$$MN^2 - LN^2 - LO = 0;$$

hence.

$$KN^2 - MN^2 = \text{gnomon KOQ}.$$




*to the gnomon* ERC, *by protheorem 6. And further, as* KL, *the sine of the arc* AK, *is to* EH *or* AH, *the whole sine, here in the eccentric circle, so is the perpendicular* δκ *in the epicycle (from the point* δ *of the arc* γδ, *which is similar to* AK, *to the diameter of the apsides* βγ*) to the semidiameter of the epicycle* βγ. *Therefore also, as the gnomon* KOQ *is to the gnomon* ERC, *so is the square on* δκ *to the square on* βγ. *But* HN *is equal to* βγ. *And the square on* HN *is equal to the gnomon* ERC, *by protheorem 7. Therefore the square on* βγ *is also equal to the gnomon* ERC, *and in addition, the square on* δκ, *the perpendicular from the point on the epicycle just mentioned, will equal the gnomon* KOQ. *But the square on that perpendicular* δκ *is the excess of the square on the circumferential distance* δα *over the square on the diametral distance* κα. *Therefore the gnomon* KOQ, *equal to it, is the excess of the square on* δα *over the square on* κα. *But* KN *is equal to* δα. *Therefore* KN *exceeds* κα *by the gnomon* KOQ. *But it also exceeds the square on* MN *by the same gnomon. Therefore, the diametral distances* MN *and* κα *are equal. Q. E. D. It will be demonstrated likewise concerning* NY, *that it is equal to* αμ, *where* ζη *is similar to* CI. *And so for all.*

XII

Again, it is also clear from the same that

The area of the circle, both as a whole and in its individual parts, is the genuine measure of the sum of the lines by which the arcs of the elliptical planetary path are distant from the sun's centre.

*For by protheorem 9, if the area of the whole circle is set equal to all the diametral distances of all the arcs of the division chosen, the parts of the area, as* KNA, *bounded at the point* N *from which the eccentricity is*
291 *measured, are made equal to those diametral distances that belong to the arc* KA *enclosing that area.*

*But by protheorem 11, preceding, the diametral distances* KT, TI, *that is,* κα, μα, *by chapter 40, are the same as the distances* MN, NY *of the points* M, Y *of the ellipse.*

*Therefore, as the area of the circle is to the sum of the distances of the ellipse, so is the part of the area of the circle* KNA *bounded at the sun's centre* N, *whence the eccentricity is measured, to the sum of the distances on the ellipse belonging to the elliptical arc* AM *having the same number of degrees as the arc of the circle enclosing the area AK.*[16]

XIII

However, the following doubt arises: if the area AKN is equivalent to all the distances of as many points on the elliptical arc AM from N as we have taken on AK, what, then, would that elliptical arc be; that is, where would it end? For it seems that it should not end at the perpendicular line KL. The reason for this is that in this way, by protheorem 4, unequal elliptical arcs correspond to equal arcs on the circle, and thus the arcs are less about the vertices A, C, and greater about B. However, it appears necessary to take equal arcs of the elliptical orbit, should we wish to estimate and compare the times of the planet to traverse them. To be specific: because it is certain that

[16] Kepler's manner of stating this theorem shows that he had still not accepted what we now know as 'Kepler's Second Law' as anything more than a convenient way of approximating the true law, stated in chapter 32, that the elapsed times over equal small arcs are proportional to the distances of those arcs from the sun. Since the total elapsed time, or the mean anomaly, is the sum of those distances, if Kepler could show that the area is an exact measure of the sum of the distances, he could then use the area instead of the sum. So the next three theorems represent an effort to show beyond a doubt that the area is such a measure if the distances are taken as described. Kepler actually used the area of the (imaginary) circle instead of that of the ellipse, which shows how far the 'area law' is removed from the physical truth as Kepler saw it.

the end of this arc should be at the distance MN from N, therefore, as in chapter 58, an arc MZ drawn about centre N with radius NM somewhere indicates a point bounding this arc of the ellipse, and it appears that that point is going to be not M but Z, at which the arc intersects the line KH, making that arc of the orbit AZ.

The reply is made that the arc of the ellipse on which the times are measured by the area AKN should by all means by divided into unequal parts, with those near the apsides being smaller.

*For suppose the planet itself to divide the path* ABC *into equal arcs. Since the planet takes a longer time on arc* A *than on arc* C *proportionally as* NA *is longer than* NC, *while* NA *and* NC *taken together equal the longer diameter of the ellipse, and* HB *is the shorter semidiameter of the ellipse, the planet's amount of time on the arc at* B *and the opposite arc together will therefore be less than on the arcs at* A *and* C *together.*[17] *Therefore, to make the amount of time at* A *and* C *shorter, and at* B *and its opposite longer, thus making the amount of time on any two opposite arcs taken together the same, the arcs at* A *and* C *should be made smaller, and at* B *and its opposite, longer. But this is accomplished by the perpendiculars* KML, *as is clear from the objection itself.*

But by this solution we only discover with certainty that the arcs about A, C should be somewhat shorter. Whether the particular arcs determined by the perpendiculars KML are exactly the required arcs, is not yet established. But it will now be made clear, in the following manner.

XIV

If someone were to divide an ellipse AMC into any number of equal arcs, assigning to each individually its distance from N, while taking
292 the areas AMN, ABN, ABCNA in place of the sum of the distances on AM, AB, ABC, by protheorem 10 he would bring about the same error that occurred in ch. 40 above when we tried to do on a perfect circle what we are here supposing to be tried on an ellipse. That is,

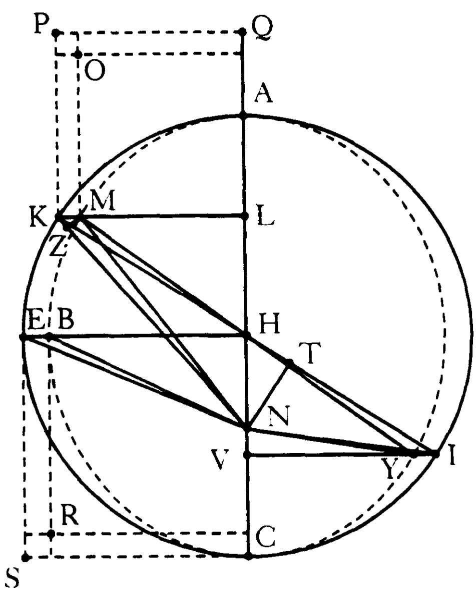
[17] Later, in the *Epitome of Copernican Astronomy* (p. 669), Kepler remarks on this line: 'The single small word "*erit*" ["will be"] introduced a great deal of obscurity. If you change it to "*computaretur*" ["could be computed"], everything will be clearer. I should, however, say that this was the more obscurely expressed, and made the more laborious, in that the distances were not considered as triangles there, but as numbers and lines.'

two lines MN, NY from two points M, Y opposite one another through H, are taken as equivalent to the shorter line MHY.

Suppose, however, that that same person were to divide an ellipse AMC into the same number of unequal arcs, contrary to protheorem 10, according to the following law: the circle AKC being first divided into equal arcs, perpendiculars KL would then be drawn to AC from the ends of the individual arcs, cutting the ellipse AM into arcs also; and the elliptical area would be taken for the distances of these arcs from N. In that case, a remedy would be provided for the error which has been committed: a most perfect compensation.

I shall prove this for the beginnings of the quadrants, A and C; for the ends, B; and the motion in between.

*At the beginnings of the quadrants* A, C, *if the two lines* NA, NC *be taken for the line* AHC, *there is no error. At the end, however, if for* BN *(that is, for* EH*) I take* BH, *the consequent error or defect is a maximum, by protheorem 10, the amount being* BE. *And by protheorem 7 of this chapter, as* HE *is to* EB, *so is the required length to the error committed at this position. Now, suppose that the sum total*



*of all the distances receives a measure erring in defect, namely, the area of the ellipse. Then when the defect is distributed among the individual distances by the force of our operation or computation, the distances* NA, NC *will be made too small with respect to this measure of all of them. We were thus deceived in thinking that all the lines err equally in defect, since* NA *and* NC *are not in fact in error. They do indeed add up to the correct sum, but when the sum is in turn distributed,* NA *and* NC *do not receive their correct value, because certain lines about* B *have defrauded the others.*

Let us now see how we can remedy this error in the same proportion.

*By protheorem 4 of this chapter, the least arcs* AK, AM *about the apsides* A *or* C *are in the same ratio as* KL *to* LM, *that is,* EH *to* HB. *This was the ratio by which the lines about* B *formerly erred in defect. And at* B, *in turn, the least arcs of the circle and the ellipse,* KE *and* MB, *say, are equal; just as, before, the straight lines* AN, NC *together were equal to the line* AHC. *Therefore, as in the previous consideration of the straight lines, so will it be here in the consideration of the arcs: when the mean and uniform measure of the arcs is conceived, the arc at the apsides* A *or* C *will be short with respect to it, and the arc at*
293 *the middle longitudes* B *will be long.* Thus where the distances are too short with respect to their erroneous sum, which gives the wrong area for the proposed ellipse, the arcs will be small with respect to their mean value, as at A, C; and where the distances are too long, the arcs are too long, as at B. And so to the extent that we accumulate too little elapsed time in our calculation, owing to the rather short distances at the apsides, there are that many more distances on that arc, it being cut into small parts each of which has its own assigned distance. And inversely, to the extent that about the middle longitudes B more time has been accumulated by the individual distances than is fitting in our calculation, where we carried over to the innocent apsides A, C, the part of the defect at this location, the calculation has collected correspondingly fewer distances, they having been obtained from the large parts of the arc by begging. At both A and C, what the individual distances could not do owing to their brevity in the calculation, they accomplish by their being closely spaced, with the result that they accumulate the correct elapsed times. And here, the error arising in the calculation occasioned by their excessive length is again removed by their being more widely and loosely spaced.

Of the beginning and end. I have said that at the beginning, at A and C, the arcs of the circle differ from those of the ellipse, and at B and the point opposite the correct distances differ from those assembled by the area of the ellipse, in the same ratio that EH has to HB; and the arcs at BE and the distances at A and C end up differing by the same ratio, namely, the ratio of equality.

We must now do the same for the motion in between.

*The lines* NA, NC, *though at first only slightly larger than the lines* AHC, *increase by swift increments so as to exceed them by a considerable amount. On the other hand, where the excess is greatest, as that of* BN *over* HB, *the increments are slowed down perceptibly. They are greatest in the middle, near an eccentric anomaly of 45°.*

*This is to some extent shown by the angle and secants of the equation. For* BN *differs from* BH *by about the same amount as the secant of the angle of the optical equation differs from the whole sine, while the opposite angles of the equations assist each other in the same ratio. Now the increments of the secants of the optical equation at about 45° are near a maximum, and are small at the beginning and end of the quadrant. Concerning these see the end of ch. 43.*

*Furthermore, the increments of the elliptical arcs marked off by the perpendiculars* KL *progress in the same ratio. For at the beginnings,* A *and* C, *the arc* AK, *always beginning from* A, *is to its increment as* LK *is to* KM. *But the whole arc itself is small, and so the increment is small as well. At the end, near* B, *the ratio of* AE *to* AB *is reduced nearly to equality, even though the arc* AB *is large, since it is near the quadrant. Therefore, the increment is again small. So it is at the middle, about 45°, that the increment of the arcs is most evident.*

It is thus clear that in the intermediate progress, too, the ratios are equal, so far as minute consideration can be carried.

Although the demonstration is most certain, it is likewise *gauche* and ungeometrical, at least in that part pertaining to the progress of the intermediate increases. As always, I would like to have this small part carried out geometrically and with *finesse*, so that even an Apollonius would be satisfied. Meanwhile, until someone else discovers and provides us with this [improvement], we should be content with what we have.

XV

But let us complete the proof.

294 The arc of the ellipse whose time is measured by the area AKN, should end on LK, so it would be AM.

For hitherto we have been proceeding on the fiction that if anyone had so much leisure as to want to compute the area of the ellipse, it would turn out that in using the area of the ellipse AMN in place of the same number of distances of AM as there are equal arcs on AK, he would not miss the mark. Let this serve us as the previously demonstrated major premise of the proposition.

*I shall now add the minor premise, derived from protheorem 3 of this chapter. Here it was shown that the area* AKN *is to the area* AMN *as the area* AKC *is to the area* AMC. *The conclusion therefore is, since the ratio of equimultiples is the same, that the area of the circle* AKN *also measures the sum of the diametral or elliptical distances (such as* KT, TI*) on* AM, *there being as many as there are parts in* AK. *Whence it is clear that I assigned more closely spaced distances to the parts of the ellipse about* A, C *correctly when I made the same number as there were intersections made by the perpendiculars* KL *coming from equal arcs of* AK.

So that no one may doubt the truth of this, confused by the subtlety and perplexity of the argument, this truth previously came to be known through experience, in the following manner. At the individual degrees of eccentric anomaly, I set up the diametral lines KT, TI in place of the distances from N. I also added each in order to the sum of the previous ones. When all were collected, the sum was 36,000,000, as is fitting. Next, when the individual sums were compared with the whole, following the rule of proportions, the sum 36,000,000 would be to 360 degrees (the nominal value of the whole periodic time) as the individual sums would be to the elapsed times they signify. This produced exactly the same results, down to the last second, as would have come out had I multiplied half the eccentricity by the sine of the eccentric anomaly, [added the area of the sector contained by the eccentric anomaly], and compared [the sum] to the area of the circle, which would be given the same value of 360 degrees (the nominal value of the periodic time).[18]

[18] In the diagram, the eccentric anomaly is KHA, whose sine is KL. The eccentricity is HN, and when its half is multiplied by KL, the product is the area KHN. When this is added to the area of the segment KHA, the sum is the area AKN, which represents the mean anomaly, which is the time. Kepler discovered empirically that the ratio of this area to the area of the circle is the same as the ratio of the distances on AM(at points determined by the perpendiculars) to the sum of all distances on the circle. In this passage he failed to mention the adding of the sector to the area of the triangle, clearly an oversight in a syntactically involved sentence. The omission has been remedied by the bracketed words.

Then, when I was of the opinion that the correct distance NM should be applied to the line KH, becoming ZN, and was thus investigating the equated anomaly ZNA, thinking it corresponded to the mean anomaly AKN, the equations disagreed obviously with my vicarious hypothesis of chapter 16. At about 45°, the difference between the equated anomaly and the true value found through experience in the observations was a defect of about 5½ minutes, and near 135° about 4 minutes. But when NM[19] was so applied as to end on KL, then when the equated anomaly MNA was applied to the mean anomaly AKN, it agreed exactly with the vicarious hypothesis, that is, with the observations. And when the fact was established, I was afterwards driven, once I had settled on the principles, to seek the cause of the matter which I have revealed to the reader in this
295 chapter as skilfully and lucidly as possible. And unless the physical causes that I had taken in the place of principles had been good ones, they would never have been able to withstand an investigation of such exactitude.

If anyone thinks that the obscurity of this presentation arises from the perplexity of my mind, I shall myself only thus far acknowledge to him my guilt, that I was unwilling to leave anything untested, no matter how utterly obscure, and no matter how irrelevant to the practice of astrology, which many deem the sole end of this celestial philosophy. But as for the subject matter, I urge any such person to read the *Conics* of Apollonius. He will see that there are some matters which no mind, however gifted, can present in such a way as to be understood in a cursory reading. There is need of meditation, and a close thinking through of what is said.

[19] Reading 'NM' instead of 'AM'.

# 60

## A method, using this physical – that is, authentic and perfectly true – hypothesis, of constructing the two parts of the equation and the authentic distances, the simultaneous construction of both of which was hitherto impossible using the vicarious hypothesis. An argument using a false hypothesis

In chapters 56, 58, and 59, the planet was assumed to approach the sun and recede from it along a diameter directed towards the sun, thus making an elliptical orbit, and further, it was assumed to spend time at each individual point in proportion to the distance of the point from the sun. Thus we happen upon a most convenient short cut through the preceding chapter 59, for evaluating the sum of any number of elapsed times all at once. For it was shown that when a line is drawn from a circle perpendicular to the longer diameter of an ellipse inscribed in that circle (in the previous diagram, let it be KL, perpendicular to AC), so as to intersect the ellipse at M, and supposing that the sun is at N, the sum of all the distances of points on the arc AM from the sun N is contained in the area AKN.

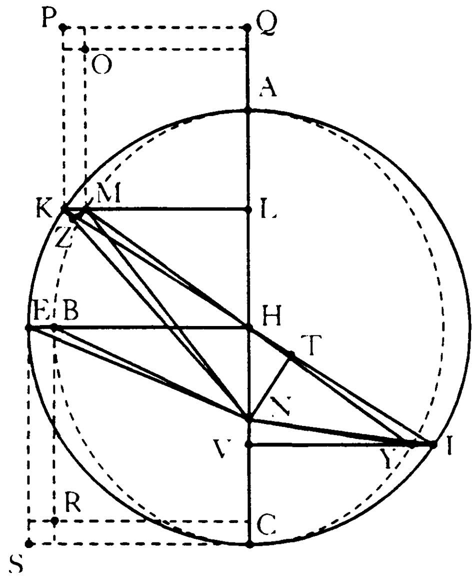
Given an eccentric anomaly, to find the mean anomaly corresponding to it.

*For an arc* AM *of the ellipse being supposed, which is defined by the arc of the circle* AK, *the area* AHK *is given, which is the sector of the arc* AK, *by which arc that sector is also measured, in units of which the whole area of the circle is 360°.*

Or the physical part of the equation.

*And because the arc* AK *is given, its sine* KL *is also given. But as* KL *is to the whole sine* EH, *so is the area* HKN *to the area* HEN, *as was proved in ch. 40. Also, since the eccentricity* HN *is given, half of it multiplied by* HE *will describe the area* HEN. *This value is found at once at the beginning, so that it may be known what this small area amounts to, when the whole area of the circle has the value of 360° of time.*

*And so, once the area* HEN *is known, it is very easy to find the area* HKN *by the rule of proportions. For as* EH *is to* KL, *so is* NEH *to the*




*area* NKH, *or its value in degrees, minutes, and seconds; and this,*
296 *added to the value of* KHA, *establishes a value for* KNA, *which is the measure of the time which the planet takes on* AM. This, then, is one of the parts of the equation, the one I call 'physical'[a], namely, the area HKN. Yet I so arrange the tables that there is no need to mention the equation, nor is there a separate column showing the optical[b] part of the equation, that is, the angle NKH. The terms, 'mean anomaly', 'eccentric anomaly, and 'equated anomaly' will be more peculiar to me. The mean anomaly[c] is the time, arbitrarily designated, and its measure, the area AKN. The eccentric anomaly[d] is the planet's path from apogee, that is, the arc of the ellipse AM, and the arc AK which defines it. The equated anomaly[e] is the apparent magnitude of the arc AK as if viewed from N, that is, the angle ANK.

Terms.
a] The physical part of the equation.
b] The optical part.
c] The mean anomaly.
d] The eccentric anomaly.
e] The equated anomaly.

Given the eccentric anomaly, to find the equated anomaly.

Now the angle of equated anomaly is found as follows. *The arc* AK *being given, the sine of its complement* LH *is given. And as the whole is to* LH, *so is the whole eccentricity to the part to be added*[1] *to 100,000 (subtracted, below 90°) to give the correct distance of Mars from the sun, namely,* NM.[2] *So in triangle* MLN, *the angle at* L *is right, and* MN

[1] That is, HT.
[2] Since NM = KT.

*is given, and* LN *is also given. For it is made up of* LH, *the sine of the complement of* AK, *the distance from apogee, or the eccentric anomaly; and the eccentricity* HN. *Below 90°, in place of the sum* LH, HN *their difference should be taken, and in place of the complement of the eccentric anomaly, its excess. Therefore, the angle of equated anomaly* LNM *will not be hidden.* Here anyone who wishes can easily figure out what has to be changed in the other semicircle.[3]

Given the equated anomaly, to find the eccentric anomaly, and thus the mean as well.

On the other hand, given the eccentricity and the equated anomaly, the eccentric anomaly is given, a little more laboriously, whether we proceed demonstratively or by analysis.[4]

Preparation for this.

It can be investigated demonstratively by this method: the angle is found under which appears the planet's incursion KM made from any point K on the circle as if seen from the centre of the sun. This method depends upon several protheorems.

I

The small lines of the planet's incursion towards the diameter of the apsides increase in proportion to the sines of the eccentric anomaly.

*For as* EH *is to* KL, *so is* EB *to* KM. *This was established in chapter 59, and demonstrated in the* Conics.

II

The ends of one of the small lines being connected to the centre, and it being supposed that the small line remains the same in quantity at all points of the eccentric, the tangent of the angle at the centre decreases approximately in proportion to the sines of the complement of the eccentric anomaly.

[3] Kepler here computes the equated anomaly u and the mean anomaly a for a given value of the eccentric anomaly b according to the formulas

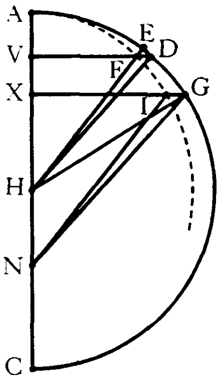
$$a = br + e \sin b$$

and

$$\cos u = (e + r \cos b)/(r + e \cos b).$$

(r is the semimajor axis of the ellipse, and e the eccentricity).

[4] In the task of computing the eccentric anomaly when the equated anomaly is given, Kepler proceeds quite haphazardly. The simplest way would proceed analytically; however, Kepler provides this analytic solution only in the second place. In the first place, he presents the unsystematic but original geometrical method of the following five theorems.




297 *Let* DF *be the small line, part of* DV, *the sine of the eccentric anomaly* AD. *Let the ends* D, F *be connected to* H, *and let* HF *be extended. And let the straight line* ED *be tangent to the circle at* D, *intersecting* HF *at* E. *Therefore, since* DVH *is right,* VDH *will be the complement of* VHD, *the eccentric anomaly. And since* EDH *is also right,* HED *will be less than a right angle by the quantity* EHD. *This is of hardly any significance, since where it is greatest it does not exceed 8 minutes. And for the same reason,* VFH, *that is,* EFD, *is greater than the complement* FDH *of the eccentric anomaly, but by the quantity* FHD, *which is clearly of no significance. And since* FED *is somewhat more acute than a right angle, the arc circumscribed about* FED *will be somewhat longer than a semicircle. Therefore,* ED *is to* DF *as the sine of an angle which somewhat exceeds the complement of the eccentric anomaly, is to the sine which is slightly – really, hardly at all – smaller than the whole sine. Now if* FD *retains this length throughout the whole quadrant,* ED *is made approximately proportional to the sines of the complement of the eccentric anomaly. For if* FD *remains the same in length, and the end* D *is at* A, *the angle* FDH *is right, and thus* FHD *is a maximum, and then* DFH *is at its most acute, and consequently the arc above* FD *is at its longest. From that point, as* FD *moves down from* A, *the arc* FED *decreases and the angle* FED *increases, until at degree 90* FD *becomes part of the line* DH. *Thus* HF *belongs to* HD, *and* ED *vanishes, and there, by analogy, the arc above* FD *is equal to the semicircle, and is at its least.*

III

The ends of the small line of the planet's incursion towards the diameter of the apsides being connected, however long the line happens to be at any eccentric anomaly, the tangents of the angles at the centre (and thus the angles themselves as well, when they are very small), increase approximately in the ratio compounded of the ratio of the sines and the ratio of the sines of the complements of the eccentric anomaly; that is, in proportion to the rectangles on the quadrant formed by multiplying the sines of the angles by the sines of their complements. Thus, the greatest rectangle at 45 degrees is to the greatest angle at the same eccentric anomaly of 45° as the remaining rectangles are to the remaining angles of eccentric anomaly.

Term. What is the rectangle of the quadrant?

*For at these angles, such as* EHD, *two factors are compounded: the length of the incursion, varying from nothing to a maximum, and its apparent magnitude, from nothing to a maximum. But, by I, the incursions increase in proportion to the sines, and by II, the tangents of the angles of apparent magnitude of these incursions, as if viewed from the centre of the eccentric, decrease in proportion to the sines of the complement. By the former cause it happens that the angle is nothing at* A *when the sine is nought, and by the latter cause the angle is nothing at*
298 *an eccentric anomaly of 90, when the sine of the complement is nought; and further, at both places the rectangle has vanished entirely. But at an anomaly of about 45°,* FD *has now turned out greater than half [its maximum], because the sine, 70,711, is greater than 50,000, half the whole sine. And its angle* EHD *is greater than half by still more, because the sine of the complement is also greater than half, namely, 70,711 also. Consequently, the rectangle of the quadrant is the greatest of all, and at the same time is a square, equal to half the square on the radius, namely, 5,000,000,000.*

IV

The angle of the planet's incursion from the circumference of the circle towards the diameter of the apsides is the same at the eccentric anomaly, about the centre of the eccentric, and the circular equated anomaly, of the same number of degrees, about the centre of the sun.

*Let the equated anomaly* ANG *be constructed equal to the eccentric*

Term. This anomaly is called the 'circular equated anomaly' because it is not really the equated anomaly; rather, it is what the equated anomaly would be if the planet's orbit were a circle.

*anomaly* AHD *at the circumference of the circle* G; *that is, let* NG *be drawn parallel to* HD. *And from* G *let* GX *be drawn perpendicular to* AC, *and on it let* GI *be the correct incursion of the planet. And let* I *and* N *be joined. Now* XG *is to* GI *as* VD *is to* DF, *by I, while* XG *is to* GN *as* VD *is to* DH, *because of the similarity of the triangles. Therefore,* IG *is to* GN *as* FD *is to* DH. *Also,* FDH *and* IGN *are equal. So* FHD *and* ING *are also equal. And* H *is the centre of the eccentric, while* N *is the centre of the sun. Therefore, the angle, et cetera. Q.E.D.*

V

The authentic and truest measure of the angle by which the fictitious equated anomaly, which depends upon the circle, differs from the true equated anomaly, which ends on the ellipse, is the rectangle contained by the sine of the ficititous equated anomaly and the sine of the complement of the true equated anomaly.

*In the same diagram, when the sine of the angle* AHD *is multiplied by the sine of the angle* VFH, *the authentic measure of the angle* FHD *is going to result, by III. But by IV, the sines of the equal angles* VHD *and* XNG *are the same, and also, the sines of* VFH, XIN *are the same. Therefore, when the sine of the angle* XNG, *the fictitious equated anomaly, is multiplied by the sine of the angle* XIN, *the complement of* XNI, *the true equated anomaly, there results the authentic measure of the angle* FHD, *that is, by IV, of the angle* ING, *which is the difference between* XNG *and* XNI.

COROLLARY

Because the difference ING is small, and is never greater than 8 minutes, the difference between the rectangles of the sines of XIN and XGN is going to be still smaller in effect.

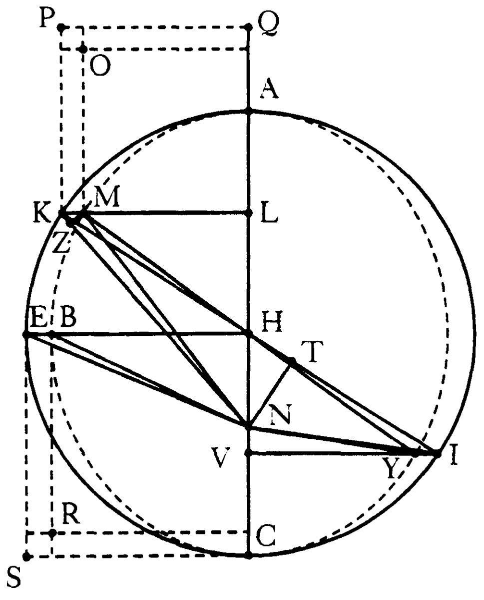
299 From this, the following procedure arises. *The angle of the true equated anomaly being given, let its sine be multiplied by the sine of its complement. Let double the product, with the last five digits dropped, be multiplied by the maximum angle of incursion, at an anomaly of 45°. The product will be the angle of incursion at the given anomaly. This, added to the true equated anomaly* XNI, *gives the fictitious,* XNG. *By this angle, and the known sides* NH, HG, *the eccentric anomaly* AHG *is found, and the value of the triangle* HGN*, *as before.*

Given the equated anomaly, to find the corresponding eccentric anomaly.

*And the mean anomaly.

Moreover, it is not difficult to find the maximum angle at anomaly 45°. *Let* VHD *be 45°. Therefore, as the whole sine is to 70,711, so is the maximum incursion, or maximum breadth of the lunule, of 429 (or, more correctly, 432) to* FD, *305. And since at 45°* HV, VD *are now equal, subtract FD, 305, from* VD, *70,711. The remainder,* VF, *is 70,406. This, with* HV, *gives the angle* VHF, *44° 52′ 34″, which differs from 45° 0′ 0″ by only 7′ 26″. And this is the maximum of the angle* ING.

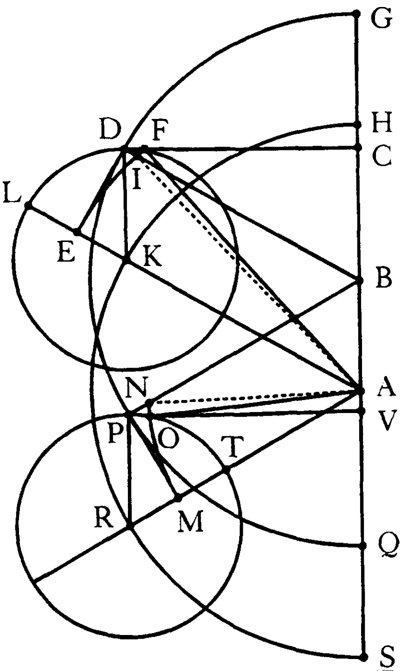
The following is another method using analysis, whose fundamentals are these. *In the diagram of ch. 59, given the angle* MNL, *the ratio of the lines* MN, NL *is given, and I know that* MN *and* LN *are composed of parts in a known permuted proportion. For* MN *contains the (known) whole sine, and* LN *contains the known eccentricity* HN. *The remainder of* MN *has the same ratio to the remainder of* LN, *which is* LH, *as the eccentricity* HN *has to the whole sine*[5]. *If you prefer, you may also refer to the diagram in chapter 58. Therefore, let* MN *be 100,000 + 1x,*[6] LN *from the angle* MNL, *30°, be (8,660,300,000 + 86,603x)/100,000, and* NH *be 9265 or 926,500,000/100,000, so that*




[5] Since MN = KT = KH + HT, the remainder of MN is HT. The remainder of LN, as Kepler says, is LH. And because the triangles KHL and NHT are similar, HT:HN::LH:KH, or the remainder of MN is to the remainder of LN as the eccentricity HN is to the radius (or 'whole sine') KH.

[6] Letting the unknown, x, be the magnitude of HT.

HL would be (7,733,800,000 + 86,603x)/100,000. But as HN, 9265, is to 1x, so is 100,000 to LH. Therefore, in the second instance, HL is (100,000/9265)x, that is, (1,079,320/100,000)x. Previously, it was (7,733,800,000 + 86,603x)/100,000. With the denominators removed, as well as whatever can be subtracted equally from both sides, what remains is that 992,717x is equal to the number 7,733,800,000. Therefore, the single root is 7790. And MN is 107,790. And because as HN is to this root, so is the whole to LH, LH is therefore 84,084, which is the sine of KE, 57° 14′, the complement of the eccentric anomaly AK, 32° 46′. Now that this is found, the area AKN, the measure of the time or the mean anomaly, is found as before. These things are clearest in the diagram of ch. 58. Let GQ be the eccentric, AB the eccentricity, GD or LD the eccentric anomaly, FAC the equated anomaly, FA or EA the distance. So as AK is to AB, so is BC to KE. And at the equated anomaly CAO, as AR is to AB, so is BV to RM[7]. So EK or RM is supposed to be the root. The rest is as above.




[7] As Kepler says, it is clearer here. In modern terms,

$$r:e::r\cos\beta:e\cos\beta,$$

where r is the radius, e the eccentricity, and β the eccentric anomaly.

Given the mean anomaly, to find the eccentric anomaly and thus the equated anomaly.

But given the mean anomaly, there is no geometrical method of proceeding to the equated, that is, to the eccentric anomaly. For the mean anomaly is composed of two areas, a sector and a triangle. And while the former is numbered by the arc of the eccentric, the latter is numbered by the sine of that arc multiplied by the value of the maximum triangle, omitting the last digits. And the ratios between the arcs and their sines are infinite in number. So, when we begin with the sum of the two, we cannot say how great the arc is, and how great
300 its sine, corresponding to this sum, unless we were previously to investigate the area resulting from a given arc; that is, unless you were to have constructed tables and to have worked from them subsequently.

This is my opinion. And insofar as it is seen to lack geometrical beauty, I exhort the geometers to solve me this problem:

Given the area of a part of a semicircle and a point on the diameter, to find the arc and the angle at that point, the sides of which angle, and which arc, encloses the given area. Or, to cut the area of a semicircle in a given ratio from any given point on the diameter.[8]

[8] Here the 'Keplerian Problem' is formulated with perfect clarity.

In the Kepler manuscripts at the USSR Academy of Sciences in Leningrad (vol. XIV p. 422) there appears a fragment of a letter (in the form of a copy) to an unknown correspondent, in which Kepler remarks upon the computation of the equated anomaly from the mean anomaly, as follows:

'Now something from my Astronomy. Ch. 59 and 60 are thick with errors in the letters, not all of which are corrected in your copy. For me the anomalies are three: the mean, given by the time, which I number by the area AKN; the eccentric, which, improperly, is the area of the circle AKH, or the arc AK, or the angle AHK, or properly, the arc of the ellipse AM; and the equated, which is the angle MNA. Given the mean anomaly AKN, I have no way of finding the eccentric anomaly AK other than by trial and error. For I suppose an arc AK, and multiply its sine KL by the value of the maximum area EHN, which is 19,110 seconds (this is provided by multiplying EH by half the eccentricity HN, and comparing it to the area of the circle, which I arbitrarily suppose to be 360°). Thus I obtain the area KHN in seconds, which I add to the supposed arc AK, or the area AHK (since the measure of the two is now the same), to give AKN. If this is equal to the given mean anomaly, then I supposed AK well. When tables are constructed, as I have indeed done, there is no further difficulty here, for it is extracted immediately. But I am now concerned with a method of computing some single equation. Through the given arc AK and LH, to which I add the eccentricity HN, there results LN. KH being drawn, and its perpendicular NT. KT will be the measure of the distance of the planet M from the sun N. For KT and MN are equal. I did in fact say this in the commentaries, but did not explain it in this diagram in ch. 59 and 60. And now MN, NL are known, and L is right, and therefore, the equated anomaly MNL is given.'

The procedure outlined here is still the best way to compute the equated anomaly from the mean. A method of iteration that converges quickly is given by Peter Duffet-Smith, *Practical Astronomy With Your Calculator*, Second Edition (Cambridge, 1981), p. 85.

It is enough for me to believe that I could not solve this *a priori*, owing to the heterogeneity of the arc and the sine. Anyone who shows me my error and points the way will be for me the great Apollonius.

PART V

# On the latitude

# 61

## An examination of the position of the nodes

The ratio of the orbs of Mars and the earth, the eccentricity of each, and the shape of their paths, have all been found with great certainty in the preceding chapters. Therefore, we can now easily accomplish here what we sought out in an approximate way in chapters 11, 12, 13, and 14.

Let us begin with the nodes. On 1593 December 10, at $7^h$ $0^m$ in the evening, Mars was observed at 4° 44′ Aries, with latitude 0° 1′ 15″ south, parallax not accounted for. Its altitude being 35½°, it was not subject to refraction. After the 687 days of one complete revolution of Mars, on 1595 October 28 at $11^h$ $30^m$ pm Mars was found at an altitude of 51° in 18° 35′ Taurus, with latitude 4½′ south, parallax not accounted for. And again, 687 days previously, on 1592 January 23, at $10^h$ pm, it again had a southern latitude of 2′, with an altitude of 25°. And finally, subtracting another 687 days, so that we come to 1590 March 7, Mars was observed on March 4 at $7^h$, at an altitude of 14°, to have a latitude of 3′ 20″ south. This would have appeared larger, except that Mars was low enough to be refracted, and appeared too high. For the refraction at this altitude is 3½′, of which about 2′ is accounted to the latitudes; thus, the apparent latitude would be 5′ south. But since we are anticipating by three days the date corresponding to the others, the approach to the node of 1½° made in this space of time removes three minutes from the inclination. When this is converted into latitude, however, the effect is somewhat less, so that the latitude remaining to Mars on the 7th

would be $2\frac{1}{2}'$, and perhaps a little less, if the refraction were less. For its quantity is not perfectly constant.

Let it thus be concluded that in 1590 the latitude was 1 minute; in 1592, $1\frac{1}{2}'$; in 1593, $2\frac{1}{2}'$; in 1595 at $11^h$, $4\frac{1}{2}'$, as we might allow an error from one source or another of one minute either way. These latitudes will indicate to us an inclination of $1\frac{1}{2}'$, which requires a distance from the nodes of about 40′. This is only for the sake of consensus.

We will nevertheless accomplish our aim more accurately using the year 1595. For while on October 28 at $12^h$ the latitude was $4\frac{1}{2}'$ south, six days later, on the following November 3, at the same time, the latitude was 19′ 45″ north. Therefore, over 6 days the latitude was changed by 24′. So it changed 4′ per day. And since on October 28 at $12^h$ its eccentric position was 16° $8\frac{1}{3}'$ Taurus, and the remaining latitude was $4\frac{1}{2}'$, this would be traversed in one day and one eighth, after which time 37′ would be added to Mars's position. Therefore, the node will be at 16° $45\frac{2}{5}'$ Taurus, at the beginning of November of 1595.

About the other node, there was not such a crowd of observations.
303 Therefore, the year 1589 alone will uphold the trustworthiness of this operation. For since on 1589 May 6 Mars had $6\frac{2}{3}'$ of northern latitude, it would traverse this in $2\frac{1}{3}$ days, according to the proportion of the latitudinal motion of the preceding days, [arriving at the node on] May 8 at $20^h$, at which time its eccentric position is found to be 16° 42′ Scorpio. In 1595, this would be 16° 47′ Scorpio, the position of the ascending node, while previously we found the ascending node to be at 16° $45\frac{2}{5}'$ Taurus. Therefore, at the end of 1595, the nodes are at 16° $46\frac{1}{3}'$ Taurus and Scorpio.

# 62

## An examination of the inclination of the planes

On 1593 August 25 at $17^h\,27^m$, Mars was observed at opposition to the sun at 12° 16′ Pisces[1]. On the 23rd its latitude was 6° 7′ 30″. On the 24th it was 6° 5′ 30″. On the 29th it was 5° 52′ 15″. Therefore, in 5 days the latitude decreased by 13′ 15″, while during one day before opposition, by 2′. Therefore, according to this proportion, if the latitude on the day and hour of opposition is taken to be 6° 2′ 30″, there will not be half a minute's error.

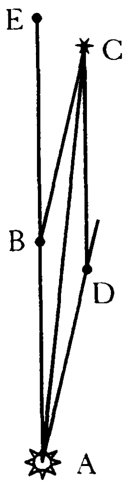
These latitudes were observed when Mars was at an altitude of 22°, which is now thought to be enough to free the fixed stars from refraction. Now since the equated anomaly was 166° 36′, the distance between Mars and the sun was 138,556, and between the earth and the sun, 100,666. *Hence, in the diagram of chapter 13, if* A *is the sun,* B *the earth,* C *Mars, and* AB *is 100,666,* AC *138,556, and* EBC *6° 2′ 30″, the declination* BAC *of the orbit from the ecliptic at this point is shown to be 1° 39′ 22″*. And since the node is at 16° 43′ Taurus[2], I subtract from this 12° 16′ Pisces. There remains an arc of 64° 27′. And as the sine of this is to the inclination here of 1° 39′ 22″, so is the whole sine to 1° 50′ 10″, the inclination of the southern limit[3].

But since the position is rather far from the limit, in order to cut off any opportunity for suspicion, let us consult observations at positions

[1] This is Observation 7 from the table at the end of chapter 15.

[2] According to the position given at the end of the preceding chapter, together with the motion of the node given in chapter 17, this should be 16° $44\frac{3}{4}$′ Taurus: the effect upon the computation, however, is negligible.

[3] The translator finds Mars's distance to be somewhat less (138,547). The resulting inclination is 1° 39′ 11″, and the inclination at the limit would accordingly be 1° 49′ 56″.

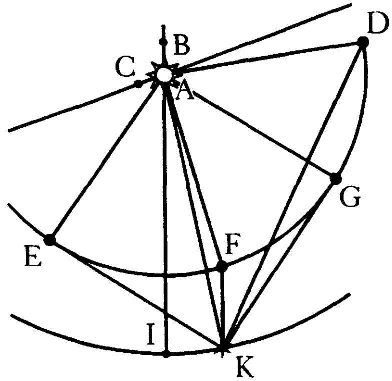


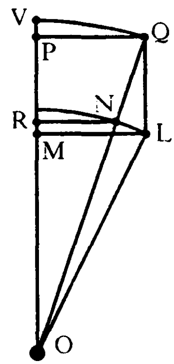

other than acronychal, where Mars is near the limit. In undertaking this, I shall also present a more universally applicable demonstration of the ratio between the inclination and the observed latitude. On 1593 July 21 at $14^h$ (in astronomical terms)[4], the planet was observed at 17° 45¾′ Pisces, with latitude 5° 46¼′ south. At this hour the eccentric position of Mars is found to be 20° 1½′ Aquarius, while the sun's position was 8° 26′ Leo.

*In the present diagram, let* EA *be at 8° 26′ Leo, and* KA *at 20° 1½′ Aquarius.* EAK, *the true angle of relative motion, will be 11° 35½′. Also, let* EK *be at 17° 45¾′ Pisces. I say that the sine of* AEK *is to the sine of* EAK *as the sine of the inclination of* K *is to the sine of its observed latitude. For let the inclination of* K *be understood as a straight line dropped perpendicularly from the body of the planet to the ecliptic. So, as the distance* EK *is to the distance* AK, *so will the sine of the apparent angle of the line* K *as seen from* A *be to the sine of its apparent angle as seen from* E. *But as the sine of* EAK *is to the sine of* AEK, *so is the distance* EK *to the distance* AK. *Therefore, as the sine of* EAK *is to the sine of* AEK, *so is the sine of the apparent angle of the line* K *as seen from* A *to the sine of its apparent angle as seen from* E.

304 *The minor premise is known from trigonometry, and specifically, from Book 3 Number 14 of Lansberg's trigonometry*[5]. The major

[4] That is, at 2 am on July 22.
[5] Philip Lansberg, *Triangulorum geometriae libri IV* (Leiden 1591).




premise requires proof. *Therefore, let there be the straight line* VO, *from two points of which,* P *and* M, *let two perpendicular and equal lines* PQ *and* ML *be set up. And let the ends* Q *and* L *be joined with a point on the line* VO, *and let this be* O. *Now, about centre* O, *with radius* OL, *let an arc be described intersecting* QO *at* N, *and from* N *let a perpendicular* NR *be dropped to* VO. *Therefore, as* PQ *is to* QO, *so is* RN *to* NO. *But* PQ *is equal to* ML. *Therefore, as* ML *is to* QO, *so is* RN *to* LO. *Now* ML *is the sine of the angle* LOM, *under which the magnitude* PQ *or* LM *is observed from nearby, when* LO, *which is the shorter distance of the end* L, *is the whole sine. But* QO *is the longer distance of the magnitude* ML, *or of the end of* PQ, *namely,* Q. *And* RN *is the sine of the angle* NOR, *under which* LM *is observed, or the more remote line* PQ, *where* NO, *or* LO, *is again the whole sine. Therefore, as the sine of the apparent angle from nearby is to the longer distance, so is the sine of the apparent angle from afar to the shorter distance. And, permuted and converted, as the shorter distance is to the longer, so is the sine of the apparent magnitude from afar to the sine of the apparent magnitude from nearby.* And in the present investigation, and universally as well, as the distance of Mars from the earth is to its distance from the sun, so is the sine of the latitude to the sine of the inclination of the planes. And, in turn, as the distance from the sun is to the distance from the earth, so is the inclination to the latitude. *Q.E.D.*

*Since these things are certain, and since the line designated by* K *appeared to be 5° 46¼′ from* E, *multiplying the sine of this by the sine of* EAK, *and then dividing by the sine of* AEK, *results in the sine 3188, whose arc is 1° 49′ 37″. And this is the inclination of the point* K *as it*

*would appear from* A. And since Mars is at 20° 1½′ Aquarius, and the node is at 16° 43′ Taurus[6], and thus the elongation of Mars from the node is 86° 42′, therefore, as the sine of this elongation is to the whole sine, so is the sine of 1° 49′ 37″ to the sine of the maximum inclination, 3200[7]. Therefore, as before, this again gives 1° 50′ 2″ south.

For the northern inclination, at midnight following 1585 January 31, at an altitude of 53°, the latitude of Mars, now decreasing, was 4° 31′ north. But the true opposition was 16 hours 46 minutes previously, at 21° 36⅙′ Leo[8]. Accordingly, the latitude would have been 4° 31′ 10″. And since the [full-circle] complement of Mars's equated anomaly was 7° 6′ 23″[9], its distance from the sun was 166,334, and the sun's distance from the earth, 98,724. *So, again in the diagram of chapter 13, if* AC *is 166,334,* AB *98,724, and* EBC *4° 31′ 10″,* BCA *comes out to be 2° 40′ 50″. This, subtracted from* EBC, *leaves* BAC, *1° 50′ 20″*. But because we are 5° from the limit, the inclination of the limit will be about 25″ greater, namely, 1° 50′ 45″. Before, the southern inclination was 1° 50′ 8″. The difference of 37 seconds is clearly of no significance. The average of the two is 1° 50′ 25″, the perfectly correct inclination, the same amount found in ch. 13 above with various methods and operations, to which I again draw your attention here.

Now, if I compute the latitudes of Mars at opposition to the sun using this inclination of the limit, I find the following[10].

305

| | Year | Distance of Mars | Distance of sun | Inclination | Apparent latitude | Our table in ch. 15 |
|---|---|---|---|---|---|---|
| 1 | 1580 | 152,976 | 98,223[11] | 0° 37′ 42″ | 1° 45½′ N. | 1° 40′ |
| 2 | 1582 | 162,255 | 98,233 | 1° 36′ 6″ | 4° 3⅓′ N. | 4° 6′ or 4° 3′ |
| 3 | 1585 | 166,335 | 98,724 | 1° 50′ 3″ | 4° 30½′ N. | 4° 31⅙′ |
| 4 | 1587 | 164,635 | 99,641 | 1° 25′ 42″ | 3° 37′ N. | 3° 37′ or 3° 41′ |
| 5 | 1589 | 157,045 | 100,860 | 0° 23′ 20″ | 1° 5⅓′ N. | 1° 7⅓′ or 1° 12¾′ |
| 6 | 1591 | 144,744 | 101,777 | 1° 11′ 9″ | 3° 59⅙′ S. | 4° 1½′ or 3° 56′ |
| 7 | 1593 | 138,556 | 100,666 | 1° 39′ 40″ | 6° 3¾′ S. | 6° 2½′ or 5° 58′ |
| 8 | 1595 | 148,817[12] | 98,756 | 0° 1′ 39″ | 0° 5⅕′ N. | 0° 8′ approx. |
| 9 | 1597 | 159,200 | 98,203 | 1° 19′ 17″ | 3° 20′[13] N. | 3° 33′ |
| 10 | 1600 | 165,406 | 98,478 | 1° 49′ 24″ | 4° 30¼′ N. | 4° 31′ |
| 11 | 1602 | 166,004 | 99,205[14] | 1° 39′ 35″ | 4° 7⅖′ N. | 4° 8′ or 4° 10′ |
| 12 | 1604 | 160,705 | 100,359 | 0° 52′ 9″ | 2° 18⅗′ N. | 2° 21½′ or 2° 26′ |

6 This position is about two minutes low – an insignificant error, as was remarked above.
7 Recalculation shows the sine to be 3193, and the corresponding angle 1° 49′ 26″.
8 This is Observation 3 from the table at the end of chapter 15.
9 This is the exact figure obtained using the aphelial position for the beginning of 1584. Kepler apparently used the wrong row in the table of ch. 17. The correct anomaly is 7° 7′ 32″. This has no significant effect upon the distance, however.

In the first, an observation on the day was lacking, as you have seen in ch. 15. In the second there was an uncertainty of three minutes in the observation, since they occasionally used 34° 7′ as the altitude of the pole, which was 34° 5½′[15]. The third has served as our foundation. The fourth agrees to a hair, if you neglect parallax, correction for which would incorrectly make the observed latitude 3° 41′, as you have seen in ch 15. In the fifth, we are wanting 2 minutes. It is surely the observation that is too high, on account of refraction, since Mars was no higher than 22½°, as you know from ch. 15. In the sixth, you may note a slight defect of about two minutes. But the quantity of the refraction is not so reliable: what if it was two minutes higher? The seventh, again, served us as a foundation. In the eighth, the declination was without doubt obtained erroneously, because at that time ($8^h$) Mars was not at the meridian. And the armillary spheres, by which the declination is measured elsewhere than at meridian, err more easily than the quadrants. Furthermore, a comparison with nearby dates, as in ch. 15, shows that the latitude was 0° 5′ N., the same as we have computed. The ninth observation is not worthy of trust[16]. However, the accurately examined calculation for December 10 closely agrees with the Fabrician latitude of 3° 23′, for it gives 3° 21⅔′ N. The tenth comes close to the calculation. The eleventh corresponds to a hair when refraction is excluded. The twelfth is barely two minutes greater than the calculation. I believe this is because there is that much uncertainty in my instruments. For in my quadrant of six cubits, two minutes are not easily discerned. We therefore have the acronychal latitudes determined accurately enough throughout the entire circumference of the circle, using this inclination of 1° 50′ 30″. An examination of the remaining latitudes at observations at positions other than acronychal, of which there are many closely spaced examples in this book, I leave to more diligent scholars.

[10] Where recomputation reveals a substantial difference, the translator's figure is given in a footnote.
[11] 98,415
[12] 148,321
[13] 3° 26′
[14] 99,268
[15] Kepler inadvertently gives the zenith distances of the pole rather than the polar altitudes here.
[16] Note, however, that Kepler's computed latitude is in error, and should have been 3° 26′.

# 63

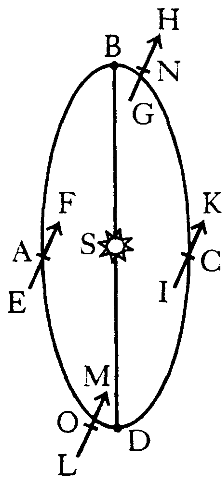
## Physical hypothesis of the latitudes

306 It was said in chapter 57 that if the diameter of the body or globe of Mars is supposed to possess a magnetic force, and to be directed towards the middle longitudes, and also to remain parallel to itself in that disposition throughout its entire circuit, the physical hypothesis of the eccentricity is complete.

This supposition is all the more probable in that now the reason for the latitude too is explained using a closely related theory: that there be supposed some diameter of latitude in the body or globe of Mars that is directed towards the sidereal position of the limit, and remains parallel to itself in this disposition throughout its entire circuit. The ratio of this power to the former is that which [the power] of direction in our magnets has to the force of attracting iron.

That is, the former seeks the sun or flees it, while the latter, rather than seeking by sailing towards, or fleeing, those sidereal positions beneath which the limits of the latitudes are reached, is only directed towards those positions, as a magnet towards the pole. For likewise, a magnet does not sail towards the polar region even if it floats freely.

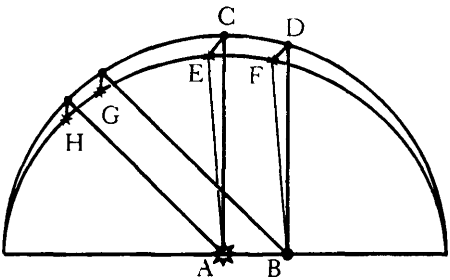
In fact, the excursion of the planet from the plane of the ecliptic to either side follows the direction towards which this axis of inclination, and specifically the part of it which leads in the motion of its body, is pointed. Let CBAD be the ecliptic, A, C the nodes; B, D the limits. And let the axis of latitudes in the body of the planet be GNH, EAF, LOM, ICK. Now since we are supposing this axis to remain equidistant from itself throughout its circuit, it will happen that as the body moves from the ascending node C to the northern limit B, the axis IK




of the body, which initially, at the node C, was tangent as it were to the imaginary circle of its circulation through CNAO, later intersects it at right angles to the limits N, O, being directed towards the centre of the world S, that is, towards the sun. But also, owing to a certain amount of declination from the royal road[1] CBA, this axis has thus far been enticing the body of the planet to leave that path in the direction of N, towards which the leading part K was pointed. Now, at the limits, although it has indeed remained inclined to the plane of the ecliptic CBS (for we have said that it remains equidistant from itself in all positions, and so once it is inclined to the plane of the ecliptic it will always be inclined), it nevertheless does not continue to decline from the royal road, that is, from the circumference of the plane of the ecliptic CBAD, once it is placed at GH. For it does not incline forward towards A, nor back towards C, but only to the side or towards the pole, places to which its path is not directed. So when the planet is moved forward beyond B, the other part of the axis, G, which inclines towards the south, is now in front, and it thus leads the planet from its greatest northern inclination N through the descending node A to the greatest southern inclination O.

[1] Here, 'royal road' means the ecliptic. In chapter 68, however, Kepler uses this term in a special sense. Since it appeared that over the ages the ecliptic had changed its position relative to the fixed stars, Kepler introduced a 'mean ecliptic' under the name 'royal road', in order to avoid giving the plane of the earth's orbit a privileged status. There, the royal road is the equatorial region of the sun, which he assumed to rotate.

This axis of inclination is very much like an oar, in that just as boatsmen use oars to move from one bank to the other, this attends the planet through the inclination of the axis, moving it from north to
307 south and back, while the river, that is, the immaterial *species* of the sun, proceeds along the direct path CBAD.




As for geometrical dimensions, there is no need for verbosity. A straight line moved on a rectilinear course while remaining parallel to itself creates a plane through its motion. This axis is itself a straight line, and it is moved in the direction it points (this pointing, moreover, presupposes a straight course). It therefore describes a plane. And if this plane be extended, it intersects the sphere of the fixed stars in the shape of a great circle, FEGH in the diagram of chapter 13, because it intersects the plane of the ecliptic DC at the centre of the world or the sun, A. To further convince yourself of this, you should consider that the points of intersection or nodes are in opposite positions about the centre of the sun A, as you see in the diagram. This is shown by experience: see ch. 62. And so since there is a plane in which the orbit of Mars is moved about, its inclination to the plane of the ecliptic will follow a pattern. That is, when two equal circles are described, one, DC, in the plane of the ecliptic, and the other, FE, in the plane of Mars's orbit, about a common centre A of the sun (i.e., on one and the same sphere of the fixed stars, concentric with the sun), the sine of the arc BD between the intersection of the circles and some point on Mars's circle, D say, is to the whole sine as the sine of the inclination DF of the point F is to the sine of CE, the greatest inclination, at the limit E. Furthermore, it was proved in ch. 13 above, using an ingenious treatment of the observations, that the

declinations of all the points of the circuit from the plane of the ecliptic are subject to the same measure. So no instance can be urged against our hypothesis.

But there are still two difficult questions to be answered. One concerns how this declination of the axis originates, and the other concerns the axis itself. First, is the inclination of the axis natural or rational, the work of the body's nature or of an angel? And second, are the axis of inclination and the magnetic axis that seeks the sun the same? And if they are different, how do they exist in the same globular planetary body? The two questions are interrelated.

I might almost believe it to be natural, owing to the similarity between the natural power in the magnet and this one, except that there is in addition the successive transposition of the nodes, which clearly seems the work of a reasoning (although not discursive) faculty, or at least an instinct. For to maintain its equidistant position is less marvellous and more in accord with nature than previously in the matter of the eccentricity. There, we said that it is the sun that is sought by the axis of power, while here, it is the position beneath the far distant fixed stars. There, the axis was to have been turned about by the force of this magnetic power as the body is carried around, and would not remain equidistant to itself, if it were not restrained by a stronger directional force, or by an animate force, either unassisted or capable of reasoning in some manner. Here, the axis holds to this equidistant position by the force of our directional power itself, with no need of an animate power or of reasoning. Someone might, however, consider it the act of a mind, that the diameter that effects the latitudes points directly towards the centre of the sun when the
308 planet is located at the limits, thus making a great circle of the planet's orbit, and causing the nodes to be at opposite positions with respect to the sun.

A response to this argument was made in ch. 39 above. I affirmed that the planet moves with respect to the sun. However, it is not just any kind of motion with respect to the sun that argues for the assistance of reason. It is of course true that he who first ordained the heavenly motions so directed this axis as to point at the sun when at the appointed position, and did so deliberately and with perfect rationality. But this relationship with the sun can now be maintained without a mind, by the constancy of the magnetic faculty alone. For it is more like rest than motion, and hence is material, not mental.

Therefore, it is only the variation of this inclination which we call

the translation of the nodes over the ages that still makes a case for a motive force that is more than natural, or physical, as are magnetic powers.

Nevertheless, I would prefer to think that the two must be conjoined, rather than to suppose that the rational faculty acts alone. The magnetic faculty would be subordinate, while the rational would be in charge, ruling over it, just as we said before in chapter 57 concerning the power of seeking the sun.

Terms. The diameter that causes the eccentricity is one thing, the diameter of the reciprocation another. The former is something real, while the latter is imaginary, devised in order to imagine its effect. The one is the same everywhere, being directed perpendicular to the line of apsides, or towards the sidereal position of the middle longitudes; the other, as was said in ch. 39, is always directed towards the body of the sun.

Once this question is settled, there follows the other. If this directive power arises from magnetic, physical, natural [powers], its subject will be a body. Could it therefore happen that the same diameter that seeks the sun or flees from it also governs the planet's inclination with respect to the ecliptic, by being inclined to it? If the nodes were connected to the apsides and the limits to the middle longitudes then the diameter would be the same in all respects, administering both the eccentricity and the latitudes.

For it was said in ch. 57 that the diameter that causes the eccentricity is directed towards the middle longitudes, while it was just now said that the diameter that causes the latitude is directed towards the limits. Therefore, if the limits were connected to the middle longitudes, both diameters would have the same direction, and, their positions thus being in agreement, nothing would prevent their being identical. However, the nodes, or intersections with the true ecliptic, do not coincide with the apsides. For Mars, the northern limit is 12 degrees before the aphelion, for Jupiter, the northern limit and the apsides coincide exactly, for Saturn, the node follows the aphelion by 24 degrees, and for the moon, owing to its short orbit, everything becomes interchanged with everything else. For now the node is at apogee, now at the middle longitudes, now at perihelion [sic]. So since these two powers differ in time and position, it follows that they are not identical.

There is, however, nothing to impede their residing in one and the same planetary body as a whole, except the motion or rotation of the globe. Thus if the planets are moved like the moon, which does not rotate, but always shows us the same face, nothing prevents our saying that the two are interwoven, as the weft is interwoven with the warp. For since the entire body of the planet would then stay in the
309 same sidereal position as it is carried about the sun, any of its rectilinear parts, among which are numbered those two diameters, will stay in the same sidereal position. If, on the other hand, it is the

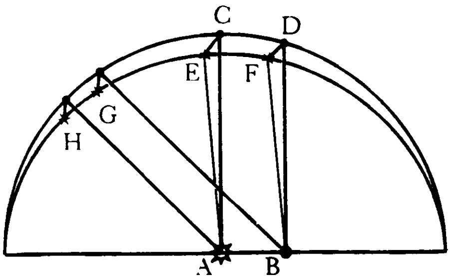
earth's globe that is in question, which has a daily rotation in addition to its annual revolution, we are left in great doubt, no less than before, in ch. 57. For if the body rotates, only one single diameter of power, that which is parallel to the axis of its rotational motion, remains constant and equidistant from itself. So if you were to say that in addition to that diameter, there is interwoven with it another altogether different one, a power of another sort, which causes the latitudes, it will observe the same direction as the axis of rotation, since it circumscribes a cone about that axis, successively traversing each of its parts, and since it inclines, now to the right, now to the left side, it finally leads the body towards the middle position, whither the axis of rotation points.

Therefore, if the globe rotates, the subject of this declinational power is either not a body but something spiritual, or is not the same body. If it is something spiritual, how does it look to certain regions of the world, which are corporeal? And how does it impart this kind of motion (declination from the royal road) to the body? Is it perhaps that the body is more easily inclined, and departs from the royal road more easily (meanwhile receiving its translational motion extrinsically, from the sun), than it is carried from place to place by the force of its own proper mover? If, on the other hand, we prefer a corporeal subject, some mechanism has to be brought into being for us, like those spherical oil lamps which, though thrown and spun around, do not spill any oil. For within is enclosed a little flask which, being drawn down by a weight in its belly[2], and held there, does not follow the convoluted motion of the surrounding sphere.

Is there then some interior globe within this globe of the earth, to which the diurnal motion of the earth's exterior does not penetrate, but which is held in place by a very strong inclination towards certain sidereal positions, so as not to follow the revolving exterior of the body? For as we shall see in chapter 68, this question pertains to the earth as well. We shall also see there whether, if some mean ecliptic be proposed for the six planets, that which we were requiring a little earlier is accomplished, namely, that the nodes of each of the planets correspond to the apsides.

Or is it rather to be believed that there are some possible modes of celestial motion which, though physical like the magnetic [powers], cannot be comprehended by anyone on earth owing to the lack of

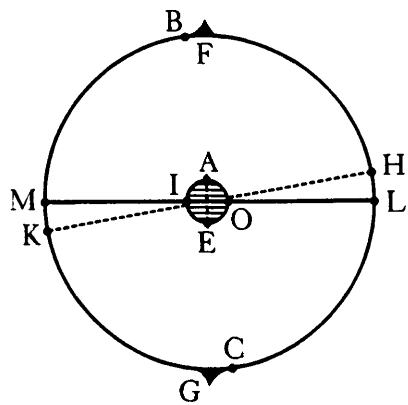
[2] Reading *ventriculoso* instead of *ventricoso*.




examples? For if we had lacked the example of the magnet (which was indeed unknown at one time), we would have been ignorant of many of the causes of the celestial motions.

Those who believe in solid orbs can easily set everything right, following what was said in ch.13. For they will attribute to the plane of Mars's eccentric FE an inclination to the plane of the ecliptic DC that does not librate, but is fixed and constant. The diameter of intersection of these planes BA will pass through the centre of the world (the centre of the sun, for Brahe), and they will say that over the ages it rotates about the centre A beneath the ecliptic DC.

310 And since the poles (F, G and B, C in the present diagram) of two great circles (ML and KH) are distant by an amount equal to their maximum declination MK, LH, the poles of Mars B, C will therefore



describe small circles about the poles of the ecliptic F, G, with radius FB, GC, of 1° 50′ 25″. These people will also say that the poles of the Martian sphere B, C revolve forward with a motion quantitatively the same as that expressed above in ch. 17, and corrected below in ch. 69.

# 64

## Examination of the parallax of Mars through the latitudes

In chapter 61, the two nodes were found to be at positions exactly opposed; a marvellous agreement and one which excludes all parallax.

Let it be the case that Mars's parallax is at least 1′ and 2′ [respectively] when at opposition to the sun (and nearer to the earth than the sun) in 1595 and 1589, and that on the former date Mars was about 38° from the zenith, and on the latter about 66°. Accordingly, in 1589, when it was thought to be at the node, it would still have been nearly 2′ to the north. Therefore, it would still have been one degree before the node. So the node would be, not 16° 46′ Scorpio, but 17° 46′ Scorpio. In 1595, on the other hand, it would have 1′ of parallax. Therefore, on the day on which it was thought to be at the ascending node, it would now actually have had a latitude of 1′, and it would thus now have been about 30′ beyond the node. Therefore, the ascending node would be, not at 16° 46′ Taurus, but at 16° 16′ Taurus. You see that the descending node is at $17\frac{3}{4}$° Scorpio, and the ascending at $16\frac{1}{4}$° Taurus, if you make use even of the least parallax. Let us therefore conclude, as in ch. 11, that Mars's diurnal parallax is entirely imperceptible, if it is indeed true that the two observations of latitude are correct with 2′.

There is another argument for no parallax, which is not dissimilar, arising out of ch. 62. It rests upon the investigation of the inclination of the planes, perfectly true unless refraction throws something off.

Let it be the case that in 1593, at an altitude of 22°, Mars had a parallax of at least 2′, while in 1585, at an altitude of 53°, it had a

parallax of one minute. The observed southern latitude would therefore be smaller than the northern, and so the inclination would also be smaller. But just now, without parallax, it appeared somewhat smaller, by an amount attributable to a small error in observation or to a certain amount of refraction at an altitude of 23°. Therefore, when parallax is considered, the observation is charged with a greater error, and conversely, if the observation stands, the parallax is entirely eliminated, if it is indeed true that the orbit of Mars is contained in a perfect plane which intersects the plane of the ecliptic at the very centre of the sun.

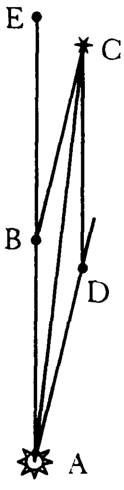
311 But the same is proved much more certainly from the latitudes observed at other acronychal positions, especially those which the circumstances of observation or refraction did not render dubious. As I began to say in ch. 15, this matter has so far been impossible to settle. For in 1587, when Mars was 55 degrees from the zenith, if it had a parallax of 4′ its latitude of 3° 37′ would have been increased to 3° 41′. But in chapter 62, it was found to be no greater than 3° 37′. And in 1589, when the nonagesimal was 64° from the zenith, if Mars's parallax had been $5\frac{1}{2}'$ (judging from the sun's horizontal parallax of 3′), then the northern latitude, instead of the observed 1° 7′, would have been $1° 12\frac{1}{2}'$ freed of parallax. But we have computed no more than $1° 5\frac{1}{3}'$, although a slight error of 2′ could have occurred in the observation, such as if Mars at an altitude of 22° was subject to enough refraction to have appeared 2′ higher (to the north) than was correct, as was said in both ch. 62 and ch. 15. And in 1602, when with a parallax correction the observed latitude was found to be 4° 10′, and without the correction, $4° 7\frac{1}{2}'$, we computed $4° 7\frac{2}{5}'$, very precisely. Similarly in 1604 we did not agree perfectly with the observed quantity of northern latitude. Therefore, we shall complain that it is much less when it is increased through a correction for parallax.

By these three procedures, we have overcome our uncertainty about Mars's parallax. However, we have not completely proved that it is utterly imperceptible, since the matter of refraction eludes us, and besides, the observations do not descend to within 2 or 3 minutes. So if anyone wishes to attribute to Mars a maximum latitudinal parallax of 2 or $2\frac{1}{2}$ minutes, these Brahean observations do not significantly disagree with him. For the inclination, too, will be accommodated to this view, becoming 1° 51′ 0″.

# 65

## Investigation of the maximum latitude at both conjunction with the sun, and opposition to it

Once the inclination is established, it is easy to define the maximum latitude, and this can be done in two ways. For one can find the maximum for all time, or how great it could be in our time. Today the two hardly differ, since the limits are the midpoints between the apsides of Mars and of the sun or earth, and they are no more than 54 degrees from one another, and the eccentricity of the sun or earth is not great. Nonetheless, let it be the case (as it once was) that the apsides of Mars and the sun coincide, along with the limits of Mars's latitudes. And let the ecliptic maintain its sidereal position. Now



312 since, in the diagram of ch. 13, Mars's greatest distance AC is 166,465, the sun's least distance AB is 98,200, and BAC is 1° 50½′, the maximum northern latitude at opposition to the sun computed from these data is 4° 29′ 10″. At conjunction with the sun, when the sun's distance from the earth is 101,800, this is decreased to 1° 8′ 34″. But the southern latitude, from Mars's distance of 138,234, and the sun's of 101,800, is computed to be 6° 58′ 24″, a little less than 7°. At conjunction with the sun, when the sun's distance is 98,200, this is decreased to 1° 4′ 36″. If, however, one considers the contrary case, in which the sun's apogee coincides with Mars's perihelion, the maximum northern latitude at opposition comes out to be 4° 44′ 12″, and at conjunction 1° 9′ 32″, while the southern latitude at opposition is 6° 20′ 50″, and at conjunction 1° 3′ 32″.

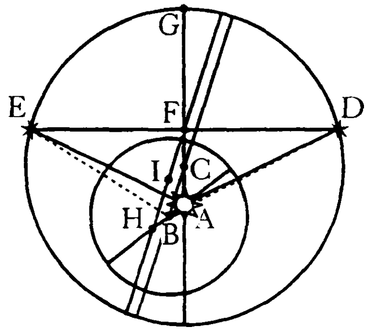
And this is how it would be if the apsides and the limits were to coincide at some time, but whether this is going to happen before the whole fabric comes to ruin is uncertain. It is certain that Ptolemy attributed the same motion to the apsides and nodes, and if this were so, that conjunction would never happen. But even though today they appear to undergo different motions, the observations of the ancients are not sufficiently reliable, and the difference of these motions even in modern astronomy is not sufficiently great, for us to conclude how many myriads of years apart these conjunctions of the apsides and the limits occur.

Therefore, let us return to our era, that which extends between us and Ptolemy. And here, one who is looking for geometrically precise determinations is presented with a manifold obstacle to computation.

First, the apsides of the sun and Mars are not in conjunction, and second, the planets' orbits are not perfect circles. So even if we project a new line of apsides through the centres of the circles of Mars and the earth (through B, C in the diagram of chapter 52), it will still be possible for the nearest approach of the celestial bodies to occur elsewhere than on this line.

Finally, even if the position of the nearest approach were established, the position of the northern and southern limit is different. For example, the limit is at 16° 50′ Leo. But the straight line BC projected through the centres of the circles is directed towards 24½ Leo and Aquarius, approximately; in the same direction, that is, in which Brahe put his line of apsides HF, to which our line BC runs parallel, both eccentric cities being bisected, AF at C, and AH at B.

And I was now about to choose the mean between 17° Leo and 25°




Leo, namely, 21° Leo, but the year 1585 gave me pause, since in that year the latitude observed at 21° 36′ Leo was clearly not a maximum. For while the opposition was on the night following January 30, the latitude observed on the 24th, preceding the opposition, was 4° 31′, still increasing, while on January 31, 16 hours past opposition, the observed latitude was again 4° 31′. It therefore appears that on the 24th, if the opposition had occurred at that eccentric position, the latitude observed would have been greater than 4° 31′, for two reasons: first, because the celestial body is nearer the earth than it was when it was not at its acronychal position, and second, because Mars was farther from apogee, and was lower.

313 Therefore, let the maximum latitude occur about 19° Leo and Aquarius, where Mars was on January 24. And since the supplement of the equated anomaly was 10°, the distance of Mars will be 166,200, and of the sun, 98,670. And so the maximum northern latitude will be about 4° 31¾′[1]. At conjunction with the sun, since its distance was 101,280, this will appear to be 1° 8′ 30″[2].

For the maximum southern latitude, Mars's equated anomaly of 170° shows us a distance of about 138,420, and the sun at 19° Leo has a distance of 101,280. Hence it is concluded that the maximum southern latitude will be about 6° 52′ 20″[3], and at conjunction it will appear to be 2° 4′ 20″[4].

[1] This number is consistent with a position at the limit. However, since the position given is 2° from the limit, the angle is about one minute less, and the computed latitude about 4° 29′, which does not fit the observations so neatly.

[2] The correction mentioned in the preceding footnote would make this angle 1° 8′.

[3] 6° 47′, with the correction mentioned above.

[4] This should have been 1° 4′ 20″, or 1° 3′ 55″ with the corrected inclination.

# 66

## The maximum excursions in latitude do not always occur at opposition to the sun

Concerning the maximum latitude that can occur in any particular period of Mars, however, it is a much more complicated business to define its exact positions geometrically, and also involves this great paradox, which I found expressed among the observations of 1593 in Tycho Brahe's hand, in the following words:[1]

> 'It is worthy of consideration that on about the tenth day of August Mars had its maximum southern latitude, and that it decreased afterwards, so that at opposition on the 24th it was about one fourth of a degree nearer to the ecliptic. However, the Canons[2] do not show this at 18 Aquarius, even when the position of maximum latitude is corrected, no matter how that maximum latitude is found in either case. The cause of this needs to be looked into carefully.'

When I later had come to him in Bohemia, and frequently inquired about how the latitudes are arranged, he answered that the nodes are at opposite positions, and the line of intersection passes through the point of the sun's mean position, or through the centre of its epicycle (for which see ch. 67 below), and recounted many other things. Of this present matter, he said most emphatically, 'this is remarkable, that the latitudes reach their maximum before or after opposition to the sun.' Mention was also made of this above in chapter 15.

314 The cause of this occurrence is in fact contained in the true

[1] TBOO 12 p. 291 (Citation from KGW 3 p. 483.)

[2] This may refer to planetary models, or to the tables constructed from them.

hypothesis of the latitude established in this fifth part; however, you would have almost as much trouble finding the boundaries of the maximum latitudes geometrically, as Apollonius of Perga had in finding the boundaries of the stations.

On the station points.

For in the business of the stations, a certain known thing can be described through which the position of the stations may be known (and that known thing is this, that the line of vision of Mars, the earth being in motion, remains parallel to itself). But the position of the stations cannot be demonstrated *a priori* from this known thing without multiple calculations, owing to the confluence of many causes. And matters stand just the same with the maximum latitude for any given occurrence. For the latitude is greatest when the distance of Mars from the earth is increasing or decreasing in the same ratio in which the lines of Mars's inclinations increase or decrease. And the latitude is increasing when the ratio of the distance decreases more than the ratio of the lines of inclination, or when the former is decreasing while the latter, on the contrary, is increasing. And, in turn, the latitude is decreasing either when the distance of Mars from the earth increases more than the lines of inclination, each in its own proportion, or when the distance is increasing while the lines are decreasing.

On the points of the maximum latitudes.

These conditions are satisfied indiscriminately, now at opposition, now before, now after, depending on whether the opposition falls at the limit, or before, or after the limit.

That these results follow from the hypothesis of this work, my ephemerides prove. In 1604, about Feb. 25 or March 6, the northern latitude was a maximum, while opposition followed by an entire month. On Sept. 27 or October 7, in turn, the southern latitude was a maximum, while Mars was between its quintile and sextile aspects to the sun[3]. Again, at the end of 1605 the northern latitude was maximum, while the sun was moving from quintile to quadrature with Mars. And, in turn, at the end of July of 1606, the southern latitude was maximum when the sun was trine with Mars[4]. But in 1607, the maximum northern latitude occurred a little after the conjunction of Mars with the sun.

The reason why these things would appear remarkable in ancient astronomy is chiefly that Ptolemy and his imitators had fabricated the extremely intricate motions of inclination, deviation and reflection.

[3] Quintile is 72°, and sextile is 60°.
[4] That is, 120° from Mars.

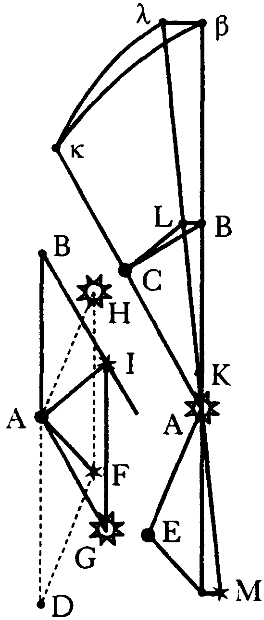
For since Ptolemy clung to his invention of the epicycle, as soon as he saw that when the planet was at opposition to the sun (and was thus visible) the epicycle went out to one side, he immediately indulged in conjecture, asserting that at conjunction with the sun, when it is not visible, the epicycle goes out in the other direction, and generally, that at conjunction the epicycle does the opposite of what he observed it to do at opposition. For there is thus some compensation, both in its periodic returns and its coherence with the sun. However, this is not discovering the true by observing, but fabricating the observations by a falsely conceived fancy. Nevertheless, it should be condoned in him, since he had few observations. On this subject, see ch. 14 also.

But come, let us see whether our calculation gives the observed latitude on August 10. For we are sure of July 21 and August 25 of that year, since the calculation will yield the observations upon which it is based[5].

315 So, on August 10 at $13^h\ 45^m$, Mars's eccentric position on the ecliptic is computed to be 2° 41′ 18″ Pisces, the sun was 27° 37′ 49″ Leo, the angle at the sun 5° 3′ 29″, the angle at the earth 18° 25′, and from the calculation Mars was at 16° 3′ Pisces, while it was observed at 16° 7′ Pisces. And since 2° 40′ 48″ Pisces, the position on the orbit, is distant from 16° 43′ Taurus[6] by 74° 2′, the inclination will therefore be 1° 46′ 10″. From this and the two angles mentioned, using the method of ch. 62, the observed latitude is found to be 6° 21′ 14″, still two minutes more than the observation has. But lest the angle's small magnitude trip us up, let us use the true distances of Mars from the earth and the sun (as the method given above requires), or in their place, the true angles. In the diagram of chapter 20 you see that CB, BA differ from CL, LA. And our method did not say that the sine of the angle LAB is to the sine of the angle LCB as CB is to BA, but as CL is to LA. Let the ecliptic position be 2° 41′ 18″ Pisces, Mars standing beneath the point λ, and κ be the position opposite the sun, 27° 37′ 49″ Aquarius. Therefore, κβ is 5° 3′ 29″, and βλ is 1° 46′ 10″. From this and the right angle λβκ, κλ or CAL is given as 5° 21′ 36″, to which corresponds the true distance of Mars L from

[5] These observations are reported at the beginning of chapter 62, and formed the basis of the revised hypothesis of the latitudes presented in that chapter.

[6] Again, as in chapter 62, Kepler takes a position for the node that differs from the position determined in chapter 61. It should have been at 16° 45⅓′ Taurus, after correction for the motion of the node. The presence of these aberrant numbers in chapters 62 and 66 suggest that the node was originally about 2′ farther back, and that Kepler later revised chapter 61 without changing the data in other chapters.




the sun A[7]. So in triangle CAL, from the sides CA, 101,077, and AL, 138,261, and from the angle just found, LCA is sought, and is found to be 160° 33′. Its supplement is 19° 27′, to which corresponds the true distance of Mars L from the earth C.

So now, using these angles of the operation, I find the apparent latitude LCB to be 6° 19′ 10″, approximately the same as the observed value[8].

Thus the hypothesis established in this work shows this very thing whose cause Brahe advised was diligently to be sought, and which ancient astronomy, for all its apparatus, could not show. And, I would add, it shows this in all its simplicity, in that the plane of the eccentric is given a constant inclination or obliquity, and this is variously increased or diminished, not in reality, but optically only, insofar as our sighting approaches it or recedes from it, or (for Brahe and Ptolemy) it approaches or recedes from our sighting.

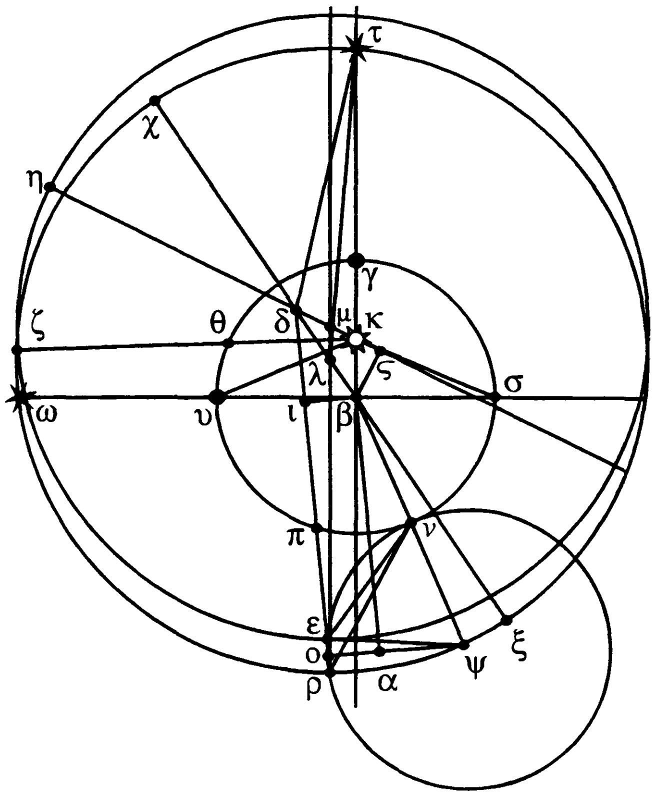
[7] Here and in the final sentence of this paragraph, Kepler appears to have gotten the sun and the earth mixed up: surely the distance corresponding to angle CAL is CL, the distance of Mars L from the earth C, while the distance corresponding to angle LCA is LA, the distance of Mars L from the sun A.

[8] Compare this with the latitudes for August 25 (the date of the opposition), 6° 2′ 30″ S., and July 21, 5° 46′ 15″ S.

# 67

## From the positions of the nodes and the inclination of the planes of Mars and the ecliptic, it is demonstrated that the eccentricity of Mars takes its origin, not from the point of the sun's mean position (or, for Brahe, the centre of the sun's epicycle), but from the very centre of the sun

316 The end is a reply to the beginning. In chapter 6, I argued on physical grounds that when solid orbs are denied, the eccentricities of the planets cannot take their origin from any point other than the very centre of the sun. I postponed part of the geometrical proof of this, based upon the observations, to chapters 22, 23, and 52, in which places I think I have satisfied even the sharpest-eyed critic. The other part I shall now expound. This is done first through the position of the nodes. It was proven in chapter 61 that when Mars's eccentricity is constructed from the very centre of the sun, or, what is the same, using acronychal observations taken when the planet is at opposition to the sun's apparent position, the nodes fall at positions that are very precisely opposite in relation to the sun's centre; that is, that the diameter of the apsides and the diameter of the intersection of the planes of the ecliptic and of Mars coincide, or intersect one another, at the same centre from which the eccentricity is computed, namely, at the centre of the sun. The question now is, if we use the sun's mean motion instead of its apparent motion, will this still result in the nodes' being at opposite positions about the point whence the eccentricity is computed? Not at all. Consider again the Copernican diagram in chapter 6. *In it let κδ now be the line of the limits, at $16\frac{3}{4}°$ Leo and Aquarius (not, as in ch. 6, the line of apsides at 29° Leo). Therefore, the line drawn through κ perpendicular to κδ will be the diameter of the nodes. But if we use the mean sun instead of the apparent sun, then we are given β instead of κ as the point from which*




*the eccentricity is reckoned. So from* β *let* βς *be drawn perpendicular to* κδ. *This will fall at positions exactly opposite about* β, *but will not fall at the positions of the nodes, because the former perpendicular through* κ *falls at the positions of the nodes, which are above* βς *by the distance*
317 κς. It is pertinent to enquire into the magnitude of the angles at the circumference of the eccentric when the point κ is connected to the points of intersection of the line βς with the circumference of the eccentric. *Since, by supposition,* ςκ *is at 16° 45′ Leo, and* βκ *is at about 5° 45′ Cancer, the angle* βκς *will be 41°; and since* βςκ *is right,* κβς *will be 49°. And since* κβ *is the sun's eccentricity, 3600, where the orb of the earth or the sun is 100,000, therefore, as the whole sine of the angle* ς *is to* βκ, *3600, so is the sine of angle* β *to* κς, *2717. And in the same units (where the semidiameter of the earth's orb is 100,000), the semidiameter of Mars's orb, from ch. 54, is 152,350. Therefore, where the*

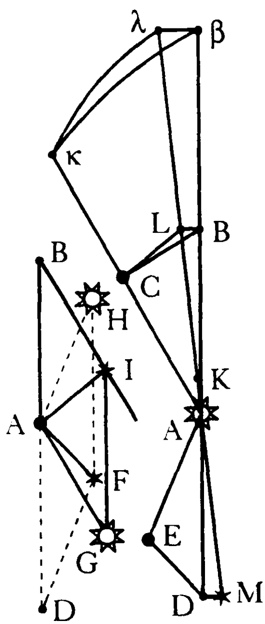
*semidiameter of Mars's orb is 100,000, κς will be 1790, showing an angle of 1° 1′ 33″ in the [table of] sines.*

The ascending node should therefore have been moved backward, and the descending node moved forward, by that number of degrees, minutes, and seconds, if I had been mistaken in taking the sun's centre κ instead of the Ptolemaic, Copernican, and Brahean point β. But if, where the observations are referred to the mean sun, and thus the point β is taken, this is done in error, and κ should have been chosen instead, the ascending node found from β should be in a place farther forward, and the descending node farther back, so as to shorten the northern semicircle by an arc of 2° 3′ 6″.

Let us see whether it happens in this way. In the observations of chapter 12, considered approximately, on 1595 October 28 Mars was considered to have been at the node. From the Brahean equations, which are referred to the point β, its eccentric position was found to be 16° 48′ Taurus. And on the morning of 1589 May 9 we supposed Mars to have been at the other, descending, node. Using the same Brahean equations, we computed Mars's eccentric position to be 15° 44½′ Scorpio at that time. So what I said should happen, does happen: there are 1 degree and 3½ minutes less in the northern semicircle. If the observations are treated more accurately, as in ch. 61, Mars arrives at the ascending node one day and 15 hours late. Therefore, about 50 minutes are added to the eccentric position, so that the planet falls at 17° 38′ Taurus, in its eccentric motion. Accordingly, the abbreviation of the upper semicircle is 1° 53½′, approximately equal to the computed value of 2° 3′.

Therefore, the point κ is entirely confirmed, and β is rejected. For why will the diameter of the intersection of the planes not intersect the diameter of the apsides in the centre from which the eccentricity originates, as above? What would be the cause of such a thing?

The same is also demonstrated through the inclination of the planes demonstrated in ch. 62, using the diagram of chapter 20. *The inclination, that is, the angle LAB under which the digression of the northern limit appears when seen from the sun A, was there found to be 1° 50′ 45″. But the angle MAD, under which the southern limit's digression from the ecliptic appears when viewed from the sun A, was found to be nearly equal to it, namely, 1° 50′ 8″. So the angles at A, above and below, are equal, and the line drawn from A to the ecliptic positions of the limits B, D, is one line (since it is in the one plane of the ecliptic). It is therefore concluded from this that the other line, drawn*




*from A to the limits themselves L, M, is also one line; and further, that what is enclosed within the orbit of Mars is a single plane. Further-*
318 *more, if the common intersection of the planes were not at κ in the former diagram (which is A in the present one), but at βς (that is, below A in the present diagram), when the limits L, M are connected with some point on the line BD below A, the angle under which LB appears from that point would be smaller, and the angle under which MD appears would be larger, by about two minutes.*

It is true that if we are allowed the liberty of making the parallax as great as we please, the arguments of this chapter are easily weakened. But it is a well-documented certainty that it is impossible to allow a parallax great enough to remove entirely the effect of this demonstration.

Also, since the point of this chapter was demonstrated most soundly in ch. 52, I can take another tack, and instead of demonstrating this point by denying parallax, I can deny parallax, as in ch. 64, by maintaining this point, which has its own demonstration in ch. 52.

It does not matter which way you do it. For both points have other demonstrations. The present way occurred to me first, and suited my purpose of showing how everything is in agreement.

# 68

## Whether the inclinations of the planes of Mars and the ecliptic are the same in our time and in Ptolemy's. Also, on the latitudes of the ecliptic and on the nonuniform circuit of the nodes

It was said in chapter 14 that in any one period of Mars whatever, the obliquity or inclination of Mars's plane to the plane of the ecliptic remains fixed. There is, however, some doubt whether this obliquity is the same, and fixed, for all ages. The reason for the doubt is this.

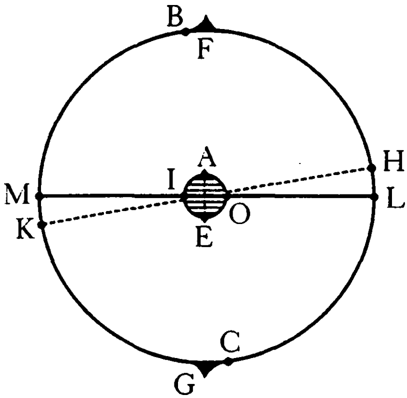
On the altered latitude of the fixed stars.

In the first volume of the *Progymnasmata*, p. 233, Brahe demonstrated that the latitudes of the fixed stars are different today than at the time of Ptolemy, the difference being this: that in the region of the summer solstice, the latitudes of the northern stars increased and those of the southern stars decreased; and, in turn, in the region of the winter solstice, the latitudes of the northern stars decreased and those of the southern stars increased. As one goes from these places towards the equinox points, the alteration of the latitudes diminishes, until near the equinox points there is none at all. This observation of our time we shall accommodate to our principles laid down in ch. 63, thus:

It is established that the sphere of the fixed stars is raised above the planets by an immense interval, and it is accordingly not affected by those motions that are in the planets. Copernicus put the matter very
319 simply: the fixed stars are not subject to any motion from place to place, and thus are truly fixed forever in the same places.

What is the ecliptic?

The ecliptic, in turn, is the great circle in the sphere of the fixed stars beneath which, for us on earth, the sun ever appears, and which it is seen to traverse annually. And whether this motion belongs to the sun or the earth, in either case it belongs to one of the planets. Therefore, the fixed stars do not themselves contain the cause of the




ecliptic: it only results from the annual motion of the earth or of the sun about the centre of the world.

The ecliptic is transported to other fixed stars.

Thus, since the ecliptic is found to have changed its position with respect to the fixed stars, it is not the fixed stars that have moved away from the ecliptic, but the latter that has moved away from the fixed stars.

The reason for the changed ecliptic.

The reason for this translation is shown beyond doubt by our principles of chapter 63, if, indeed, they are sound. *Since the sun, through its most rapid rotation in its space which, for Copernicus, is the centre of the world, sets the planets in motion through an emitted* species, *this rotation will have determinate poles. In the last diagram of chapter 63, let the body of the sun be* IO, *and the poles of rotation be* A, E, *above which stand the points* F, G *on the sphere of the fixed stars. The great circle* IO *of the rotating solar body will thus be established beneath some great circle of the fixed stars: let this be* ML. *This is doubtless one and the same circle beneath the fixed stars, the poles* F *and* G *remaining constant, and the dignity of its body declaring that it be the cause of the others' motion. Nevertheless, the planets are found to move on various circles that are inclined to one another, owing to the natural principles explained in chapter 63. Therefore, beyond doubt, the various circles of all the planets depend upon this 'royal circle'* ML, *described by the rotation of the solar body about its axis* AE, *and each of them will keep its inclination to this circle constant in quantity, though having a translational motion, since we know by experience that the nodes are transposed.*

There exists a mean ecliptic.

*Since the ecliptic, too, is one of the planetary circles, either the sun's or the earth's, it is consistent for it, too, to have some inclination to the royal circle* ML, *described among the fixed stars by the great circle* IO *of the solar body. For what would be the reasons why the other planets would decline from one another, while the ecliptic alone, standing above the solar or the terrestrial path, coincides exactly with this royal circle* ML?

From the supposition of a changeable ecliptic, all the alterations observed in the changes in the latitudes of the fixed stars follow.

*Let this therefore be granted: that the ecliptic, properly so called, is inclined to the royal solar circle. Let it be represented to us by the circle* KH *drawn among the fixed stars, and let its poles be* BC. *Under these conditions, we easily discover the occasion of the alteration of the fixed stars' latitudes: as the name suggests, these are computed from the true ecliptic and not from that royal solar circle, hitherto unknown. For the intersections or common nodes of the ecliptic, truly and properly so called (as a result of eclipses occurring only beneath that line along which the sun proceeds), with the circle* ML, *which we might call the 'mean ecliptic', are subject to a translational motion no less than the nodes of the other planets. Nevertheless, the maximum obliquity* MK *or* LH, *which is measured by the distance of the poles* FB, GC, *remains fixed and constant, as in the rest of the planets. That is, if about centres*
320 F, G, *with constant radii* FB, GC, *small circles be described upon which we suppose the poles of the ecliptic* B, C, *to revolve, then the circle* KH *as well would depart from its original position on the sphere of the fixed stars* FMG, *and over the ages would make the southern limit come to be near the same fixed stars where the northern limit once was. Over a shorter period, however, it would be as follows. Since the limits* K, H *have not moved far from their fixed stars, their latitudes will be changed by some imperceptible quantity. However, since the nodes have progressed by the same amount from their fixed stars, the latitudes of their fixed stars will be altered more evidently. This is because at the end of the quadrant, near the limit, the sines of the inclinations increase by imperceptible increments, while at the beginning, near the nodes, these increments are quite perceptible.*

Where the true ecliptic and the mean one intersect one another.

*Hence, because no change in the latitudes of the fixed stars is perceived near the equinoxes, while it is noticeable enough near the solstices, we correctly conclude that* the limits of the ecliptic's latitudes are near the equinoxes, and the nodes are near the solstices. *Therefore, the points* K, H *will be near the equinoxes. We likewise conclude this: that since the northern part of the true ecliptic flees from the north, in that the northern latitudes are increasing in Gemini and Cancer, the*

*ecliptic's northern limit is therefore either in Libra, if the nodes progress, or in Aries, if (as is more probable) they retrogress. For the moon's nodes also retrogress, traversing the zodiac in 19 years, while the apogee progresses, traversing it in $8\frac{1}{2}$ years.*

*Now the sun's apogee, or the earth's perihelion, is at $5\frac{1}{2}°$ Cancer, and thus by chapter 57, the diameter of power, causing the eccentricity, points at the sun when the earth is at $5\frac{1}{2}°$ Aries. But also, by chapter 63, the diameter of power that causes the latitude points at the sun when the earth is at the limit, which is in Aries by the present chapter 68. Therefore, by the same chapter 63, both powers can be effected by the same diameter of the earth's body.* Hence one may argue plausibly that this invisible circle or mean ecliptic and the true one known to us coincide at $5\frac{1}{2}°$ Cancer and Capricorn.

A probable examination of the mean ecliptic.

If the aphelia of all the planets were located on a single great circle, we could say that this is what we are seeking. For then it could be true of all planets, as it is here in the earth's circuits, that the nodes coincide with the apsides, and thus two sorts of phenomena – the eccentricity (in depth) and the obliquity (in latitude) – are effected by the same diameter of power. This would free us from the great difficulties with which we were left in chapter 63.

And in fact the apogees of the sun, Mars, Jupiter, and Saturn fit approximately. For the aphelia of all three superior planets are in the same semicircle, and at the same time in the same northern direction. Therefore, the southern limit of the true ecliptic would be in Libra, and the northern in Aries, which agrees with the above.

A full consideration of this question must, however, be deferred until the motions of all the planets are examined with reference to the true ecliptic, the one known to us.

Another argument for the mean ecliptic.

Further confirmation of this opinion of a hidden royal circle, projected from the sun among the fixed stars, is provided by the
321 obliquity of the ecliptic that is in common use, which is computed from the equator, but which we might more correctly call the equator's latitude from the ecliptic. Now the equator is the great circle of the earth's body that is intermediate between the poles of the earth's daily rotation on its axis. And the same name of 'equator' or 'equinoctial' is given to that region of the sphere of the fixed stars that stands above the terrestrial equator in any era. The same name of 'poles' is given to the points of the fixed stars that stand above the earth's poles in any era. And this axis, and this great circle, are inclined to the ecliptic differently in different eras. For to the extent

What is the equator or equinoctial?

that the northern latitude of the fixed stars in Cancer, and the southern latitude in Capricorn, is greater today, the equator's latitude from the ecliptic is smaller than it was once, since this obliquity is greatest in Cancer and Capricorn. It was once 23° 51½′, while today it is 23° 31½′, the difference of 20′ being the change in latitude of the fixed stars.

It is, however, reasonable to suppose that the circle of the equator with its axis and poles would have been destined forever to decline from the poles of this ecliptic HK by an equal and fixed distance, if the true ecliptic were the world's primary circle. But the ecliptic has changed, and the inclination of this axis to the ecliptic (and with it the inclination of the equator, to which this axis belongs) has been altered, so that to the extent that the ecliptic has receded from the fixed stars in Cancer, it has approached the equator. Therefore, the equator appears to maintain a constant inclination to some other circle. So a great cause, and a great dignity, belong to this hidden circle. And thus from all these plausible arguments there arises a royal circle LOM, middle among the circles of the planets, to which all the planets, and Mars with them, maintain a constant inclination.

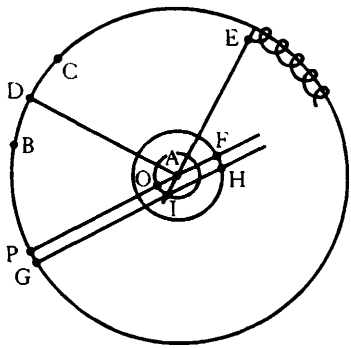
The example of the moon should not trouble us, whose inclination to the ecliptic, but not to any other great circle, is a constant 5°, both in the past and today, even though the ecliptic has been moved. For there is an enormous difference between the moon and the other planets. The orbs of the others encircle the centre of the world. The moon's orb alone (roughly speaking) is outside the centre and is transported from place to place. The others in common circle the sun, while the moon circles the earth. The eccentricities of the others and the whole theory of longitude and latitude originate from the sun, while those of the moon originate from the moving earth. The sun sweeps the others around in a circle, while the earth moves the moon. What wonder, then, if the moon keeps the limits of its latitude constant with respect to the changeable ecliptic HK, beneath which lies the terrestrial circle, while the other planets do so with respect to some other invariable circle, such as LOIM? So the moon should not prevent us from giving credence to this theory.

Whether the inclinations of the planes of Mars and the ecliptic vary.

It is therefore granted that Mars's orbit is inclined at a constant angle to some circle that maintains its position beneath the same fixed stars, such as LOIM. It follows that this same orbit of Mars has different inclinations to the ecliptic HK in different ages, since in certain of its parts it leaves the fixed stars it originally lay beneath and

moves on to others. This only follows, however, if we grant that the nodes of Mars and the nodes of the earth, that is, the intersections of these orbits with the invisible circle LOIM, are not always carried over the same intervals in the heavens, some being faster than others. 322 An authentic example of this was just given. For since the equator maintains a constant inclination to this invisible circle LOIM, while the ecliptic is meanwhile moved, the declination of the equator from the ecliptic is consequently perceived to be changeable.

The earth's pole does not proceed exactly along the small circle BCE, but in the spiral depicted at E, with one loop per year, a similar spiral being described by the opposite pole. Each depends upon the other, and from this interconnection arises the progression of the equinoxes and the solstices. But the magnitude of each of these spirals is about the same as that of Copernicus's *orbis magnus*, or of the orb of the sun, for the others. That is, the ratio of this spiral to the surface of the sphere of the fixed stars is imperceptible. It can thus be taken as the simple line BCE. However, for the correct imagining of this motion, it should be noted that the




*Let* A *be the pole of the mean ecliptic, or the point upon which the straight line falls that is drawn from the centre of the sun through the pole of the solar body. About centre* A *with radius* AB *of 23° 42' (or thereabouts) let a smaller circle be described, and let* B, C *be the positions of the north pole of the world, or the points upon which falls a line from the centre of the earth's body through the pole of the daily rotation on the same body,* B *at the time of Ptolemy, and* C *at our time. If the nodes of the ecliptic also retrogress, the northern limit must be placed near the fixed stars in the region of Aries and Pisces. For the northern latitude of the fixed stars in Gemini and Cancer has increased, as was previously said. Let the midpoint* D *between* B *and* C *be taken, marking the position of the pole of the equator at an intermediate time, and let* AD *be joined. Thus the circle* AD *extended will pass through the solstice of the intermediate time. From* A *at right angles to* AD *let* AE *be drawn, which, being extended, will pass through the vernal equinox of the intermediate time. Therefore, close to the line* AE *there would be the pole of the circle beneath which the orbit and circuit of the earth was once established. And because the northern limit is in Aries,*

*let* EA *be extended in the direction of* A, *and let the point* I *be taken on the extension below* A. *Thus the Ptolemaic pole of the ecliptic would be at* I. *About centre* A *with radius* AI *let a small circle be described, on which let another point* O *be taken, nearer to* C *than* I *is to* B. *And let* O *be the present pole of the ecliptic,* $23^\circ\ 31\frac{1}{2}'$ *from* C, *while* I, *the Ptolemaic pole of the ecliptic, is* $23^\circ\ 51\frac{1}{2}'$ *from* B. *This will be the theory of the ecliptic's altered obliquity and of the altered latitude of the fixed tars, except that the size of the small circle* OI *is not known to us. For the ecliptic's 20' alteration of obliquity can be produced in various ways.*

*And because* O *is today's pole of the ecliptic, and* OC *points towards the beginning of Cancer, let* CP *be the eighth part of the circle, and* P *the middle of Leo, where the northern limit of Mars is today. Let* PO *be extended beyond* O, *and let* GI *be drawn through* I *nearly parallel to* PO, *but slanting somewhat forward in longitude (for the sidereal position of Mars's limit was once a little farther forward than today), and let it be extended beyond* I. *And about* A *let a small circle be described, intersecting* PO *at* F *and* GI *at* H. *Let the size of the circle be sufficient to make* OF *greater than* IH. *And let the pole of the circle beneath which the circuit of Mars is established, be placed at* F *today, and at* H *in the past. Today's obliquity* OF, *or the inclination of the plane of Mars to the ecliptic, will be greater, and the Ptolemaic obliquity* HI *will be smaller. Nevertheless the pole of Mars's orbit* H, F *would have moved from* H *to* F *keeping at a constant distance* AH, AF *from* A.

*And since the pole of the Martian orbit has traversed a fairly large arc from* H *to* F, *whether forward or backward, but at the same time the pole of the ecliptic has gone from* I *to* O *about the same point* A, *the pole of Mars would appear to be nearly at rest, since* IH *and* OF *are nearly parallel.*

A great inequality in the motion of the nodes must indeed follow if it is true that the poles of the individual planets circle some common pole in different times.

323 For there is an anomaly originating in this way in the precession of the equinoxes, whose circumstances are quite like the present ones.

I have stated what would be in agreement with the principles established in this work, and by what hypotheses the inclinations of the planes might have been made different in different ages. Let us now examine the observations of Ptolemy. For since the northern latitude of Mars is with Cor Leonis, a northern star, while the

axis of the terrestrial equator, extended both ways to the fixed stars, describes a cylinder each year whose magnitude is the same as that of one of these spirals, and which has the sun's body in its middle. Further, over the ages the same axis of the earth describes two cones with equal vertices at the sun, except that they are confused by the earth's circuit about the sun, so that the vertex of each cone is contained within the other cone, owing to the coincidence of all the cylinders; while the base is BCE. Thus the cone is compounded of many cylinders.

What is the reason why the translation of Mars's nodes is so slow?

On the inequality of the precession of the equinoxes.

southern latitude is with the southern stars of Capricorn, it is reasonable that the same should have happened to Mars's maximum latitudes as happened to those stars: they both should have increased. For the stars' latitudes did increase, the northern ones around the summer solstice, and the southern ones around the winter solstice. Ptolemy therefore said that Mars's maximum observed northern latitude was 4° 20′, while today it is 4° 32′[1]. This confirms our opinion, since he shows a maximum latitude that is 12′ smaller than today's, while the nodes stayed at approximately the same distance from aphelion as they are today. On the other hand, he makes the southern latitude about 7°, while today it could also be that much, namely, 6° 52⅓′. We are therefore left undecided by his observations. For concerning those 12′ in the northern latitude, it should be noted that the smallest graduations on his instruments were 10 minutes, and that he usually supposed an error amounting to one of these parts. Also, the difference between the Greek symbols for 20′ and 40′ is very small and slippery, often neglected by translators. Nevertheless, the Arabic says 20′ here.

There is nothing besides this in Ptolemy that can lead us to a judgement of how these matters stood in antiquity. For the observation examined in the following chapter 69 is shown to be in error. Therefore, as long as we are wanting suitable observations from antiquity, circumstances compel us to leave this discussion of the motion of the nodes, along with many other matters, to posterity, if, indeed, it should please God to vouchsafe the human race a length of time in this world sufficient to work through such remaining questions thoroughly.

[1] Ptolemy, *Syntaxis*, Book 13 chapters 3 and 5.

# 69

## A consideration of three Ptolemaic observations, and the correction of the mean motion and of the motion of the aphelion and nodes[1]

From all the written records of antiquity there have survived no more than five observations of the star Mars, as well as one of extreme antiquity noted by Aristotle, who saw Mars occulted by the dark part of the half moon. However, neither the year nor the time of day were given. Nevertheless, I have discovered, using a very lengthy process of induction extending over the 50 years from Aristotle's fifteenth year to the end of his life, that this could not have happened on any other day than the evening of April 4 in the 357th year before the commonly accepted epoch of Christ, when the 21-year-old Aristotle was, as we know from Diogenes Laertius, a student of Eudoxus.[2]
324 The second observation, obtained from the Chaldeans, was preserved for us by Ptolemy. This was made on the morning of January 18, 272 B. C., when Mars occulted the northern star in the head of

[1] This chapter shows signs of having been written in great haste. The number of careless errors is extraordinary, even for Kepler. Yet the Keplerian virtuosity in handling data is also evident, and the main conclusions stand despite the mistakes.

[2] Since Aristotle was born in 384 B.C., there is an obvious error in his given age.

In the Kepler manuscripts at the USSR Academy of Sciences at Leningrad (vol. XIV fol. 295 v) there is an elaboration of the present passage, as follows:

**'There exists an observation of Mars in Aristotle**

'He once saw the moon go directly beneath Mars. For in *De caelo* book 2 ch. 12 he says, "We saw the moon, when it was so divided into two parts as to be bright on one side and darkened on the other, perceptibly come together with that star that is called 'Mars': and indeed, after it had been hidden by the dark side, we saw it at length emerge from its bright side."

**'It is found using Mars's latitudes**

'Aristotle gave neither time nor position. The time is sought.

Scorpius.[3] Here again, no particular time was given. The other four were by Ptolemy himself, using an astrolabe to measure Mars's distance from fixed stars. However, he reported only the zodiacal position at the exact moment of Mars's opposition to the sun's mean motion.[4]

Upon these few observations, arguments of the greatest moment are to be founded; or, if this is not possible, astronomy must remain incomplete. First, through the four Ptolemaic observations, the epoch of the mean motions, related to the fixed stars, belonging to Ptolemy's time, is to be found, and by a comparison of this with the modern data the mean motion itself is to be determined. Next, it seems possible to use the Chaldean observation to inquire whether the solar eccentricity was really once greater than it is today. And then, using both this observation and the Aristotelian one, if the time be known, one can hazard a guess at Mars's latitude at those times.

But by God immortal, what is this path upon which we shall be treading? For we have hardly anything from Ptolemy that we could not with good reason call into question prior to its becoming of use to us in arriving at the requisite degree of accuracy.

## I

How astronomers may investigate the beginning of the zodiac or of the ecliptic.

First, at the times set out, the mean position of the sun is given by a calculation that depends upon observation of the equinoxes and solstices. The sun reveals the beginning of Aries, not by pointing a finger at the place, but by a blind conjecture of the time. For we call

'First, he said that the moon was at its half, and thus Mars was near quadrature with the sun.

'Next, he had it moving towards Mars with its dark part forward, so the observation was in the evening.

'Third, Mars was seen. Therefore, it was an hour or (more likely) two after sunset, so that Mars could be accurately observed close to the moon. And although the beginning of this observation could have occurred two hours after sunset, it would have to end before midnight, that is, before the moon set.

'Fourth, Aristotle was born in the year 365 of Nabonassar. Let us suppose that by the time he was fifteen he was capable of making this observation. Therefore, our time is to be sought after the year 380 of Nabonassar. He lived 63 years. He departed for Chalcis in the year of Nabonassar 426, and died a little after. The observation is to be sought in this 50 year period.'

[On fol. 296 v is his series of calculations for the 50 years. For the year of Nabonassar 393 (357 B. C.), his entry reads thus:]

'Sun at the end of Pisces, Mars/moon observed in Gemini, Mars is at 25 Cancer, max. N. lat., Node at 18 Aries moon is in Gemini, considerably north, less so through parallax. This has great probability.'

According to a more recent account, the date was 4 May. See the note on p. 205 of Aristotle, *On the Heavens*, trs. W. K. C. Guthrie (Harvard University Press, Cambridge 1960).

[3] β Scorpii. The observation is in the *Syntaxis* book 10 chapter 9.

[4] The three observations considered in this chapter are in the *Syntaxis* book 10 chapter 7. The fourth is in book 10 chapter 8.

the beginning of Aries that point that the sun occupies when the day is observed to equal the nights. What if Ptolemy had been in error as to the time? We are not wanting conjectures. First, he does not give his method of observing it. My hope is that he observed the meridian altitudes, for by induction from these the moment of the sun's entering the northern hemisphere is obtained without error. But what if he had observed it using Alexandrian armillaries, where refraction could have put him off? He himself clearly suggests that he did so, when he says that the equinox was measured twice on the same day using these armillaries. He ascribes this to instrumental error; I, however, suspect that the error arose from refraction.

The Ptolemaic observation of the equinox is suspect.

Difficulty in accepting the day of the equinox given by Ptolemy.

Let it be supposed, nevertheless, that he observed it using meridian altitudes. There is another suspicion that unwelcomely but most forcefully insinuates itself: the moments of the equinoxes provided by Ptolemy do not agree within a day and a half when compared with the previous observations of Hipparchus and the later ones of Albategnius[5] and Brahe, all of which conspire in a single equality. The Ptolemaic equinoxes alone deviate. This fact has given rise to many extremely complicated opinions on the heavens, and was responsible for the motions of trepidation and libration, all of which is brought to ruin once one sees that the observations after Ptolemy always agree to the point of equality with the most ancient ones of Hipparchus.

For the Ptolemaic observation of the day of the equinox.

Ptolemy nevertheless supports himself by comparing the vernal equinoxes with the autumnal. For if, as a result of instrumental error, he had pronounced the true equinox to be on the following day, when
325 it had occurred on the previous one, the autumnal equinox would have been pronounced to be on the previous day when it belonged on the following one. If two days were thus subtracted from the length of the summer, a great alteration in the sun's eccentricity would have resulted. Nevertheless, following his observations, he left this at the same quantity found by Hipparchus. So no alternative is left us but to trust Ptolemy in believing him to have observed correctly the time at which the sun stood at the beginning of Aries.

How observations are used to learn the sun's zodiacal position, even when the zodiac's sidereal position is unknown.

## II

Once a beginning is made, and the obliquity of the ecliptic is found through observation, it is a trivial matter to use the sun's declinations

[5] al-Battani (c. 858–929), the Arab astronomer best known to Mediaeval and Renaissance Europeans, who was particularly distinguished for his work on the solar theory. He discovered the motion of the sun's apogee.

on each day to report its elongation from the point occupied by the sun at the time stated to be the moment of the equinox, whatever it might be or in whichever sphere it might lie. For various authors thought up a variety of spheres for this purpose: after the eighth and ninth spheres established by Ptolemy, some have set up a tenth, and others most recently, an eleventh and a twelfth, all through the merest of speculations, against which overabundance of mechanisms Brahe vehemently inveighed. He never, however, told me what he intended to substitute in their place, nor did he leave it in writing anywhere. Copernicus, on the other hand, acted ingeniously and wittily (in the common opinion), and wisely (in mine), in removing his eyes from the heavens and seeking that point in the globe of the earth above which a point in the sphere of the fixed stars appears in any particular age, as was said in ch. 68. However, this is not the place to discuss this matter further.

The theory of the eighth sphere is difficult to understand in the ancient account, and inconsistent: Easy in Copernicus

III

There follows a demonstration of the equation, which depends upon the sun's observed entry into the beginnings of the cardinal signs. For when the equation is subtracted from or added to the sun's apparent position, the sun's mean motion is established with respect to the point which the sun is observed to occupy at the time of the equinox. Here too, as to the quantity of the equation, there is greater uncertainty than before over the equinox or the beginning of the zodiac. For today that equation appears to be 20′ less than the quantity which Hipparchus appears to have demonstrated for himself, and which Ptolemy retained. Nor is there sufficiently good reason why we should say that the ratio of the orbs is different today from what it once was. For affirmations of the greatest moment require the most solid evidence, and this we are lacking. And those observations cannot be that accurate, especially concerning the entry into Cancer and Capricorn. If we substitute the modern equations for Ptolemy's, we shall not change his observations as much as Ptolemy himself says he can discern in observing, nor as much as the Ptolemaic observations can be vitiated by the matter of refraction. For we can with certainty name the day of the Ptolemaic equinox observation, though meanwhile remaining uncertain about the time of day. And here, the partnership of the vernal and autumnal equinoxes is no such defense against the small error we are considering here as it was before against a large one.

Were the equations of the sun once greater?

The time of the Ptolemaic equinox is uncertain.

That the equations of Ptolemy's day are equal to ours, the constancy of the modern ones clearly argues. For those found today by Brahe, and those found several centuries ago by Albategnius and Arzachel,[6] are nearly the same.

There is therefore suspicion that the equation of the sun used by Ptolemy is in error, since it is deduced from erroneous apparent positions of the sun. Consequently, Ptolemy did not relate Mars to
326 either the sun's mean or its apparent position, without the chance of error.

Nevertheless, there is this consolation, that we need to make use of the sun's apparent position: this is clear from the preceding.

With many thanks for the Ptolemaic observations, the modern equations of the sun are retained.

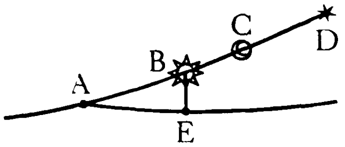
We can, however, proceed on a twofold path: either we believe Ptolemy on the equinoxes, or, using the modern equations, we apply a correction to the Ptolemaic equinoxes, making the vernal equinox three hours later than that noted by Ptolemy, and the autumnal earlier by the same amount, the result being an error of 8′ in the sun's declination at each place. Ptolemy's instruments were doubtless not calibrated with divisions any finer then 10′. And Hipparchus assigns an uncertainty of one such division. For this reason, the times, too, that the sun takes to traverse the quadrants of the zodiac, were not expressed more precisely than to within a quarter of a day. So much for the true length of summer and winter.

IV

The Ptolemaic position of the sun's apogee has an uncertainty of many degrees.

But what shall we now say of the sun's entry into Cancer and Capricorn, upon which the apogee and the setting out of the equations depends? And of how easily a quarter of a day can diminish the vernal quadrant of the zodiac, and increase the autumnal? For the sun's entry into Cancer is quite imperceptible. Nor can I be persuaded that Hipparchus and Ptolemy examined the very moment of this entry, ignoring the intermediate points. I find it more credible that throughout the summer they would have been carefully noting the sun's declinations, and would have matched up the equal ones on both sides of the solstice, taking as the true entry of the sun into Cancer the time intermediate between the moments of equal declinations. In this way, if a comparison were made of positions near to the solstice, there would indeed be little error, but there still might occur

6 Also known as Alzerkali, the earliest noteworthy astronomer of Spain (fl. ca. 1080). His name has often been associated with the Toledan Tables, although he was but one of many contributors to that work.




as much as a quarter of a day, during which time 15′ of the sun's motion elapse. Therefore, even if the equinoxes were perfectly certain, it is nonetheless possible that there be an excess or defect of a quarter of a degree in the sun's position in the two segments about the solstice, and the apogee would then fall eight degrees farther forward or farther back. So much for the sun's motion.

V

The Ptolemaic zodiacal positions of the fixed stars are not without suspicion of an error of about 20 minutes.

Now, as for the observations of Mars itself, even if Ptolemy in fact managed to line the astrolabe up accurately on the fixed stars, there is still clearly no more certainty about Mars's zodiacal position (just as in the previous consideration of the sun's position) than there is about the positions of the fixed stars. If Ptolemy committed an error in assigning a fixed star its degree of elongation from the point of the equinox, the same error will be committed in proclaiming the position of Mars. Furthermore, the elongation of the fixed stars from the sun (and thus from the point of Aries from which the sun's elongation is known through its declination) is itself not free from suspicion of error. For consider both the manner in which it is found and the argument of error. In the year 2 of Antonine Ptolemy sought it through the half-illuminated moon.[7] Using the astrolabe, he found the moon's elongation from the sun, and that of Cor Leonis from the moon. Therefore, when the sun's elongation from the point of the equinox is given, the elongation of the fixed star from the same point is also given*. Now, in measuring the elongation of the moon from the sun, an error of half a degree appears to have been committed. For the measurement was made at sunset. But the sun when setting appears higher than it should by about half a degree. Therefore, the moon's elongation appears less than it should, and so also that of Cor Leonis from the sun, and likewise from the equinoctial. Thus it appears that half a degree must be added to the positions of the fixed stars at the time of Ptolemy.

*A is the hidden point of the equinox, B is the sun, C the moon, D the fixed star, all visible; BE is the sun's declination. AB is obtained most easily by

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observations of BE at noon. BC is known through instruments during the day, and CD through instruments and by night. AB, BC, and CD being com-

7 The observation is from the *Syntaxis* book 7 chapter 2.

bined, the elongation AD of the fixed star from A is at length obtained, A being a previously invisible point which now is revealed once it is related to the fixed star D. Afterwards, the planets are related to the fixed stars through observation, and thus their elongation from the beginning of the zodiac A is known.

Therefore, when Ptolemy considered Mars (when observed in relation to the fixed stars) to be at opposition to the sun's mean position, it would now really have been half a degree beyond this opposition. So when these four observed positions of Mars are presented by Ptolemy: 21° 0′ Gemini, 28° 50′ Leo, 2° 34′ Sagittarius, 1° 36′ Sagittarius: we should take these: 21° 30′ Gemini, 29° 20′ Leo, 3° 4′ Sagittarius, 2° 6′ Sagittarius. Now Ptolemy actually defended himself against such presumption, affirming that he frequently had sought out this one thing, namely, the elongation of the fixed stars from the moon, of the moon from the sun, and hence the distance of the fixed stars from the sun and from the equinox point, and had always found it to be the same. So although he produced only one observation in order to demonstrate the method, it is nonetheless credible that he consulted several observations, at both the rising and the setting of the sun or moon, finally choosing the one which he saw to be intermediate among many operations that produced various positions.

For the Ptolemaic longitudes of the fixed stars.

Now this argument over 30′ seems to be irrelevant to Mars's mean motion, if indeed on these four occasions Mars, as it was observed with respect to the fixed stars, can be referred to them, without consideration of the equinox point, whose distance is uncertain. This is the method I used in ch. 17 above to investigate the position of the aphelion in Ptolemy's times. We are nevertheless hindered in this respect, that the observed positions of Mars are to be referred to points opposite the sun's apparent position. This task cannot proceed correctly unless the distances of both Mars and the sun from the common equinox point are previously known, since the arc of the true elongation of Mars from the sun cannot be deduced otherwise than through these components (so to speak).

The extent to which the uncertainty in the positions of the fixed stars affects the observations of Mars.

If, at the moment taken as the true opposition of the bodies, the planet should appear to be 30′ beyond the sun's true positions, the planet still has some involvement with the second inequality, and is not yet ready for an enquiry into the first inequality. And at apogee, these 30′ in the equation of the orb occupy a large arc on the eccentric, to which corresponds an even larger portion of the time or the mean motion. At perigee the opposite takes place. For this equation of the centre occupies a small arc on the eccentric, to which corresponds an even smaller portion of the mean motion. Therefore, anyone who says that on these four occasions Mars was observed 30′ farther along on the zodiac, says in effect that at the equinoctial point

Mars's mean motion was many minutes farther back at apogee, and a few minutes back at perigee. And since the arc on the eccentric is smaller than this 30′ arc resulting from faulty observation, neither Mars's eccentric position nor even its sidereal position are as far forward as Mars itself appears to have moved with respect to the fixed stars, the difference being that quantity by which the arc on the eccentric differs from the 30′ arc in the observation. And since this arc is large at aphelion, differing but little from the 30′ arc in the
328 observation, and the opposite at perigee, it therefore finally follows that at aphelion a small amount, and at perihelion more, must be subtracted from Mars's mean motion with respect to the fixed stars, if we accept that the fixed stars are 30′ farther forward on the zodiac. Thus not only is the mean motion made smaller (although by a much smaller quantity than the 30′ resulting from faulty observation), but also the relative positions of the three acronychal observations used by Ptolemy will be disturbed, whence must arise another aphelion and another eccentricity. However, this will cause us no trouble later. For we may neglect it, even if the observations introduce something large, as long as there is no suspicion of error in the fixed stars, since it is certain that they do not have the same precision as the Brahean ones. So we shall use the form of equation found through the Brahean observations, as if they remain the same throughout the ages.

Since we have encountered three forks in the road, one concerning the sun's eccentricity, another concerning the position of the sun's apogee, and the third concerning the zodiacal positions of the fixed stars and of Mars, there are therefore eight ways of establishing the mean motions and the aphelion at the moments of observation, even if, ignoring the zodiac, we compute only with reference to the fixed stars.

## Let the first investigation retain all the Ptolemaic data concerning the sun and the fixed stars.

Reduction of the Ptolemaic observations to apparent opposition to the sun.

Since the positions of the sun's mean motion were 21° 0′ Sagittarius, 28° 50′ Aquarius, and 2° 34′ Gemini, and the sun's apogee was 5° 30′ Gemini, the apparent positions of the sun were 21° 40′ Sagittarius, 1° 13′ Pisces, and 2° 41′ Gemini, all three beyond opposition.[8] The

[8] For these computations, Kepler used Ptolemy's value for the sun's eccentricity. In the *Syntaxis* (book 3 chapter 4), Ptolemy finds this to be $2^p\ 29'\ 30''$ where the radius of the eccentric is $60^p$. This amounts to 4153 where the radius is 100,000. Compare this with the Tychonic value of about 3600, which Kepler accepted (in chapter 23).

true opposition therefore preceded them. And since the diurnal motion at 21° Gemini (Cancer, today) is about 23′, and that of the sun, 61′, and the sum is 1° 24′, those 41′ therefore require 8 hours,[9] at which time Mars was visible at 21° 8′ Gemini, opposite the sun's apparent position. Likewise, at 29° Leo (Virgo, today) Mars's diurnal motion is usually taken to be 24′, the sun's diurnal motion 59′, and the sum 1° 23′. Therefore, a difference of 2° 23′ requires 1 day 17 hours 21 minutes, at which time Mars was visible at 29° 31′ Leo. Finally, at 3° Gemini (Cancer, today) Mars's diurnal motion is 23′, the sun's 57′, the sum 1° 20′, by which it is shown that for 7′ there are required $2^h\ 6^m$, at which time Mars was visible at 2° 36′ Sagittarius.

The corrected times are therefore these[10] Positions:

| | | | | | |
|---|---|---|---|---|---|
| Hadrian | 15 | Tybi | 26, | $5^h\ 0^m$ | 21° 8′ Gemini |
| Hadrian | 19 | Pharmouthi | 4[11], | $15^h\ 39^m$ | 29° 31′ Leo |
| Antonine | 2 | Epiphi | 12, | $7^h\ 54^m$ | 2° 36′ Sagittarius[12] |

| | | | | |
|---|---|---|---|---|
| Interval in Egyptian years | 4 | 68 days | $10^h\ 39^m$ | 68° 23′ |
| | 4 | 97 days | $16^h\ 15^m$ | 93° 5′ |

To the first interval there corresponds a mean sidereal motion of 80° 57′ 14″ beyond the complete cycles, and to the second, 96° 16′ 24″.

[9] The time corresponding to these figures is more nearly 11 hours. Although this change has no immediate significant effect upon the positions, Kepler repeats this error (which implies a combined diurnal motion of 2°) many times in this chapter.

[10] The translator's recomputation shows slightly different figures, partly as a result of Kepler's error in the diurnal motion. Although the changes are slight, they affect later computations, and so are given here:

| | | | | | |
|---|---|---|---|---|---|
| Hadrian | 15 | Tybi | 26, | $2^h\ 0^m$ | 21° 10′ Gemini |
| Hadrian | 19 | Pharmouthi | 4, | $16^h\ 14^m$ | 29° 31′ Leo |
| Antonine | 2 | Epiphi | 12, | $7^h\ 54^m$ | 2° 36′ Sagittarius |

| | | |
|---|---|---|
| 4 Egyptian years 68 days | $14^h\ 5^m$ | 68° 21′ |
| 4 Egyptian years 97 days | $15^h\ 40^m$ | 93° 5′ |
| Mean motion for the first interval: | 81° 5′ 20″ | |
| for the second: | 96° 19′ 17″. | |

[11] Incorrectly stated as 'VI' in the text, although the intervals are given correctly.

[12] Tybi, Pharmouthi, and Epiphi are the fifth, eighth, and eleventh months of the 365 day Egyptian year. See. R. Catesby Taliaferro's Appendix A to his translation of Ptolemy's *Almagest (Great Books of the Western World*, vol. 16 p. 466) for an explanation of Ptolemaic dates.

The Ptolemaic observations are:

| | | | | | |
|---|---|---|---|---|---|
| Hadrian | 15 | Tybi 26/7 | 1 hour after midnight | 21° | Gemini |
| Hadrian | 19 | Pharmouthi 6/7 | 3 hours before midnight | 28° 50′ | Leo |
| Antonine | 2 | Epiphi 12/13 | 2 hours before midnight | 2° 34′ | Sagittarius |

But in the former instance the apparent motion of Mars was 68° 21′ 20″ beyond the complete cycles, with precession over the interval subtracted, in the amount that it was at that time.[13] In the latter instance, the apparent motion was 93° 2′ 20″.

Now let the hypothesis previously under investigation, established on the basis of the most recent observations, be brought in, and let the question be raised, at what anomalistic position do the apparent motions on the eccentric correspond to the mean motions, as I have just now given them? After a few trials, this is found: If for the last
329 time the aphelion of Mars is placed at 0° 41′ Leo, and for the remaining times somewhat before that, owing to the precession of the equinoxes, while at the first time the mean anomaly is 46° 37′, at the second, 34° 21′, and the third, 130° 37½′; and thus the elongation from the equinox at the middle time was $5^s$ 4° 59′ 20″, then by the modern hypothesis of the equations the star Mars is placed at 21° 7′ Gemini for the first time, 29° [31′][14] Leo for the second, and 2° 37¼′ Sagittarius for the third, fortuitously precise. For the foundations are not such as to allow one to hope for such precision. Had Ptolemy made note of more oppositions of his day, we would doubtless be experiencing greater difficulty. For with three solar oppositions it is handled easily.[15] Compare this aphelion with ch. 17.

[13] Ptolemy caused later astronomers endless difficulty by adopting an annual precession of 36″, while data from all other ages yield a precession of about 51″. Kepler followed accepted procedure in accepting Ptolemy's word that the precession really was that small then.

Applying the Ptolemaic precession to the intervals of 4 years 2¼ months and 4 years 3¼ months gives 2′ 30″ and 2′ 34″, respectively. From the numbers Kepler gives, it is clear that he intended to use 2′ 40″ for both intervals, but erred in subtraction when correcting the first angle. It should have been 68° 20′ 20″, or 68° 18′ 20″ using the corrected diurnal motion.

[14] The number of minutes was omitted here. However, in case 3 below, Kepler states that the second longitude was unchanged and gives it as 29° 31′ Leo, a figure that agrees closely with the aphelion and mean anomaly given here.

[15] Since observations 1 and 2 are on opposite sides of the aphelion and at comparable distances from it, they are not much affected by changes in the position of the aphelion. Accordingly, if the angle between them does not fit the theory, there is not much that can be done. Fortunately, the angle fits fairly well, as Kepler remarks.

Observations 2 and 3, on the other hand, are on the same side of the aphelion, and the predicted heliocentric positions can be changed appreciably by moving the aphelion. So the way to adjust the aphelion is to try different positions, computing only positions 2 and 3, until the observed positions, run *back* through the theory, give mean anomalies that are at the correct interval from each other. That this was indeed Kepler's procedure is suggested by the translator's test of the method, in which the computed mean anomalies are 96° 15′ 36″ apart, almost exactly Kepler's interval. It should also be noted that (as Kepler shows in chapter 60) it is much easier to find the mean anomalies from the longitudes than the other way around.

The same procedure applied to observations 1 and 2 give mean longitudes that are too near each other by 9′. Kepler apparently compensated for this by increasing all the mean anomalies, especially that of observation 1, thus keeping 2 and 3 at a constant interval while increasing the interval between 1 and 2.

Second, the Ptolemaic equation and apogee being retained, let 30 minutes be added to the fixed stars.

The result will be slightly different. For since Mars is half a degree beyond opposition to the sun, the corrected opposition will follow. The sums of the diurnal motions were 1° 24′, 1° 23′, and 1° 20′. Therefore, for the extra 30′ the corresponding times come out approximately the same, to be added to all three, namely, about 8 hours 40 minutes. To this there corresponds 8½′ of Mars's apparent motion, which is to be subtracted from these 30′. The remaining 21½′ are to be added to the planet's positions, placing it at 21° 29½′ Gemini, 29° 52½′ Leo, and 2° 57½′ Sagittarius. The intervals both of time and of zodiacal positions will remain about the same. Thus the distribution of the mean anomaly among these observations, which was just now found, will also be the same. Only the aphelion will be transposed by the same number of minutes, so that on the last date it is at 1° 2½′ Leo. It therefore has to be moved 8½′ back among the fixed stars. And the mean motion from the equinox will be increased by the above mentioned 21½′, but it will be 8 hours 40 minutes longer. And to these hours there correspond 11′ 24″ of mean motion. Therefore, at the proposed time, the mean motion from the equinox will be only 10′ greater than before. But the positions of the fixed stars are 30′ farther removed from the equinox. Therefore, Mars's mean motion with respect to the fixed stars has proceeded 20′ less than before.

Third, the sun's apogee being transposed by 11 or 12 degrees,[16] while the longitude and equation of the fixed stars remains the same.

Then on the first date the sun will be 20′ back in position[17]; on the middle date practically nothing will be changed; and on the last date it will be 21′ farther along in position, owing to the altered equations of

[16] At the end of section IV of this chapter, Kepler mentions changing the apogee by 8°, a change that would better agree with the 20′ change in the sun's position.

[17] Here, the three observations are on the same side of the line of apsides (the sun's, this time). The first observation is near perigee, the second near the quadrant, and the third near apogee, all in the ascending semicircle. In this configuration, the equations of the second are not much affected by a change in the apsides, but those of the first and third are altered, in opposite directions. Specifically, when the apogee is moved forward, position 1 is moved back and position 3 is moved forward. This is the situation Kepler describes; hence, he has moved the apogee forward.

the sun. Therefore, the first opposition will be 4 hours later,[18] and Mars will be the same number of minutes farther back in position, while the last opposition will occur $4\frac{1}{3}$ hours earlier,[19] with Mars the same number of minutes farther along in position.

| The results[20]: | | | Positions: |
|---|---|---|---|
| Tybi 26 | H. 9 | M. 0 | 21° 4′ Gemini |
| Pharmouthi 4[21] | H. 15 | M. 39 | 29 31 Leo |
| Epiphi 12 | H. 3 | M. 37 | 2 40 Sagittarius |
| Egyptian intervals {4, 68 days | H. 6 | M. 39 | 68° 27′ |
| {4, 97 days | H. 12 | M. 0 | 93° 9′ |

330 The first interval of time becomes smaller. So also the corresponding mean motion will be 5′ 15″ smaller, so as to be 80° 53′. The second interval of time again becomes smaller. Therefore, the mean motion corresponding to it will be 5′ 40″ smaller, namely, 96° 10′ 48″. So, since to the two mean motions, both smaller, there corresponds a greater apparent motion than before, and on the supposition that the mean anomaly is the same as before for both, the apparent motion is greater by about 9′, it therefore appears that Mars has to move down from aphelion. However, the first interval is unchanged unless a great descent is made, while the second [is corrected] by a descent of about 36′.[22] Therefore, if we were to indulge in our enquiry, and not take the modern hypothesis as given, we would arrive at a completely different hypothesis with a new eccentricity.[23] And if, on the other hand, these three observations of Ptolemy were perfectly certain, this would constitute the basis for an argument that he established the sun's apogee correctly.

But when 36′ are subtracted from Mars's aphelion, placing it at 0°

[18] Here again is Kepler's odd implicit overestimation of the combined diurnal motion: 20′ in 4 hours would amount to 2° per day. The correct time would be $5\frac{3}{4}$ hours.

[19] $6\frac{1}{3}$ hours.

[20] The effect of the above changes is to increase all the differences, so that the positions are all several minutes farther apart and the times or mean anomalies are several minutes of arc closer together. However, the earlier error in the diurnal motion decreased the first longitude, so the position Kepler gives, 21° 4′ Gemini, is correct.

[21] Again given incorrectly as 'VI'.

[22] This is because the first two observations are on opposite sides of aphelion, while the second and third are on the same side, so positioned as to allow their interval to be changed easily. See footnote 15 above.

[23] The first two observations are so situated as to make it practically impossible to adjust the predictions to the times by moving the aphelion. This would accordingly have to be accomplished by changing the eccentricity, a course of action upon which Kepler was understandably reluctant to embark.

3′ Leo[24] for the last time, and when its mean motion is so adjusted that for the middle time the [mean] anomaly is 34° 58½′ with longitude from the equinoctial of $5^s$ 5° 0′ 50″, the following observations result[25]:

| | | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| First | 21° | 7′ | Gemini | should | 21° | 4′ | Gemini | 3+ |
| Second | 29 | 28 | Leo | have | 29 | 31 | Leo | 3− difference |
| Third | 2 | 37 | Sagittarius | been | 2 | 40 | Sagittarius | 3− |

Again, an accurate enough approach. For we cannot hope that the observations were of such certainty. So, whether the sun's apogee is known correctly or not, the distance of the mean motion from the equinoctial is certain with 1½′.

Fourth, the same things will be changed in the computed positions of the second case, and in establishing the mean longitude, that is, by transposing the apogee and the fixed stars.[26]

Fifth, the sun's apogee and the Ptolemaic longitude of the fixed stars remaining the same, the modern solar eccentricity is used.

Thus, while the first and last positions of the sun remain approximately the same, the sun's apparent position will be changed in the middle position by 20′. For the former fall near the sun's apsides, where the equation is small, while the latter is near the middle longitude, where the equation caused by the eccentricity is maximum. And since in Aquarius the equation is additive, when 20′ are taken away from the equation, the sun is moved back through the

[24] Subtracting 36′ from 41′ leaves 5′, a figure also in accord with the position obtained by subtracting the mean anomaly of 34° 58½′ from the mean longitude of $5^s$ 5° 0′ 50″.

[25] The corrections mentioned above tend to decrease the intervals between the predicted positions (first column), and increase the intervals between those obtained from the observations (second column). The resulting discrepancy, however, is about what Kepler thought, except that the third difference should be −7½′. This could, however, be reduced by further adjustments in the aphelion and mean longitude, so Kepler is in principle correct in his claim that the theory can be made to fit the data.

[26] This has been expressed somewhat confusedly, as the translation suggests. Kepler's meaning appears to be that, when both the apogee and the position of the fixed stars are changed, the resulting positions and times are as in case 2. This will be so provided that moving the apogee does not significantly affect the longitudes.

same number of minutes, and will be, not in 29° 31′ Aquarius, but in 29° 11′ Aquarius. The correct and truest opposition therefore follows by 4 hours.[27] The planet will then be at 29° 27′ Leo.[28] The earlier time interval and its mean motion is increased, and the apparent motion is decreased, while the later time interval is decreased, and the apparent motion is increased. So once again, more evidently than before, the application of this correction calls upon us to change the hypothesis, unless we go by the best advice we can get from the words and numbers of the hypothesis of this era. For to move the planet forward a smaller distance in a greater time near apogee, and a
331 greater distance in a smaller time near perigee, nothing will suffice but to increase the eccentricity. If everything were kept the same, as in the first case, the results for the first and third times would indeed again be the same as then, namely, 21° 7′ Gemini and 2° 37½′ Sagittarius. But in the middle position, it would come out to be 29° 36½′ Leo, where it ought to have been 29° 27′ Leo, a difference of 9½′.[29] To eliminate this, the aphelion ought to remain in about the same place, but the mean motion should omit 3½′. Then the results will be[30]:

| | | | | | | |
|---|---|---|---|---|---|---|
| First | 21° 4′ | Gemini | should | 21° 8′ | Gemini | | −4 |
| Second | 29° 33½′ | Leo | have | 29° 31′ | Leo | Difference | +2½ |
| Third | 2° 38½′ | Sagittarius | been | 2° 36′ | Sagittarius | | +2½ |

## Sixth, the same change of the second case will occur, if we at once change both the sun's eccentricity and the longitude of the fixed stars.

[27] Here again is Kepler's erroneous diurnal motion of 2°. The correct time is 6 hours.

[28] When the diurnal motion is corrected, this becomes 29° 25′ Leo.

[29] Because of the error in the diurnal motion, the computed longitude is 2′ greater, the longitude of true opposition 2′ less (as was noted above), and the difference is therefore 13½′, much worse than Kepler thought. There is no way to accommodate the theory to so great a difference, as the corrected figures below show.

[30] The numbers in the first column are very curious indeed. Although the first longitude is in accord with Kepler's assumptions, the second and third are actually greater than those in case 1. This would be the result of an increase in equated anomalies (since both are on the descending semicircle). However, a decrease in the mean longitude at these positions would result in a decrease in the mean anomalies. It is something of a mystery how Kepler managed to decrease the mean anomalies while increasing the equated anomalies!

The longitudes should have been 29° 35¼′ Leo and 2° 34′ Sagittarius. The figures in the second column are also in error. Because of the error in the diurnal motion in case 1, the first longitude should be 21° 10′ Gemini. For the second longitude, Kepler gives the original position from case 1, apparently forgetting that he had changed the position of true opposition to 29° 27′ Leo (29° 25′ corrected). The differences are thus −6, +9½, and −2: obviously no adjustment of aphelion or mean longitude could accommodate such differences.

Seventh, if on the other hand we at once change both the eccentricity and apogee of the sun, combining the third and fifth cases, the fundamentals will be these.[31]

| | | | | | | |
|---|---|---|---|---|---|---|
| Tybi | 26 | H. 9 | M. 0 | 21° | 4′ | Gemini |
| Pharmouthi | 4[32] | H. 19 | M. 39 | 29 | 27 | Leo |
| Epiphi | 12 | H. 3 | M. 37 | 2 | 40 | Sagittarius |
| Intervals | D. 68 | H. 10 | M. 39 | 68° | 23′ | |
| | D. 97 | H. 8 | M. 0 | 93 | 13 | |

The first interval remains the same as in the first case, while the last is much altered, And because more of the path is traversed in a smaller time, it must be moved farther towards perigee. To 8 hours of mean motion correspond 10′ 30″, to which add the extra 8 minutes of travel.[33] The total is thus 18½′, which we shall make up if we move the aphelion back by 1° 12′, putting it at 29° 29′ Cancer for the last time, with a mean anomaly of 131° 45′.[34] Therefore, [the position of] its mean motion is 11° 4′ Sagittarius,[35] which, in the first case, was 11° 18½′ Sagittarius. From this we compute[36]:

[31] Because of Kepler's errors in the diurnal motion and in the change resulting from moving the sun's apogee, nearly all the times and positions in this table are wrong. The correct figures for the first and third dates are obtained from the corrected table under case 3, and the correct figures for the second date are from case 5, corrected. The result is:

| | | | | | |
|---|---|---|---|---|---|
| Tybi | 26 | $7^h\ 54^m$ | 21° | 4′ | Gemini |
| Pharmouthi | 4 | $22^h\ 14^m$ | 29 | 25 | Leo |
| Epiphi | 11 | $23^h\ 54^m$ | 2 | 42 | Sagittarius |
| Inter- | D. 68 | $14^h\ 20^m$ | 68° | 21′ | |
| vals | D. 97 | $3^h\ 20^m$ | 93 | 17 | |

The mean motion corresponding to the second interval is 96° 3′ 8″.

[32] Changed from 'VI' in the text.

[33] The correct figures are: 12⅓ hours of mean motion, to which 16′ 10″ correspond, extra travel being 12′. The total is thus 28′ 10″.

[34] These figures are corrected as follows. The first requisite is to find two mean anomalies at an interval of 96° 3′ 8″ which yield equated anomalies at an interval of 93° 17′. After several trials, the following are found:

| | | |
|---|---|---|
| Mean anomalies: | 36° 11′ | 132° 14′ 8″ |
| Equated anomalies: | 30° 27′ 43″ | 123° 44′ 43″. |

If we now place the aphelion at 28° 56′ 0″ Cancer and 28° 58′ 40″ Cancer, respectively, the required longitudes are obtained, approximately (the slight discrepancy results from neglecting motion of the aphelion in the first computation). Thus the aphelion is moved back 1° 42′ 20″, not 1° 12′ as in Kepler's computations.

[35] Kepler's figures would require this to be 11° 14′. The translator's revision results in nearly the same value, 11° 12¾′.

[36] Again, almost everything changes. The table should be:

| | | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| For the first time: | 21° | 1′ | Gemini | should | 21° | 4′ | Gemini | +3′ |
| Second | 29 | 24 | Leo | have | 29 | 25 | Leo | −1 |
| Third | 2 | 43 | Sagittarius | been | 2 | 42 | Sagittarius | +1 |

Note that the second longitude on the right has inadvertently been increased by 2′.

| | | | | | | |
|---|---|---|---|---|---|---|
| For the first time: | 21° 3½′ | Gemini | should | 21 | 4 | Gemini |
| Second | 29° 26½′ | Leo | have | 29 | 29 | Leo |
| Third | 2° 41′ | Sagittarius | been | 2 | 40 | Sagittarius |

Finally, with alterations in all three data that we have taken from Ptolemy, the result is the combined effect of the seventh and second cases.

It is therefore apparent that the epoch of the mean motion with respect to the equinox and the fixed stars is not much changed by an alteration in the sun's eccentricity, or in the apogee, or in both at once, but that it is changed when the positions of the fixed stars are altered. For the third case adds 1′ 30″, the fifth subtracts 3′ 30″, and the seventh subtracts 4′ 30″. The second case alone subtracts 10′[37] from the mean motion measured from the equinoctial, and 20′ measured from the fixed stars.[38]

332 As a result of this, two epochs of motion at the time of Ptolemy are established.

*How it happens that through two errors that cancel one another the Ptolemaic elongation of the fixed stars from the beginning of Aries remains the same.*

But what if we make something suitable by combining the second and fifth cases, by which we can hold simply to the Ptolemaic longitude of the fixed stars, eliminating any need for us to suspect that there might be two epochs of Mars's mean motion? For Ptolemy explicitly affirms that in his observation he found the moon's distance from the sun to be 92° 8′, the same amount he computed from his hypothesis of the moon's motions. Ptolemy would have spoken truly: he would have been skilful enough in observing, and would have plainly seen this distance on his instrument to be the same as that prescribed by his hypothesis of the moon's motions, which was not in error near the quadratures. From this, I argue thus. If the sun had been at 3° 5′ Pisces,[39] where Ptolemy placed it using his eccentricity, the moon could not have appeared exactly 92° 8′ from it, according to the measure of the hypothesis, for the reason that the setting sun reaches the eye by refraction, and appears higher than it actually is (and thus 30′ farther to the east). But because the arc from the moon

[37] In the second case Kepler *added* 10′ to the mean motion measured from the equinox, and subtracted 20′ from the mean motion measured with respect to the fixed stars. Here again one can see the haste with which this chapter was composed.

[38] With the data corrected, the third and fifth cases could not produce any satisfactory theory. However, Kepler's evaluation (with corrections) holds for the remaining cases.

[39] The longitude given by Ptolemy is 3° 3′ Pisces, as Kepler has it below.

to the sun was observed to be 92° 8′, and because of refraction, this was in actual fact 92° 38′, the sun was therefore not at 3° 3′ Pisces, but at 2° 33′ Pisces. And this is in agreement with the fifth case, where we said that Ptolemy's maximum additive equation (which occurs at 5° Pisces) becomes 20′ smaller when today's eccentricity is used, thus putting the sun at 2° 43′ Pisces instead of 3° 3′ Pisces. And so, on the supposition that refraction is universal throughout all places and times, as is discussed in the *Optics*, and supposing this observation to stand, we arrive at the conclusion that the sun's eccentricity is less than that reckoned by Ptolemy.

It should not trouble you that I spoke of a refraction of 30′, while this diminution is but 20′. For if you consider well, since 30° Taurus was culminating, 1° Pisces was then setting at Alexandria, and the sun, being at 3° Pisces, consequently had an altitude of two degrees, or perhaps even more, and therefore the refraction was less than 30′; nor was all the refraction simply longitudinal. Thus these two causes[40] were about the same in quantity, and cancelled one another.

Ptolemy's values for the positions of the fixed stars are not scrupulously accurate.

However, anyone who knows anything of the Ptolemaic reckoning of the fixed stars will not consider this difference of ten minutes worth mentioning. For example, Ptolemy comes up with an interval of 54° 10′ between Cor Leonis and Spica Virginis, although in the heavens themselves it is not more than 53° 59′.

Establishment of the mean motion.

Epoch of the mean motion at the time of Ptolemy.

Let us therefore follow whither our inclinations and our arguments lead: as in the first case, in the second year of Antonine, on the 12th day of Epiphi, at the 8th hour, at Alexandria in Egypt, let Mars's mean motion from the equinoctial be 11° 18′ 30″ Sagittarius. This time corresponds to the common year of Christ 139 May 27. The difference of meridians between Hven and Alexandria is nearly two hours, from the most recent geographical tables. Therefore, at Hven in the year of Christ 139 May 27 at $6^h$ the mean motion was $8^s$ 11° 18′ 30″. But in that year, Cor Leonis had a longitude of 2° 30′ Leo, that is, $4^s$ 2° 30′ 0″. Therefore Mars's mean motion was $4^s$ 8° 48′ 30″ from Cor Leonis. But on 1599[41] May 27 at $6^h$ Mars's mean motion was $0^s$ 0° 47′
333 30″ from the equinoctial, while the distance of Cor Leonis from that point, as demonstrated by Brahe, was $4^s$ 24° 15′ 45″. Therefore Mars was $7^s$ 6° 31′ 45″ from Cor Leonis.

40 That is, refraction and the altered eccentricity.

41 This year was clearly chosen advisedly: it is 1460 Julian years from Ptolemy's date, and also exactly 1461 Egyptian (that is, 365 day) years from that date, because 1460 times 365¼ is equal to 1461 times 365.

| | | | |
|---|---|---|---|
| On | 139 | May 27 at $6^h$: | $4^s$ 8° 48′ 30″ |
| | 1599 | May 27 at $6^h$: | 7 6 31 45 |
| An interval of {1460 Julian / 1461 Egyptian} Years | | | 2 27 43 15 |
| | | the Prutenics give | 2 28 5 56 |
| | | Difference | 22 41 |

For each year nearly one second must be subtracted. Therefore, at noon on 1 January in the first year of Christ, at Hven, it is elongated in its mean motion by $5^s$ 8° 52′ 45″ from Cor Leonis.[42]

And so much for Mars's mean motion with respect to the fixed stars.

The motion of the aphelion will come out a little different from what it was above in ch. 17. For in the year of Christ 139 May 27 it was at 0° 41′ Leo, while Cor Leonis was at 2° 30′ Leo. It therefore preceded the latter by 1° 49′. But today, in 1599 May 27, it is at 28° 58′ 50″ Leo, while Cor Leonis is at 24° 15′ 45″ Leo.

| | |
|---|---|
| Therefore, today's aphelion follows by | 4° 43′ 5″ |
| While for Ptolemy it preceded by | 1° 49′ 0″ |
| a progress of | 6 32 5 |

over an interval of 1460 Julian years, which makes an annual motion of a little greater than 16″. So at the root of the Christian era, at noon on January 1, this puts the aphelion 2° 27′ before Cor Leonis.

## On the mean motion of the sun with respect to the fixed stars, treated in passing, for future reference.

In the year of Christ 139 Pharmouthi 9, which is February 23, at sunset at $5^h$ $30^m$, $3^h$ $30^m$ at Hven, the apparent position of the sun was computed as 3° 3′ Pisces; therefore, the mean position was 0° 43′

[42] This should have been $9^s$ 8° 52′ 45″, as the following computations show:

First, Kepler's value for Mars's mean diurnal motion is required. In 1460 Julian years Mars completes about 776.25 sidereal cycles. By Kepler's figures here (which are consistent with the mean longitudes and dates elsewhere in this work), the total mean motion is thus 87° 43′ 15″ beyond 776 full cycles in 533,265 days. So the mean diurnal sidereal motion is 0.52403162°.

Next, the total mean motion must be counted from a known position to the epoch. On 139 May 27 at 6 pm the elongation from Cor Leonis was $4^s$ 8° 48′ 30″. From this date back to 1 A. D. January 1 at noon is 50,550.25 days, to which corresponds a mean motion of 209° 55′ 45″ beyond full cycles. Since we are counting back in time, this must be subtracted from the elongation in 139; or its full circle complement, 150° 4′ 15″, must be added. The sum is 278° 52′ 45″, or $9^s$ 8° 52′ 45″, exactly as given by Kepler except for the number of signs.

Incidentally, Kepler's value for Mars's sidereal period, computed from the above mean sidereal motion, is 686 days 23h 33m 17s.

Pisces. But the longitude of Cor Leonis was found to be 2° 30′ Leo. Therefore, the mean sun preceded Cor Leonis by $5^s$ 1° 47′ 0″. But on 1599 February 23 at $3^h$ $30^m$ at Hven the mean sun was at 12° 47′ 41″ Pisces, and Cor Leonis 24° 15′ 30″ Leo. So the mean sun preceded Cor Leonis by $5^s$ 11° 27′ 49″.

Over 1460 Egyptian years,[43] 9° 40′ 49″ were removed.[44]

Our conclusion is 2′ 42″ less than that from the Prutenics over the same number of years, and the epoch will be $5^s$ 7° 14′ 36″ from Cor Leonis at the root of the Christian era on January 1.[45]

Similarly, the progression of the sun's apogee is found to be 8° 23′, and at the root of the Christian era it was $1^s$ 27° 48′ 0″ before Cor Leonis.[46]

[43] This must be 1461 Egyptian years: see footnote 41.

[44] From this, one can compute a Keplerian value for the sidereal year. The total number of cycles (1460) is multiplied by 360, and 9° 40′ 49″ are subtracted, to find the total angular motion in degrees. This is then divided by the number of days (533,265), the quotient being the mean diurnal motion, 0.9856081°. The inverse of this, multiplied by the 360° in the full circle, gives the number of days in a sidereal year as 365.2567. The modern value is 365.2564.

[45] This is computed as follows:

The time interval from Ptolemy's date back to 1 January of the year 1 is 50,456d 15½h. Using the mean diurnal sidereal motion computed from Kepler's data in the preceding footnote, and subtracting full cycles, one finds the remaining angle to be about 50° 28′ 45″. This must be subtracted from the position in Ptolemy's observation, so that the sun's mean position is now $6^s$ 22° 15′ 45″ before Cor Leonis, or $5^s$ 7° 44′ 15″ beyond it. It would thus appear that the figure in the text is incorrect: very likely the minutes were inadvertently changed from 44 to 14.

[46] The computation would have been something like this:

| Date | Position of Apogee | Position of Cor Leonis | Elongation |
|---|---|---|---|
| 1599 | 95½° | 144° 15′ 45″ | −48° 45′ 45″ |
| 139 | 65½° | 122° 30′ 0″ | −57° 0′ 0″ |
| 1461 Egyptian years | | | +8° 14′ 15″. |

Motion over the additional 139 years and several months would be about 46′ 50″, resulting in an elongation of −57° 46′ 50″ at the beginning of the Christian era. This agrees well with Kepler's elongation; however, it is not clear how he arrived at the figure of 8° 23′ for the progression.

# 70

## Consideration of the remaining two Ptolemaic observations, in order to investigate the latitude and ratio of the orbs at the time of Ptolemy

334 It is true, as I have more than once remarked, that Ptolemy had at his disposal many more observations that were presented in his Opus. This may be seen in the presentation of the method for investigating the ratio of the orbs, where a single observation is used that is within three days of the opposition. For it was said in chapter 53 that observations that are so close result in a very large error if they are off by even one minute. Nevertheless, let us follow his footprints, and upon the hypothesis just established, erected upon the foundation of the first case, let us compute this fourth position[1] as well.

| | | | | |
|---|---|---|---|---|
| Epiphi 12, | $8^h$[2] | – Anomaly | | 130° 37′ 30″ |
| 15, | $9^h$ | | | |
| 3 days | 1 hour | Mean motion | | 1° 35′ 39″ |
| | | [Mean] Anomaly | | 132 13 9 |

| | |
|---|---|
| Equated [Anomaly] | 123° 43′ 34″ |
| Aphelion | 120 41 0 |
| Eccentric position | 4° 24′ 34″ Sagittarius. Distance 143,660 |

The sun's true position on the 12th was 2° 36′ Gemini[3]. Add the motion of the three days and one hour, near apogee, which is 2° 53′ 40″ from modern experience. This makes it 5° 29′ 40″ Gemini, and let the present distance of the apogee be used, 101,800. Therefore, the point opposite the sun and Mars's eccentric position differ by 1° 5′ 6″.

[1] This is the observation from the *Syntaxis* book 10 chapter 8, of 2 Antonine, 15/16 Epiphi, 3 hours before midnight (139 May 30 at 9 pm), when the sun's mean position was 5° 27′ Gemini and Mars was observed at 1° 36′ Sagittarius.

[2] This is the time of the true opposition, from 'case 1' of chapter 69.

[3] This is the position of true opposition, from chapter 69.

This arc appears to be 3° 43′ 14″, so that Mars would appear at 1° 46′ 26″ Sagittarius.

If, however, we use the Ptolemaic eccentricity of the sun, the sun's motion over the three days will be 1′ smaller, and the sun will be at 5° 28′ 40″ Gemini, The difference is thus 1° 4′ 6″. The apparent magnitude of this arc, using the Ptolemaic distance of the sun and earth, 102,100, will be 3° 45′ 45″[4], so the planet will fall at 1° 43′ Sagittarius. But Ptolemy said that it was seen at 1° 36′ Sagittarius. We have therefore come out 7′ to 10′ beyond the correct figure. But the least division of the Ptolemaic instrument, which can always be considered as the uncertainty, has the value 10′.

You should also note that if we have erred by two minutes in the eccentric position, we shall now err by seven minutes in the observed position. For if Mars is moved back to 4° 22′ Sagittarius on the eccentric, it will now appear at 1° 36′ Sagittarius.

Above, on Epiphi 12, there was also an excess of $1\frac{1}{2}'$. So these results are in agreement.

Through the more ancient observation, an investigation of the ratio of the orbs.

And because at such proximity to opposition a difference in the eccentricity has little effect, let us also consult the more ancient observation[5]. Between the morning of January 18 during the year 272 before Christ[6], and noon on January 1 of the year 1 of Christ, there are 272 Egyptian years, 51 days, and several hours. For since at Alexandria, the sun at 25° Capricorn rises at $7^h$, and the morning observation of Mars was made one hour earlier, as dawn was breaking, it was therefore made at the sixth hour, which is the fourth hour at Hven, from which time there are eight hours until noon. From this time interval, and the principles laid down above, the sun's mean motion is found to have gone $5^s$ 25° 32′ 50″ beyond Cor Leonis[7], with
335 an anomaly of 234° 54′ 34″. The corresponding equation from Ptolemy is 2° 0′ 30″, and from Brahe, 1° 42′ 54″, additive; and for the

[4] The apparent arc resulting from this distance is 3° 41′ 16″. It is not clear how Kepler obtained his figure, which is consistent with a distance of 102,900. However, a complete recomputation from Ptolemy's data yields an arc of 3° 43′ 5″, which would place the planet at 1° 46′ 18″ Sagittarius, a change that does not affect Kepler's conclusions.

[5] That is, the Chaldean observations from the *Syntaxis* book 10 chapter 9, of 476 Nabonassar, on the morning of Athyr 21 (18 January 272 B. C., historical style) when Mars was observed to occult β Scorpii (which, according to Ptolemy, was at 6° 20′ Scorpio).

[6] This is according to the historical style; in the astronomical reckoning it would be −271.

[7] This shows two things: first, that Kepler used the elongation given in chapter 69, $5^s$ 7° 14′ 36″, and thus that the incorrect number of minutes given there (the correct number being 44′) was not a printer's error; and second, that he used a diurnal motion here that differs slightly from the one resulting from his data in chapter 69 (see footnote 44 of that chapter). These data result in a figure of 0.985607°, corresponding to a sidereal year of 365.2572 days. The elongation should have been about 30′ greater.

former, the sun's distance from the earth is 98,790, and for the latter, 98,976. But Mars's mean motion was then $2^s$ 6° 7′ 12″ beyond Cor Leonis. Also, since the aphelion is 3° 40′ 20″ before Cor, Mars's anomaly will be 69° 47′ 32″, the equated anomaly 60° 15′ 27″, and the distance 158,320.

Here we shall follow a twofold path to the end of the calculation. The first is through the Ptolemaic eccentricity and equation. Then the sun's elongation from Cor Leonis is $5^s$ 27° 33′ 20″[8], differing from Mars's eccentric longitude of $1^s$ 26° 35′ 7″ by $4^s$ 0° 58′ 13″. This arc length, and the distances of the earth and Mars from the sun, show an apparent elongation from the sun of 82° 43′ 46″. Thus the apparent elongation of Mars from Cor Leonis is $3^s$ 4° 49′ 34″[9].

And the second path is through the Brahean eccentricity and equations, if they are assumed to have been the same then as well. The sun's apparent position will be 17′ 36″ farther back, or $5^s$ 27° 15′ 44″[10]. Thus the angle of commutation will be $4^s$ 0° 40′ 37″. Through this, together with our value for the sun's distance from earth, as if it too were the same then, Mars's apparent elongation from Cor Leonis is shown as $3^s$ 4° 51′ 28″[11]. The difference between the two calculations is very small and of no significance. Is it then true that

> Mars was observed as if placed upon or fitted to
> the northern star in the brow of Scorpius[12],

as the description of the observation says? Let us see. For Ptolemy, Cor Leonis is at 2° 30′ Leo, and the Northern Bright Star in the Brow of Scorpius is at 6° 20′ Scorpio, at an elongation of $3^s$ 3° 50′ 0″. For Brahe, Cor Leonis is at 24° 17′ Leo. The brow of Scorpius is at 27° 36′ Scorpio. The elongation is $3^s$ 3° 20′ 0″[13]. But Mars's elongation was just computed to be $3^s$ 4° 51′ 28″. The difference is a degree and a half.

That Ptolemy, although he pretended to have tested it with another observation, here seems to have demonstrated an erroneous ratio of the orbs using an erroneous observation.

Since he had confidence in this observation, it being the most ancient of those upon which he could have depended, Ptolemy doubtless established that ratio of the orbs which we have hitherto discovered in his numbers, and which this observation appeared to

[8] This is again about 30′ too small, owing to the error in chapter 69, and subsequent figures should be adjusted accordingly.

[9] Recomputation using corrected data shows this to be about $3^s$ 4° 56′.

[10] Again, too small: this elongation should have been about 30′ greater.

[11] Recomputation using corrected data shows this to be about $3^s$ 4° 58′. It is therefore true, as Kepler says, that the difference between the two methods of computation is insignificant. However, the elongations are greater than Kepler's by an amount not entirely negligible.

[12] β Scorpii.

[13] This is what appears in the text, although the difference between the longitudes is obviously 1′ less.

require. For in the mean motion computed for this time, he differs from me by no more than 20 minutes. The remaining discrepancy therefore comes from the ratio of the orbs. Now his pretence of investigating this ratio using an observation three days from opposition was adopted so that he might seem to deduce various things from various phenomena. And so since the ancient observation was to be reserved for investigating the mean motions, he substituted the other one for finding the ratio of the orbs, which had already been found using the ancient one. As was just said, it is absurd to test the ratio of the orbs using an observation as close to opposition as the one by which Ptolemy pretends to have demonstrated this ratio.

336 No one should therefore wonder at our differing by a degree and a half from the observation that Ptolemy summoned from antiquity. He should rather examine Ptolemy's ratio of the orbs, so different from those proven by present day observations, and consider that in order to keep this observation, Ptolemy corrupted the ratio of his orbs.

Ptolemy did not correctly understand the words describing the observation.

As for the observation itself, of which this is the verbal description: ἑῷος ὁ τοῦ 'Αρέως 'εδόκει προστεθεικέναι τῷ βορείῳ μετώπῳ του σκορπίου (In the morning, the [star] of Mars appeared to have just come upon the northern brow of Scorpius), I believe that an error was committed by Ptolemy, who understood the first star of Scorpius, while the observer was referring to the fifth[14]. This is proven from the words themselves. For the brow of Scorpius has six bright stars. Of these, there are three prominent ones, of third magnitude, or better, second. The remaining three are of fourth, or, by my estimate, third magnitude, and one is higher than the three bright ones, and farther north. Now if the observer called the 'Bright Star in the Brow' (which Brahe correctly pronounced to be of second magnitude, and which Ptolemy understood to be the intended star) the 'Northern Brow', did he not speak ambiguously in saying simply 'northern' rather than 'brightest of the northern', since the star was not the northernmost? Thus I, much more prudently, will take it to be the northernmost, the fifth in number, that was described by the observer.

Furthermore, my computed longitude of Mars agrees with this, and not with the Bright Star of the Brow; and this on the assumption of the hypothesis which the modern Brahean observations have generated. For Brahe places the northernmost star at 29° 3½′ Scorpio.

[14] The numbering represents the order in which the stars appear in both Ptolemy's and Tycho's tables. The fifth star is $\nu$ Scorpii.

Subtract Cor Leonis at 24° 17′ Leo. The difference will be its elongation from Cor, 94° 46½′. But our calculation puts Mars at an elongation of 94° 49½′ or 94° 51½′ from Cor Leonis[15]. The difference is 3 or 5 minutes, not greater.

*Whether Mars's latitude would have allowed it to cover up the star?*

I do not deny that I have to show the effect in the latitude, since I interpret the words, 'ἐδόκει προστεθεικέναι, as if he said, 'It appeared to approach so near that the two stars could be taken as if they were one, that they appeared to touch one another.' The Arabic, however, translates it [with a word signifying] 'to have covered up', as if the Greek had read, ἐπιπροστεϑεικέναι'. Accordingly, in the *Optics*, p. 304, I used the word, 'superimposed'[16]. The best word in German is 'drangesetzt'. From this I reasoned as follows. Whether Mars ran beneath it centrally, or grazed its northern or southern margin, it could not have been removed from the star latitudinally by any great distance. For indeed, the latitudes are less uncertain than the longitudes, since the manner of their variation is simpler and more consistent, as is proven in this book. We now know that the node retrogresses with respect to the fixed stars, by 4° 15′ during one 'year of the Dog'[17], as was proven in ch. 17. For Ptolemy, the northern limit was considered to precede Cor Leonis by 3½ degrees. For us, over the intervening 410 years, it had retrogressed one degree, so that at the time of observation it would be 2½° before Cor Leonis. Therefore, the node is 87½° past Cor Leonis. But Mars is 56° 35′ past Cor Leonis. Therefore, it is 31° from the node, making the
337 inclination 57½′, which, by the parallax of the orb, results in a latitude of 1° 7′[18]. But now it is clear from Brahe that the latitude of the Bright Star of the Brow is 1° 5′, while that of the Northernmost Star of the Brow is 1° 42′. So the latitude appears to refute me concerning the Bright Star of the Brow, leading me to believe that this star was occulted by Mars, and not the other.

[15] The recomputed elongations were 94° 56′ and 94° 58′. The difference is thus somewhat greater than Kepler claims, although his argument retains its plausibility.

[16] In the passage cited, Kepler used 'placed beneath' (*suppositum*) rather than 'superimposed' (*superpositum*).

[17] Since the Egyptians reckoned their year as 365 days, the seasons are displaced in their time reckoning. They found that after 1461 such years, that is, after 1460 tropical years, the first appearance of Sirius, which was most important to them, fell once again at the same season. Hence, this length of time was called an '*annus cynicus*' (year of the dog). Exactly one such dog-star period lies between the times of writing of the *Syntaxis* and of the *New Astronomy*, a coincidence that proved useful to Kepler in the previous chapter (see chapter 69 footnote 41).

[18] That is, Mars's inclination from the ecliptic has been optically lengthened by the earth's proximity to Mars. The method for computing this 'parallax of the orb' is to be found in chapter 62.

But this collusion of numbers is fortuitous. For in the latitude of the Northernmost Star of the Brow Brahe and Ptolemy are in agreement, the former pronouncing it to be 1° 46′, and the latter, 1° 42′. In the latitude of the Bright one they differ. Ptolemy has 1° 20′; Brahe, 1° 5′. But the former numerical equality results from an error, and the latter difference is really more like an agreement. For the latitudes of the northern stars in Scorpius, Sagittarius, Capricorn, and Aquarius are smaller today than they once were by about 16′ 20″, and those of the southern stars are greater by the same amount, since the ecliptic has been transposed and the declinations of the degrees of the ecliptic have been altered by the same amount, as Brahe proved and as we have said in ch. 68. Thus if it is true – and it is very true – that the latitude of the Bright Star in the Brow of Scorpius is 1° 5′ today, at the time of Ptolemy and Hipparchus it was no less than 1° 20′, probably greater. And so Mars has a smaller northern latitude than either of the stars mentioned, and passed beneath both. For it is certain that even if we were off by a whole degree in [the position of] the node, the latitude in the calculation would be off by no more than three minutes. Also, it has now been shown in ch. 64 to be entirely uncertain whether the northern latitude for Mars was also once greater in the southern signs. Therefore, my clever interpretation of the word 'προστεθεικέναι' was in vain. It can only be explained as denoting the stars' being placed side by side in the same longitude; and on this ground, the one that I favour is just as good a candidate as the Bright, its greater latitude notwithstanding.

The words of the observation have their common signification.

Consider whether the meaning could be this: that since in the northern part of the brow there are three stars in the form of a triangle, Mars was sighted in the middle of them, and was thus 'placed upon the northern brow' of Scorpius, it having simply been made one of that number of stars that are in the northern part of the brow of Scorpius.

This interpretation is furthered by the observer's having said 'northern brow' rather than 'northern star of the brow', since he is describing, not one single star, but an entire part of the constellation.

So these two ancient observations are of no use to us in estimating either the latitude or the ratio of the orbs at that time. Therefore, since there are no observations to the contrary to impede us, while the extreme likelihood of our position confirms us in it, let us conclude that the ratio of the orbs is also the same as it was once, while the maximum latitudes today are somewhat altered.